## Dr. Saeed A. Dobbah Alghamdi

Department of Statistics Faculty of Sciences Building 90, Office 26F41
King Abdulaziz University http://saalghamdy.kau.edu.sa


## Chapter 5

## Discrete Probability Distributions

## Objectives

$\square$ Constructing a probability distribution for a random variable.
$\square$ Finding the mean, variance, and expected value for a discrete random variable.
$\square$ Finding the exact probability for $x$ successes in $n$ trials of a binomial experiment.
$\square$ Finding the mean, variance, and standard deviation for the variable of a binomial distribution.

## Introduction

$\square$ Many decisions in business, insurance, and other real-life situations are made by assigning probabilities to all possible outcomes pertaining to the situation and then evaluating the results.
$\square$ This part explains the concepts and applications of probability distributions. In addition, a special probability distribution called binomial distribution is explained.

## Discrete Probability Distribution

A random variable is a variable whose values are determined by chance.
A discrete probability distribution consists of the values a random variable can assume and the corresponding probabilities of the values. The probabilities are determined theoretically or by observation.

## Discrete Probability Distribution

## 5-4 $\quad$ Example

Construct a probability distribution for rolling a single die.

Since the sample space is $S=\{1,2,3,4,5,6\}$ and each outcome has a probability $\frac{1}{6}$, the distribution will be

| Outcome X | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability P(X) | $1 / 6$ | $1 / 6$ | $1 / 6$ | $1 / 6$ | $1 / 6$ | $1 / 6$ |

## Discrete Probability Distribution

## 5-5 Example

Represent graphically the probability distribution for the sample space for tossing three coins.

| Number of heads X | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Probability P(X) | $1 / 8$ | $3 / 8$ | $3 / 8$ | $1 / 8$ |


© All rights are preserved for Dr. Saeed A. Dobbah Alghamdi, Department of Statistics, Faculty of Sciences, KAU

## Discrete Probability Distribution

During the summer months, a rental agency keeps track of the number of chain saws it rents each day during a period of 90 days. The number of saws rented per day is represented by the variable $X$. The results are shown here. Compute the probability $P(X)$ for each X and construct a probability distribution and graph for the data. \begin{tabular}{l|ccccc}
$X$ \& 0 \& 1 \& 2 \& Total <br>
\cline { 2 - 6 } \& \# of days \& 45 \& 30 \& 15 \& 90

 

$P(X=0)=\frac{45}{90}=0.5, P(X=2)=\frac{30}{90}=0.333, P(X=2)=\frac{15}{90}=0.167$ <br>
X <br>
\hline $\mathrm{P}(\mathrm{X})$
\end{tabular} O

## Requirements for a Probability Distribution

$\square$ The sum of the probabilities of all the events in the sample space must equal 1;

$$
\sum X=1
$$

- The probability of each event in the sample space must be between or equal to 0 and 1 ;

$$
0 \leq P(X) \leq 1
$$

## Requirements for a Probability Distribution

## Example

Determine whether each distribution is a probability distribution.
a.

| X | 0 | 5 | 10 | 15 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $1 / 5$ |


| X | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | $1 / 4$ | $1 / 8$ | $1 / 16$ | $9 / 16$ |

C. | X | 0 | 2 | 4 | 6 |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | -1.0 | 1.5 | 0.3 | 0.2 |

d.

| X | 2 | 3 | 7 |
| :--- | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | 0.5 | 0.3 | 0.4 |

a. Yes, it is a probability distribution.
b. No, it is not a probability distribution, since $\mathrm{P}(\mathrm{X})$ cannot be 1.5
or -1.0.
c. Yes, it is a probability distribution.
d. No, it is not, since

## Mean of a Probability Distribution

In order to find the mean for a probability distribution, one must multiply each possible outcome by its corresponding probability and find the sum of the products.

$$
\mu=X_{1} P\left(X_{1}\right)+X_{2} P\left(X_{2}\right)+\cdots+X_{n} P\left(X_{n}\right)=\sum[X \times P(x)]
$$

## Mean of a Probability Distribution

## 5-10 Example

In a family with two children, find the mean of the number of children who will be girls.
The probability distribution is

| \# of girls X | 0 | 1 | 2 |
| :--- | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | $1 / 4$ | $1 / 2$ | $1 / 4$ |

Hence,

$$
\mu=\sum[X \times P(x)]=0 \times \frac{1}{4}+1 \times \frac{1}{2}+2 \times \frac{1}{4}=1
$$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## Mean of a Probability Distribution

## 5-14 Example

If three coins are tossed, find the mean of the number of heads that occur.

The probability distribution is

| \# of heads X | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | $1 / 8$ | $3 / 8$ | $3 / 8$ | $1 / 8$ |
| $\mathrm{X} \mathrm{P}(\mathrm{X})$ | 0 | $3 / 8$ | $6 / 8$ | $3 / 8$ |

Hence,

$$
\mu=\sum[X \times P(x)]=0+\frac{3}{8}+\frac{6}{8}+\frac{3}{8}=1.5
$$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## Mean of a Probability Distribution

## 5-18 Example

The probability distribution shown represents the number of trips of five nights or more that American adults take per year. Find the mean.

| \# of trips X | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | 0.06 | 0.70 | 0.20 | 0.03 | 0.01 |
| $\mathrm{X} \mathrm{P}(\mathrm{X})$ | 0 | 0.70 | 0.40 | 0.09 | 0.04 |

Hence,

$$
\mu=\sum[X \times P(x)]=0+0.7+0.4+0.09+0.04=1.23
$$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## Variance of a Probability Distribution

$\square$ The variance of a probability distribution is found by multiplying the square of each outcome by its corresponding probability, summing those products, and subtracting the square of the mean.
$\square$ The formula for calculating the variance is:

$$
\sigma^{2}=\sum\left[X^{2} \times P(X)\right]-\mu^{2}
$$

$\square$ The formula for the standard deviation is:

$$
\sigma=\sqrt{\sigma^{2}}
$$

## Variance of a Probability Distribution

## 5-23 Example

Find the mean, variance and standard deviation for the probability distribution for the number of spots that appear when a die is tossed is

| Outcome X | 1 | 2 | 3 | 4 | 5 | 6 | $\Sigma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability $\mathrm{P}(\mathrm{X})$ | $1 / 6$ | $1 / 6$ | $1 / 6$ | $1 / 6$ | $1 / 6$ | $1 / 6$ | 1 |
| $X \times P(X)$ | $1 / 6$ | $2 / 6$ | $3 / 6$ | $4 / 6$ | $5 / 6$ | $6 / 6$ | $21 / 6$ |
| $X^{2} \times P(X)$ | $1 / 6$ | $4 / 6$ | $9 / 6$ | $16 / 6$ | $25 / 6$ | $36 / 6$ | $91 / 6$ |

Hence,

$$
\begin{gathered}
\mu=\sum[X \times P(x)]=21 / 6 \\
\sigma^{2}=\sum\left[X^{2} \times P(X)\right]-\mu^{2}=\frac{91}{6}-\left(\frac{21}{6}\right)^{2}=2.917 \\
\sigma=\sqrt{\sigma^{2}}=\sqrt{2.917}=1.708
\end{gathered}
$$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## Application Using Excel



## Application Using Excel



## Variance of a Probability Distribution

## 5-29 Example

Five balls numbered $0,2,4,6$ and 8 are placed in a bag. After the balls are mixed, one is selected, its number is noted and then it is replaced. If this experiment is repeated many times, find the variance and standard deviation of the numbers on the balls.

| \# on ball X | 0 | 2 | 4 | 6 | 8 | $\Sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(X)$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | $1 / 5$ | 1 |
| $X \times P(X)$ | 0 | $2 / 5$ | $4 / 5$ | $6 / 5$ | $8 / 5$ | $20 / 5$ |
| $X^{2} \times P(X)$ | 0 | $4 / 5$ | $16 / 5$ | $36 / 5$ | $64 / 5$ | $120 / 5$ |

Hence,

$$
\begin{gathered}
\mu=\sum[X \times P(x)]=\frac{20}{5}=4 \\
\sigma^{2}=\sum\left[X^{2} \times P(X)\right]-\mu^{2}=\frac{120}{5}-4^{2}=8 \\
\sigma=\sqrt{\sigma^{2}}=\sqrt{8}=2.828
\end{gathered}
$$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## Application Using Excel



## Application Using Excel

## 5-34 Example "Variance and Standard deviation"



## Expected Value

- Expected value or expectation is used in various types of games of chance, in insurance, and in other areas, such as decision theory.
$\square$ The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable. The formula is:

$$
E(X)=\mu=\sum[X \times P(x)]
$$

$\square$ The symbol $E(X)$ is used for the expected value.

## Expected Value

## 5-36 Example

One thousand tickets are sold at $\$ 1$ each for a color television valued at $\$ 350$. What is the expected value of the gain if a person purchases one ticket?

|  | Win | Lose |  |
| :---: | :---: | :---: | :---: |
| X | $\$ 349$ | $-\$ 1$ | $\Sigma$ |
| $P(X)$ | $1 / 1000$ | $999 / 1000$ | 1 |
| $\mathrm{X} \times P(X)$ | $349 / 1000$ | $-999 / 1000$ | $-650 / 1000$ |

Hence, $E(X)=\sum[X \times P(x)]=-\frac{650}{1000}=-0.65$
The meaning of this value is that if a person purchased one ticket each week over a long time, the average loss would be $\$ 0.65$ per ticket, since theoretically, on average, that person would win the set once for each 1000 tickets purchased.

## Application Using Excel



## Application Using Excel



## Application Using Excel



## The Binomial Distribution

$\square$ Many types of probability problems have only two possible outcomes or they can be reduced to two outcomes.

- Examples:
- When a coin is tossed it can land on heads or tails,
- When a baby is born it is either a boy or girl,
- A multiple-choice question can be classified as correct or incorrect.


## The Binomial Experiment

$\square$ The binomial experiment is a probability experiment that satisfies these requirements:

1. Each trial can have only two possible outcomes; success or failure.
2. There must be a fixed number of trials.
3. The outcomes of each trial must be independent of each other.
4. The probability of a success must remain the same for each trial.

## The Binomial Distribution

$\square$ The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a binomial distribution which is the probability of exactly $x$ successes in $n$ trials

$$
P(x)={ }_{n} C_{x} \times p^{x} \times q^{n-x}
$$

where
$p$ the symbol for the numerical probability of success
$q$ the symbol for the numerical probability of failure

$$
p+q=1
$$

$n$ the number of trials
$x$ the number of successes; $x=0,1,2, \ldots, n$

## The Binomial Distribution

## 5-43 Example

A coin is tossed 3 times. Find the probability of getting exactly two heads.

- This can solved using the sample space

ННН,HНТ,НТН,THH,НTT,THT,TTH,TTT
There are three ways of getting 2 heads.
So, $P($ getting 2 heads $)=\frac{3}{8}=0.375$

- Using the binomial distribution as following
- we have fixed number of trials (three), so $n=3$
- there are two outcomes for each trial, H or T
- the outcomes are independent of one another
- the probability of success (heads) is $1 / 2$, so $p=1 / 2$
here $\mathrm{X}=2$ since we need to find the probability of getting 2 heads.
Thus, $P($ getting 2 heads $)={ }_{3} C_{2} \times 0.5^{2} \times 0.5^{3-2}=\frac{3}{8}=0.375$
© All rights are preserved for Dr. Saeed A. Dobbah Alghamdi, Department of Statistics, Faculty of Sciences, KAU


## Application Using Excel



## Application Using Excel



## Application Using Excel



## The Binomial Distribution

## 5-47 Example

A survey found that one out of five Americans says he or she has visited a doctor in any given month. If 10 people are selected at random, find the probability that exactly 3 would have visited a doctor last month.

In this case $n=10, X=3, p=1 / 5$. So $\mathrm{q}=4 / 5$ and

$$
P(3)={ }_{10} C_{3} \times\left(\frac{1}{5}\right)^{3} \times\left(\frac{4}{5}\right)^{10-3}=0.201
$$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## The Binomial Distribution

## 5-51 Example

A survey from Teenage Research Unlimited found that 30\% of teenage consumers receive their spending money from part-time jobs. If 7 teenagers are selected at random, find the probability that exactly 3 of them will have part-time jobs. the probability that less than 3 of them will have part-time jobs. the probability that at most 3 of them will have part-time jobs. the probability that 5 or 6 of them will have part-time jobs. the probability that greater than 3 of them will have part-time jobs. the probability that at least 3 of them will have part-time jobs.

## Application Using Excel



## Application Using Excel



## Application Using Excel



## The Binomial Distribution

$\square$ The mean, variance, and standard deviation of a variable that has the binomial distribution can be found by using the following formulas.

$$
\begin{aligned}
\text { mean } & \mu=n \times p \\
\text { variance } & \sigma^{2}=n \times p \times q \\
\text { standard deviation } & \sigma=\sqrt{\sigma^{2}}
\end{aligned}
$$

## The Binomial Distribution

## 5-56 Example

A survey from Teenage Research Unlimited found that 30\% of teenage consumers receive their spending money from part-time jobs. If 7 teenagers are selected at random, find the mean , variance and standard deviation of the number of teenagers who will have part time jobs.
mean
variance
standard deviation $\sigma=\sqrt{\sigma^{2}}=\sqrt{1.47}=1.212$

## Application Using Excel



## Application Using Excel



## Application Using Excel



## Summary

$\square$ A discrete probability distribution can be graphed using bar chart.
$\square$ The mean, variance, and standard deviation can be found for a probability distribution.
$\square$ The mathematical expectation can also be calculated for a probability distribution.
$\square$ The binomial distribution is used when there are only two outcomes for an experiment, a fixed number of trials, the probability is the same for each trial, and the outcomes are independent of each other.

