Experiment 5: Optical Pyrometer

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Abstract

In Experiment 5: Optical Pyrometer, some of the fundamental operating principles of pyrometry are explored. In addition, the thermal radiative heat transfer property of surface emissivity is thoroughly investigated through the determination of the emissivity of a heated object. The measured emissivity of the object does not fully agree with expected results. As a result, the possibility that either the body is not a black body or that the optical pyrometer is not adequately calibrated is explored.

Objective

The goal of this experiment is to investigate some of the fundamentals of radiation heat transfer using an optical pyrometer.

Background and experimental method

Theoretical background

(Please note that in general this is not required, but it was felt that the laboratory manual was deficient and therefore this section is necessary to review some of the fundamentals to fully appreciate the analysis.)

Pyrometry is a means by which the temperature of a body is determined through the measurement of thermal radiation emitted by that body (Beckwith et al., 1993). The intensity of the emitted radiation depends upon the temperature of the body and the wavelength of the electromagnetic radiation.

If a body internally absorbs all incident radiation, then it is called a black body. As a consequence of energy conservation, a black body also emits the maximum amount of energy. Hence a black body is also a perfect emitter of thermal radiation. The hemispherical emissive power of a black body is given by Planck's law,

$$e_{\lambda_\mathrm{b}} = \frac{c_1}{\lambda^5 \left[\exp\left(\frac{c_2}{\lambda T_\mathrm{b}}\right) - 1\right]}. \tag{1}$$

When $\lambda T_\mathrm{b} < 3000\,\mu$mK, Eq. (1) can be approximated within 1% (Siegel and Howell, 1992) by Wien's formula,

$$e_{\lambda_\mathrm{b}} \approx \frac{c_1}{\lambda^5 \exp\left(\frac{c_2}{\lambda T_\mathrm{b}}\right)}. \tag{2}$$

Optical pyrometers measure the emissive power of a body at a single wavelength. This measurement is accomplished by matching the radiative intensity of an electrically-heated tungsten filament to the radiative intensity emanating from the body. When the intensities are matched, the temperature of a black body is then determined from Eq. (2).

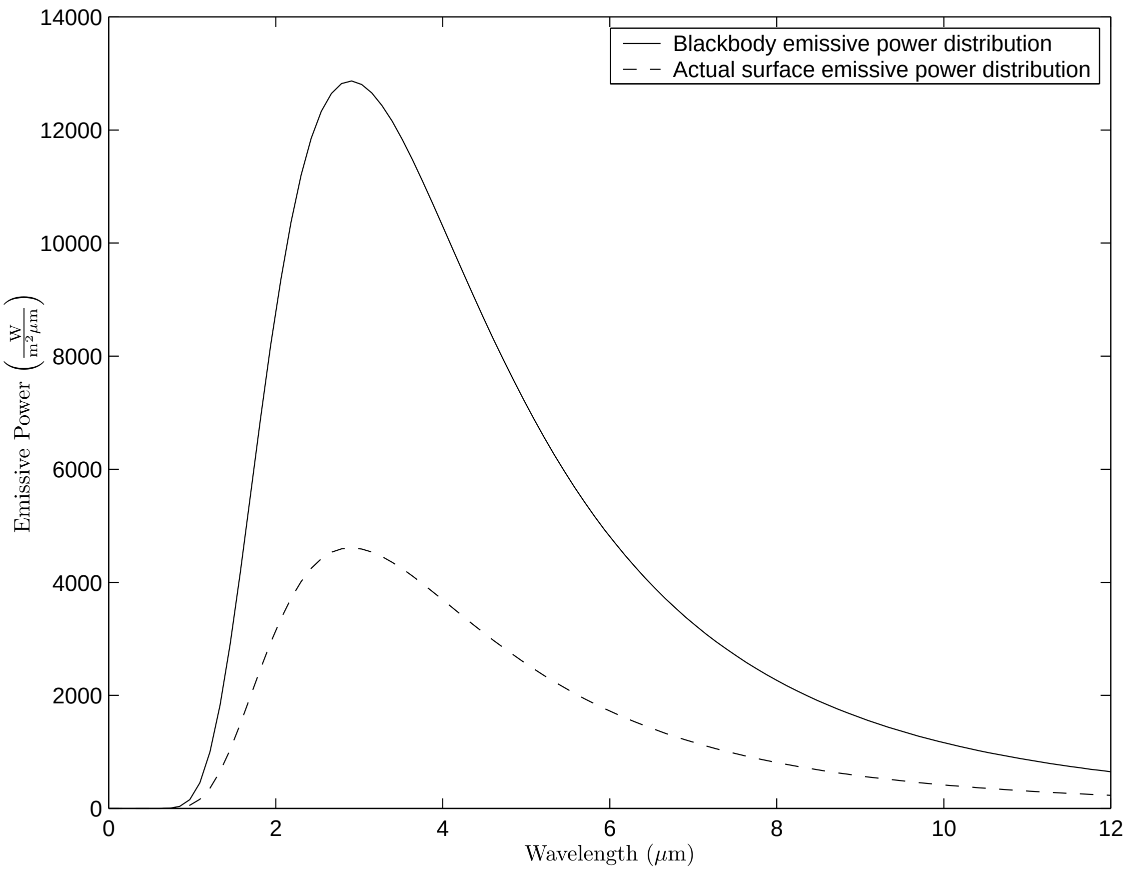

Figure 1. A comparison of emissive powers from a Black body and a "real" surface. Both bodies are assumed to be at 1000 K

Real surfaces, however, are not black bodies, i.e. perfect emitters. Consequently, the emissive power of a real surface is less than that of a black body. Figure 1 compares the emissive powers of a black body and a "real" real body as a function of wavelength. The optical pyrometer, however, only detects radiative intensity. Therefore it is not able to distinguish between a black body and a "real" surface. As a consequence, if the optical pyrometer were used to measure the temperature of a non-black surface such as the one in Figure 1, the pyrometer would measure the emissive power of the non-black body and output the temperature of a black body corresponding to the measured emissive power. From Figure 1, it is evident that the "apparent black body" temperature indicated by the pyrometer would be substantially lower than the actual temperature of the surface.

To compensate for the non-ideal behavior of real surfaces, the spectral emissivity defined as

$$\epsilon_\lambda = \frac{e_\lambda}{e_{\lambda_\mathrm{b}}}, \tag{3}$$

is used to relate the actual spectral emissive power, e_λ to the spectral emissive power of a black body at the same temperature. Unfortunately, the spectral emissive power of a real surface is not given by Planck's law.

In the case of monochromatic thermal radiation, however, the actual temperature of the real surface can be estimated by approximating e_λ with Eq. (2). Thus the actual temperature, T_{A} can be found from

$$e_\lambda \approx \frac{c_1}{\lambda^5 \exp\left(\frac{c_2}{\lambda T_{\mathrm{A}}}\right)}. \tag{4}$$

Substituting Eq. (4) and Eq. (2) into Eq. (3), yields

$$\frac{1}{T_{\mathrm{A}}} = \frac{1}{T_{\mathrm{b}}} + \frac{\lambda}{c_2} \ln\left(\epsilon_\lambda\right), \tag{5}$$

which can be used to compute the actual temperature of the body, provided that the spectral emissivity of the actual surface is known.

Experiment methodology

To investigate the basic operating principles of the optical pyrometer, a ceramic object is placed inside a laboratory furnace and its temperature is measured over a range of furnace power settings. The ends of the furnace are open, consequently the object is completely visible while it is being heated. The optical pyrometer is then aimed at the object and is used to measure the temperature of the body. Figure 2 in the laboratory manual (Liu et al., 1999) is a schematic diagram of the experiment set-up.

To determine the spectral emissivity of the body, Eq. (5) may be used if the actual temperature of the body is known. The actual temperature of the body is measured with a Platinel II thermocouple. This thermocouple is referenced to a pair of isothermal terminals located in the base of the furnace. The reference temperature of the terminals is measured with a thermometer. The thermocouple output is measured with a $6\frac{1}{2}$-digit digital multimeter.

The experiments are conducted by first heating the body to approximately 1000 °C. After the body reaches this temperature, the furnace power is reduced and the body is allowed to reach a steady state. The steady-state temperature is measured with both the optical pyrometer and the thermocouple. Following these measurements, the furnace power is reduced and the body is again allowed to reach steady state. Again, temperature measurements are obtained. This process is repeated to obtain approximately ten data points which span the temperature range of 750 °C to 1000 °C.

Results and discussion

1. Compilation of temperature measurements and the determination of the thermocouple temperature.

The data listed in Table 1 were obtained from measurements of the optical pyrometer, thermocouple, and the reference junction thermometer. An uncertainty of ±3 °C is assumed for the temperature indicated by

Table 1. Raw data obtained from the optical pyrometer, thermocouple, and thermometer.

Measurement	Pyrometer (°C)	Thermocouple emf (mV)	Reference junction (°C)
1	933 $\pm$ 3	39.224 $\pm$ 0.006	32.5 $\pm$ 0.5
2	922 $\pm$ 3	38.865 $\pm$ 0.005	33.2 $\pm$ 0.5
3	892 $\pm$ 3	37.523 $\pm$ 0.005	35.2 $\pm$ 0.5
4	862 $\pm$ 3	36.015 $\pm$ 0.005	37.8 $\pm$ 0.5
5	858 $\pm$ 3	35.833 $\pm$ 0.005	38.2 $\pm$ 0.5
6	819 $\pm$ 3	34.038 $\pm$ 0.005	39.8 $\pm$ 0.5
7	806 $\pm$ 3	33.547 $\pm$ 0.005	40.2 $\pm$ 0.5
8	781 $\pm$ 3	32.088 $\pm$ 0.005	40.2 $\pm$ 0.5
9	774 $\pm$ 3	31.737 $\pm$ 0.005	40.2 $\pm$ 0.5

the optical pyrometer. This estimate is based upon the smallest division of the optical pyrometer scale. The thermocouple emf is measured with a Keithley 2000 digital multimeter, hence the uncertainty is determined from the DC accuracy specifications for this device given by Liu et al.. A sample calculation to determine the uncertainty in the thermocouple emf is given by Eq. (12) in the Appendix. The uncertainty in the reference junction temperature is estimated to be the smallest division on the thermometer, i.e. $\pm 1\,^\circ$C.

To compare the actual temperature of the body to the temperature indicated by the optical pyrometer, it is necessary to determine the actual temperature of the body from the thermocouple and reference junction data presented in Table 1 using the law of intermediate temperatures. First, the thermocouple emf corresponding to the reference temperature is determined using the thermocouple table supplied in the laboratory. Next, the emf of the reference junction is added to the measured emf of the thermocouple. Finally, the thermocouple table is used to determine the temperature of the body from the combined emf. The results of these computations are presented in Table 2. Sample calculations which demonstrate this procedure are presented in the Appendix.

2. The determination of neutral density filter transmisivities.

The answer to this question was omitted for brevity. The analysis is identical to the determination of the surface emissivity of the body. In general, students are required to answer all bolded questions in the laboratory manual.

Table 2. Optical pyrometer and body temperatures.

Measurement	Pyrometer (K)	Thermocouple (K)
1	1206 $\pm$ 3	1240 $\pm$ 10
2	1196 $\pm$ 3	1230 $\pm$ 10
3	1165 $\pm$ 3	1200 $\pm$ 9
4	1135 $\pm$ 3	1164 $\pm$ 9
5	1131 $\pm$ 3	1160 $\pm$ 9
6	1092 $\pm$ 3	1117 $\pm$ 8
7	1079 $\pm$ 3	1106 $\pm$ 8
8	1054 $\pm$ 3	1071 $\pm$ 8
9	1047 $\pm$ 3	1063 $\pm$ 8

3. Determination of a calibration equation to relate the temperature of the test body to the pyrometer output.

In Table 2, one immediately notices that the temperatures indicated by the optical pyrometer and the thermocouple differ. There are two possible explanations: (1) the body is not a perfect emitter of thermal radiation., and (2) the pyrometer is not properly calibrated. If one desired to measure the temperature of the body with the optical pyrometer and was not concerned about the cause of the discrepancy between the pyrometer and actual body temperature, then a "calibration" equation may be developed to directly relate the readings on the optical pyrometer to the actual body temperature.

To develop the "calibration" equation, Eq. (5) (Eq. (6) in Liu et al.) is rearranged to yield,

$$T_{\mathrm{A}} = \left(\frac{T_{\mathrm{b}}}{1 + \frac{\lambda T_{\mathrm{b}}}{c_2} \ln \epsilon_\lambda} \right). \tag{6}$$

While Eq. (6) is non-linear,

$$\frac{\lambda T_{\mathrm{b}}}{c_2} \ln \epsilon_\lambda \approx 1, \tag{7}$$

for the range of temperatures encountered. Therefore Eq. (6) may be approximated by

$$T_{\mathrm{A}} = m T_{\mathrm{b}} + b. \tag{8}$$

The data listed in Table 2 is plotted in Figure 2. The fitted line and confidence intervals are obtained from linear regression. The confidence intervals are determined as $\pm t_{\alpha,\nu} S_{\mathrm{ee}}$. The "calibration" equation for the body of interest is then

$$T_{\mathrm{A}} = 1.11 T_{\mathrm{b}} - 100 \pm 6\,\mathrm{K}. \tag{9}$$

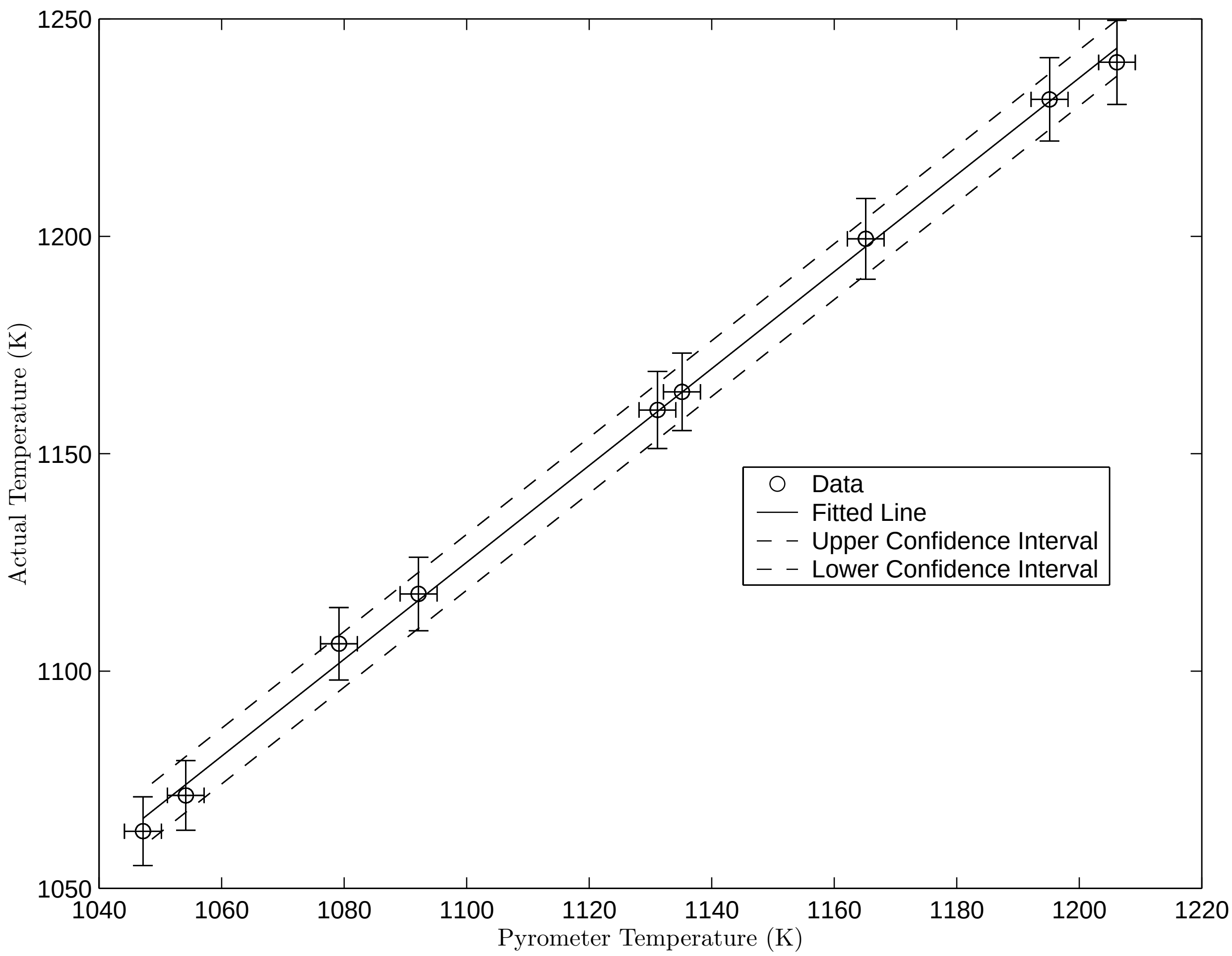

Figure 2. Determination of the calibration equation for the body.

4. Discussion of the emissivity of the body.

(a) Determination of the body surface emissivity.

If it is desired to use the optical pyrometer to measure the temperature of an object which is only slightly different from the one used in this experiment, then Eq. (9) is of little use. Further, if a different sensor were used to measure the temperature of the test body in this experiment, Eq. (9) would be useless. Therefore, a more useful approach would be to use Eq. (5) to determine the emissivity of the body. Hence Eq. (5) could be used to compute the actual temperature of the body regardless of the sensor used.

To find the emissivity of the test body, one notes that Eq. (5) is linear in $\frac{1}{T_\mathrm{A}}$ and $\frac{1}{T_\mathrm{b}}$. Hence, $\frac{1}{T_\mathrm{A}}$ versus $\frac{1}{T_\mathrm{b}}$ is plotted in Figure 3. The uncertainties in $\frac{1}{T_\mathrm{b}}$ and $\frac{1}{T_\mathrm{A}}$ are computed with Eq. (22) and Eq. (23) in the Appendix. From a linear regression analysis,

$$\frac{1}{T_\mathrm{A}} = \frac{1.07}{T_\mathrm{b}} - 7.95 \times 10^{-5}\,\mathrm{K}^{-1}. \tag{10}$$

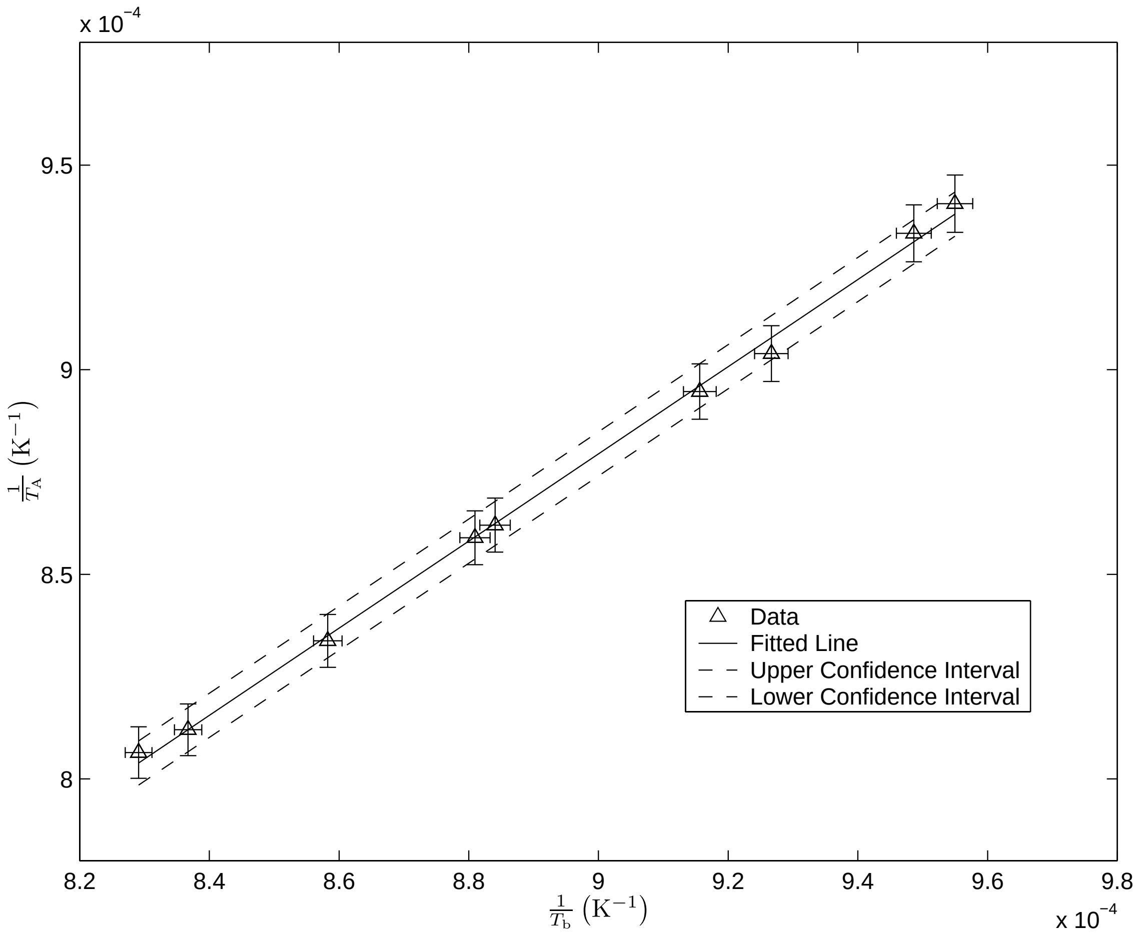

Figure 3. Determination of the emissivity of the body.

By comparison to Eq. (5),

$$\frac{c_2}{\lambda} \ln \epsilon_\lambda = -7.95 \times 10^{-5}\,\mathrm{K}^{-1}. \tag{11}$$

Using $c_2 = 14384\,\mu\mathrm{mK}$ and $\lambda = 0.66\,\mu\mathrm{m}$ yields $\epsilon_\lambda = 0.2 \pm 0.1$.

(b) Explanation of the body and apparent temperature discrepancy.

There exist two possible explanations for the discrepancy between the actual body temperature and the temperature registered by the pyrometer: The object may not be a black body, or the pyrometer may not be correctly calibrated. In addition, it is also possible that the object is a non-black body and the pyrometer is not correctly calibrated.

First, if the object is not a black body, then it will have an emissivity $0 < \epsilon_\lambda < 1$. Using Eq. (5), the

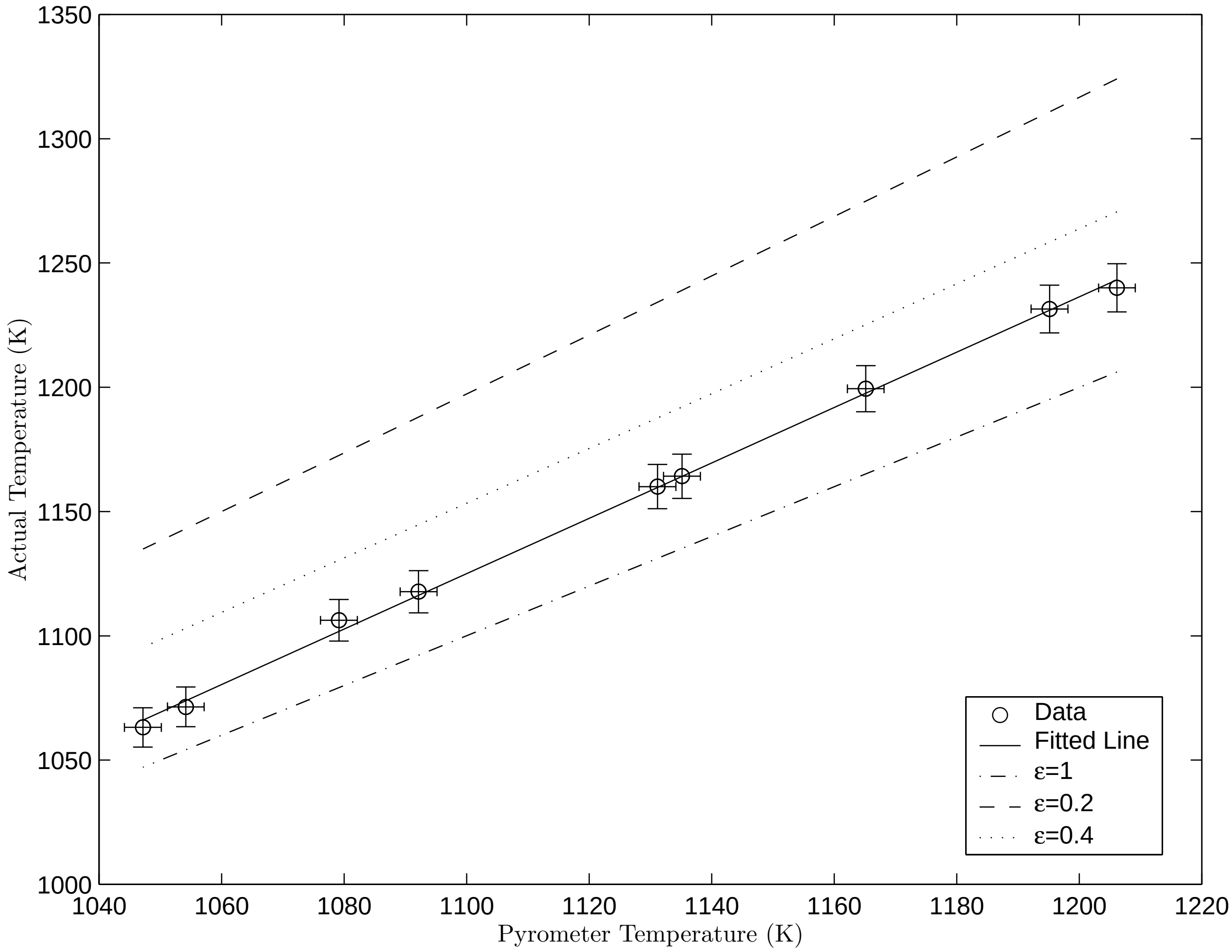

Figure 4. Comparison of the actual data and possible emissivities of the body.

range of possibilities for the body are plotted in Figure 4. From Figure 4, it is clear that it is physically possible for the body to not be a black body. In Figure 4, the data obtained for this experiment fall below the line corresponding to $\epsilon_\lambda = 0.4$, indicating that the actual emissivity is much higher. From the data analysis of this experiment, the emissivity was found to be $\epsilon_\lambda = 0.2 \pm 0.1$, consequently, the temperature predicted by tempeq and the actual body temperatures do not strictly agree. Upon consideration of the 80% relative uncertainty in ϵ_λ, however, it is possible that the agreement is much better than indicated in Figure 4.

Second, if the pyrometer were not correctly calibrated, then physically unrealistic values of $\epsilon_\lambda > 1$ would probably be observed. Because the results in Figure 4 are physically possible, one cannot conclude that the pyrometer is not calibrated correctly.

Nonetheless, the fact that perfect agreement is not obtained between the experimental data and Eq. (5) when $\epsilon_\lambda = 0.2$ does not support the conclusion that the pyrometer is calibrated correctly. Moreover, one has also to consider *how* the pyrometer calibration would be disturbed. The temperature scale on the pyrometer cannot be changed; the monochromatic filter also cannot change. The only possibility left is that the tungsten filament has eroded. This could cause the relationship between the applied current and the emissive power to change slightly, thereby giving causing the pyrometer to indicate a slightly inaccurate temperature.

Conclusions and recommendations

In this experiment, measurements of the temperature of a body heated in a furnace are compared to measurements obtained from an optical pyrometer. These measurements are then used to develop a calibration equation for the optical pyrometer and the body which will yield the actual temperatures of the body directly. Additionally, an attempt was made to determine the emissivity of the body from the experimental data. From this analysis, the emissivity of the body was found to be $\epsilon_\lambda = 0.2 \pm 0.1$. When the experimental results are compared to those predicted by an equation typically used in pyrometry, it is found that $\epsilon_\lambda > 0.4$. Consequently, it is possible that either the body is not black, or that the pyrometer is not correctly calibrated. Neither of these possibilities have been conclusively identified as the cause of the discrepancy.

Obviously, the only way to resolve this controversy is to obtain a carefully designed black body and repeat this experiment. The need for a better black body is apparent because the uncertainty in the temperature of the body used in this experiment is much larger than the pyrometer resolution. Consequently, the body in this experiment is not suitable as a calibration standard. Therefore the any further experiments with this body will yield results which will be inconclusive.

References

Beckwith, T. G., Marangoni, R. D., and Lienhard, J. H., 1993. *Mechanical Measurements.* Addison-Wesley, Reading, MA, 5th edn.

Liu, B. Y. H., Wilson, J. C., Pui, D. Y. H., and Davidson, J., 1999. *Basic Mechanical Measurements Laboratory M. E. 3031.* Mechanical Engineering Department, The University of Minnestota, Minneapolis, MN 55455.

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Nomenclature

λ Wavelength of emitted radiation, (μm), Eq. (1).

T_b Black body temperature, (K), Eq. (1).

emf Thermocouple emf, (mV), Eq. (14).

b Intercept of a fitted line, Eq. (8).

c_1 Constant, $c_1 = 374.18\,\frac{\mathrm{MW}\mu\mathrm{m}^4}{\mathrm{m}^2}$, Eq. (1).

c_2 Constant, $c_2 = 14388\,\mu\mathrm{mK}$, Eq. (1).

m Slope of a fitted line, Eq. (8).

S_{ee} Standard error of estimate, Eq. (8).

T Temperature, (°C), Eq. (14).

$t_{\alpha,\nu}$ Student-t variable, Eq. (8).

e_λ Spectral hemispherical emissive power, $\left(\frac{\mathrm{W}}{\mathrm{m}^2\mu\mathrm{m}}\right)$, Eq. (3).

e_{λ_b} Black body spectral hemispherical emissive power, $\left(\frac{\mathrm{W}}{\mathrm{m}^2\mu\mathrm{m}}\right)$, Eq. (1).

emf_1 Tabulated thermocouple emf, (mV), Eq. (13).

emf_2 Tabulated thermocouple emf, (mV), Eq. (13).

$\mathrm{emf}_{\mathrm{M}}$ Measured thermocouple emf, (mV), Eq. (16).

$\mathrm{emf}_{\mathrm{REF}}$ The emf of a virtual thermocouple at the reference temperature, (mV), Eq. (13).

$\mathrm{emf}_{\mathrm{T}}$ Combined thermocouple and reference junction emf, (mV), Eq. (16).

ϵ_λ Spectral emissivity, dimensionless, Eq. (3).

T_{A} Actual surface temperature, (K), Eq. (4).

T_1 Tabulated thermocouple temperature (°C), Eq. (13).

T_2 Tabulated thermocouple temperature, (°C), Eq. (13).

$U_{\mathrm{emf}_{\mathrm{TC}}}$ Uncertainty in the thermocouple emf, (mV), Eq. (12).

Appendix

Sample calculations

Thermocouple emf uncertainty

From Liu et al. (1999), the DC accuracy specifications for the Keithley 2000 digital multimeter are: $\pm\,(50\text{ppm RDG} + 35\text{ppm RANGE})$, where ppm represents parts per million $\left(10^{-6}\right)$. To illustrate, 39.2241 mV was measured on the Keithley 2000. Hence the uncertainty in this measurement is determined as:

$$U_{\mathrm{emf_{TC}}} = \pm 50 \times 10^{-6}\,(39.2241\,\mathrm{mV}) + 35 \times 10^{-6}\,(100.0000\,\mathrm{mV}) = \pm 0.005\,\mathrm{mV}. \tag{12}$$

Reference temperature emf and uncertainty

In order to use the law of intermediate temperatures, the thermocouple emf corresponding to the reference junction temperature must be determined. This is accomplished by using linear interpolation in the thermocouple table. To illustrate, the reference temperature was measured to be $32.5 \pm 0.5\,^{\circ}\mathrm{C}$. Using the values in Table 3, the reference junction emf is computed as

Table 3. An excerpt from the Platinel II thermocouple table used to determine the reference junction emf.

Temperature (°C)	Thermocouple emf (mV)
32	0.9888
33	1.0208

$$\begin{aligned}\mathrm{emf_{REF}} &= \left(\frac{\mathrm{emf}_2 - \mathrm{emf}_1}{T_2 - T_1}\right)(T_{\mathrm{REF}} - T_1) + \mathrm{emf}_2 \\ &= \left(\frac{1.0208 - 0.9888}{33 - 32}\right)(32.5 - 32) + 0.9888 = 1.0048\,\mathrm{mV}. \end{aligned} \tag{13}$$

The uncertainty in the reference junction emf is calculated from

$$U_{\mathrm{emf_{REF}}} = \frac{d\mathrm{emf}}{dT} U_T. \tag{14}$$

The derivative in Eq. (14) is approximated from the data in Table 3. Consequently, the uncertainty in the reference junction emf is then given by

$$U_{\mathrm{emf_{REF}}} = \left(\frac{1.0208 - 0.9888}{33 - 32}\right) \times 0.5 = 0.02\,\mathrm{mV}. \tag{15}$$

Hence the emf of a thermocouple at the reference junction is $1.01 \pm 0.02\,\mathrm{mV}$.

Thermocouple temperature and uncertainty

The temperature of the thermocouple is determined as follows: First, the emf of the thermocouple and reference junctions are combined,

$$\mathrm{emf}_{\mathrm{T}} = \mathrm{emf}_{\mathrm{REF}} + \mathrm{emf}_{\mathrm{M}} = 1.0048 + 39.2241 = 40.2289\,\mathrm{mV}. \quad (16)$$

Next, linear interpolation is used to find the corresponding temperature. Using data from Table 4,

Table 4. An excerpt from the Platinel II thermocouple table used to determine the thermocouple temperature.

Temperature (°C)	Thermocouple emf (mV)
966	40.1933
967	40.2327

$$T = \left(\frac{T_2 - T_1}{\mathrm{emf}_2 - \mathrm{emf}_1}\right)(\mathrm{emf}_{\mathrm{T}} - \mathrm{emf}_1) + T_1$$
$$= \left(\frac{967 - 966}{40.2327 - 40.1933}\right)(40.2289 - 40.1933) + 966 = 966.9\,^{\circ}\mathrm{C}. \quad (17)$$

To estimate the uncertainty in the thermocouple temperature, the uncertainties in the emf of the reference junction and thermocouple are first combined to yield,

$$U_{\mathrm{emf_T}} = \left(U^2_{\mathrm{emf_M}} + U^2_{\mathrm{emf_{REF}}}\right)^{\frac{1}{2}} = \left(0.005^2 + 0.02^2\right)^{\frac{1}{2}} = \pm 0.02\,\mathrm{mV}. \quad (18)$$

Next, the uncertainty calculated in Eq. (18) is propagated through the thermocouple table to yield,

$$U_T = \frac{dT}{d\mathrm{emf}} U_{\mathrm{emf}} = \left(\frac{967 - 966}{40.2327 - 40.1933}\right) \times 0.02 = \pm 0.5\,^{\circ}\mathrm{C}. \quad (19)$$

Finally, the thermocouple calibration table is assumed to have an accuracy of ±1%. Consequently, the calibration uncertainty of the thermocouple table is

$$U_{\mathrm{CAL}} = \pm 0.01 \times 966.9 = \pm 9.7\,^{\circ}\mathrm{C} \approx \pm 10\,^{\circ}\mathrm{C}. \quad (20)$$

Therefore, the uncertainty in the temperature of the body is

$$U_{T_{\mathrm{A}}} = \left(U_T^2 + U^2_{\mathrm{CAL}}\right)^{\frac{1}{2}} = \left(0.5^2 + 10^2\right)^{\frac{1}{2}} = \pm 10\,^{\circ}\mathrm{C}. \quad (21)$$

Consequently, the temperature of the body is $T_{\mathrm{A}} = 970 \pm 10\,^{\circ}\mathrm{C}$.

Surface emissivity calculations

When plotting $\frac{1}{T_{\mathrm{A}}}$ and $\frac{1}{T_{\mathrm{b}}}$, the uncertainties in these variables are computed through error propagation. Hence the uncertainty in $\frac{1}{T_{\mathrm{b}}}$ is given by

$$U_{\frac{1}{T_{\mathrm{b}}}} = \frac{d\left(\frac{1}{T_{\mathrm{b}}}\right)}{dT_{\mathrm{b}}} U_{T_{\mathrm{b}}} = \frac{1}{T_{\mathrm{b}}^2} U_{T_{\mathrm{b}}} = \frac{3}{1206^2} = 2 \times 10^{-6}\,\mathrm{K}^{-1}. \quad (22)$$

Similarly, the uncertainty in $\frac{1}{T_\mathrm{A}}$ is computed as

$$U_{\frac{1}{T_\mathrm{A}}} = \frac{d\left(\frac{1}{T_\mathrm{A}}\right)}{dT_\mathrm{A}} U_{T_\mathrm{A}} = \frac{1}{T_\mathrm{A}^2} U_{T_\mathrm{A}} = \frac{10}{1240^2} = 6 \times 10^{-6}\,\mathrm{K}^{-1}. \tag{23}$$

Additionally, because the uncertainties are assumed symmetric, only the absolute values are taken.

The emissivity of the body is found from

$$b = \frac{c_2}{\lambda} \ln \epsilon_\lambda, \tag{24}$$

where b is the intercept of a fitted line obtained through regression analysis. The emissivity is then directly computed from

$$\epsilon_\lambda = \exp\left(\frac{c_2 b}{\lambda}\right). \tag{25}$$

To compute the uncertainty in ϵ_λ, error propagation through Eq. (25) is used. Hence,

$$U_{\epsilon_\lambda} = \frac{d\epsilon_\lambda}{db} U_b = \exp\left(\frac{c_2 b}{\lambda}\right) \frac{c_2}{\lambda} U_b = \frac{c_2}{\lambda} \epsilon_\lambda U_b = 0.2\left(\frac{14384}{0.66}\right) \times 3.67085E - 05 = \pm 0.1, \tag{26}$$

where the uncertainty in the intercept is

$$\left(\frac{\text{Upper } 95\% - \text{Lower } 95\%}{2}\right) = 3.67085E - 05. \tag{27}$$

Data for the optical pyrometer experiment

Setting	Ref. Junction Temp	UT	ref emf calculation emf2	emf1	T2	T1	slope	emf ref	Uemf-ref
5	32.5	0.5	1.0208	0.9888	33	32	0.032	1.0048	0.016
5	33.2	0.5	1.0529	1.0208	34	33	0.0321	1.02722	0.016
4.5	35.2	0.5	1.1172	1.085	36	35	0.0322	1.09144	0.016
4	37.8	0.5	1.1817	1.1494	38	37	0.0323	1.17524	0.016
4	38.2	0.5	1.2141	1.1817	39	38	0.0324	1.18818	0.016
3.5	39.8	0.5	1.2465	1.2141	40	39	0.0324	1.24002	0.016
3.5	40.2	0.5	1.279	1.2465	41	40	0.0325	1.253	0.016
3	40.2	0.5	1.279	1.2465	41	40	0.0325	1.253	0.016
3	40.2	0.5	1.279	1.2465	41	40	0.0325	1.253	0.016

Setting	Hot junction emf	Uncertainty in emf
5	39.2241	0.0055
5	38.8651	0.0054
4.5	37.5225	0.0054
4	36.0149	0.0053
4	35.8326	0.0053
3.5	34.0376	0.0052
3.5	33.5471	0.0052
3	32.0876	0.0051
3	31.7374	0.0051

Setting	Combined emf	uncertainty	Temperature calculation T2	T1	emf2	emf1	slope	Temp (C)	Utemp	Calibration Utotal
5	40.2289	0.017	967	966	40.2327	40.1933	25.38071	966.9036	0.429	9.678552
5	39.89232	0.017	959	958	39.9172	39.8777	25.31646	958.3701	0.429	9.593301
4.5	38.61394	0.017	927	926	38.6433	38.6032	24.93766	926.2678	0.423	9.272345
4	37.19014	0.017	892	891	37.2286	37.1879	24.57002	891.055	0.418	8.920332
4	37.02078	0.017	887	886	37.0247	36.9839	24.5098	886.9039	0.418	8.87887
3.5	35.27762	0.017	845	844	35.2949	35.2533	24.03846	844.5846	0.409	8.455744
3.5	34.8001	0.017	834	833	34.8369	34.7951	23.92344	833.1196	0.408	8.341181
3	33.3406	0.017	799	798	33.3721	33.3297	23.58491	798.2571	0.402	7.992672
3	32.9904	0.017	790	789	32.9901	32.9476	23.52941	790.0071	0.401	7.910223

Figure 5. Excel worksheet used for data analysis.

Pyrometer Data

Setting	Pyrometer Temp. (C)	Uncertainty
5	933	3
5	922	3
4.5	892	3
4	862	3
4	858	3
3.5	819	3
3.5	806	3
3	781	3
3	774	3

Data for analysis

Setting	Actual T (K)	Utact (K)	Pyrometer	Upy (K)
5	1240.053553	9.678552106	1206.15	3
5	1231.520127	9.593300856	1195.15	3
4.5	1199.41783	9.272345056	1165.15	3
4	1164.205037	8.920332118	1135.15	3
4	1160.053922	8.878870062	1131.15	3
3.5	1117.734615	8.45574384	1092.15	3
3.5	1106.289617	8.341181104	1079.15	3
3	1071.407075	7.99267245	1054.15	3
3	1063.157059	7.910223449	1047.15	3

Pyrometer (K)	Pred	Pred+1See	Pred-1See
1206.15	1243.283289	1249.700667	1236.866
1195.15	1231.025905	1237.443282	1224.609
1165.15	1197.596674	1204.014051	1191.179
1135.15	1164.167443	1170.584821	1157.75
1131.15	1159.710213	1166.12759	1153.293
1092.15	1116.252213	1122.66959	1109.835
1079.15	1101.766213	1108.18359	1095.349
1054.15	1073.90852	1080.325898	1067.491
1047.15	1066.108366	1072.525744	1059.691

Figure 6. Excel worksheet used for data analysis.

Determination of emissivity

1/Tpy	Unc	1/Tact	Unc	Predicted	+tSee	-tSee
0.000829084	2.06214E-06	0.000806417	6.29E-06	0.0008039	0.000809	0.000798
0.000836715	2.10028E-06	0.000812005	6.33E-06	0.000812	0.000817	0.000807
0.000858259	2.20982E-06	0.000833738	6.45E-06	0.000835	0.00084	0.00083
0.000880941	2.32817E-06	0.000858955	6.58E-06	0.0008591	0.000865	0.000854
0.000884056	2.34467E-06	0.000862029	6.6E-06	0.0008625	0.000868	0.000857
0.000915625	2.51511E-06	0.000894667	6.77E-06	0.0008961	0.000901	0.000891
0.000926655	2.57607E-06	0.000903939	6.82E-06	0.0009078	0.000913	0.000902
0.000948632	2.69971E-06	0.000933352	6.96E-06	0.0009313	0.000937	0.000926
0.000954973	2.73592E-06	0.000940595	7E-06	0.000938	0.000943	0.000933

c2= 14384 λ= 0.66

Emissivity= 0.176873338 +- 0.1415028 High= 0.318376

Relative uncertainty= 80.00% Low= 0.035371

Figure 7. Excel worksheet used for data analysis.

SUMMARY REGRESSION OUTPUT TO DETERMINE THE CALIBRATION EQUATION

Regression Statistics			
Multiple R	0.999245304		
R Square	0.998491177		
Adjusted R Square	0.99827563		
Standard Error	2.713911899	t=	2.364623
Observations	9	tSee=	6.417377

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	34118.9279	34118.93	4632.377	3.88527E-11
Residual	7	51.55722458	7.365318		
Total	8	34170.48513			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-100.7389334	18.40506143	-5.473436	0.000932	-144.2599569	-57.21790989	-144.2599569	-57.21790989
X Variable 1	1.114307692	0.016372055	68.06157	3.89E-11	1.075593962	1.153021422	1.075593962	1.153021422

Slope Uncertainty=	0.03871373
Intercept Uncertainty=	43.52102349

Figure 8. Linear regression output.

SUMMARY OUTPUT FOR THE DETERMINATION OF EMISSIVITY

Regression Statistics	
Multiple R	0.999071338
R Square	0.998143538
Adjusted R Square	0.99787833
Standard Error	2.28207E-06
Observations	9

t= 2.36462256

tSee= 5.39624E-06

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regression	1	1.96003E-08	1.96003E-08	3763.614	8.03E-11
Residual	7	3.64549E-11	5.20785E-12		
Total	8	1.96368E-08			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-7.94864E-05	1.55241E-05	-5.120207566	0.001368	-0.000116	-4.27779E-05	-0.000116195	-4.27779E-05
X Variable 1	1.06548068	0.017367729	61.3483017	8.03E-11	1.024413	1.106548804	1.024412556	1.106548804

Uncertainty in b= 3.67085E-05

Uncertainty in m= 0.041068124

Figure 9. Linear regression output.