



## (5.5) The Substitution rule

Home work 7–47 (odd),  
53–73 (odd), 44, 48

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Students

# مقدمة

في الأبواب القادمة نقدم طرق لحساب التكامل تساعد في الوصول بالتكامل إلى صورة يمكن معها استخدام القوانين الأساسية:

التكامل بالتحويض The Substitution Rule

التكامل بالتجزئي Integration by parts

التحويض بالدوال المثلثية Trigonometric Integrals

التكامل بالدروال الجزئية Integration of rational function by partial fractions

# The Substitution Rule

Because of the Fundamental Theorem, it's important to be able to find antiderivatives. But our **antidifferentiation formulas** don't tell us how to evaluate integrals such as

$$\int 2x\sqrt{1+x^2}dx$$

$$\int e^{5x}dx$$

$$\int x^3 \cos(x^4 + 1) dx$$

$$\int \sqrt{1+x^2} x^5 dx$$

$$\int \sqrt{2x+1}dx$$

$$\int \tan x dx$$



note

لكل طريقة حساب المثلث يقابلها صيغة في المثلث.

طريقة التغيرات مماثلة صيغة  
The Chain Rule

$$If u = f(x) \Rightarrow \frac{du}{dx} = f'(x) \Rightarrow du = f'(x)dx$$

To solve these kind of integrals:

$$\int 2x\sqrt{1+x^2}dx$$

In general, this method works whenever we have an integral that we can write in the form

$$\int f(g(x))g'(x)dx$$

Observe that  $F' = f$ , then

$$(3) \int F'(g(x))g'(x)dx = F(g(x)) + C$$

Because by the chain rule:

$$\frac{d}{dx} [F(g(x)) + C] = F'(g(x)) \cdot g'(x) + 0$$

#### (4) The Substitution Rule

If  $u = g(x)$  is a differentiable function whose range is an interval  $I$  and  $f$  is continuous on  $I$ , then

$$\int f(g(x))g'(x)dx = \int f(u) du = F(u) + C$$

Example (1): Find  $\int x^3 \cos(x^4 + 1) dx$

Example (2):

$$\int \sqrt{2x + 1} dx$$

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Example (3):

$$\int \frac{x}{\sqrt{1 - 4x^2}} dx$$

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Example (4):

$$\int e^{5x} dx$$

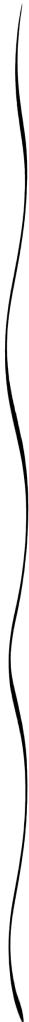
Check the answers by differentiating it.

Example (5):

$$\int \sqrt{1+x^2} x^5 dx$$

Example (6):

$$\int \tan x dx$$



bis

(5)

$$\int \tan x dx =$$

# Definite Integrals

**One method** is to evaluate the indefinite integral first and then use the Fundamental Theorem.

For instance, using the result of Example 2, we have

$$\int_0^4 \sqrt{2x+1} dx = \int \sqrt{2x+1} dx \Big|_0^4$$

$$= \frac{1}{3}(2x+1)^{3/2} \Big|_0^4$$

$$= \frac{1}{3}(9)^{3/2} + \frac{1}{3}(1)^{3/2} = \frac{26}{3}$$

**Another method**, is to change the limits of integration when the variable is changed.

## (6) The Substitution Rule for Definite Integrals

If  $g'$  is continuous on  $[a, b]$  and  $f$  is continuous on the range of  $u = g(x)$ , then

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

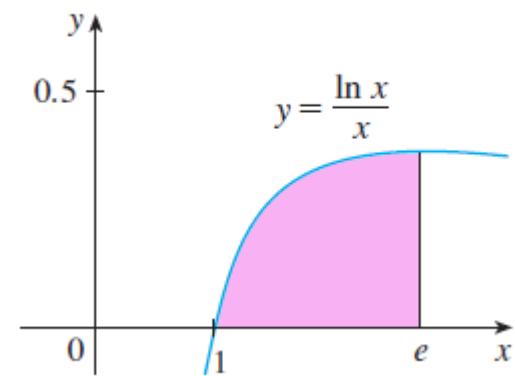
Evaluate  $\int_0^4 \sqrt{2x+1} dx$  using (6)



Observe that when using (6) we do not return to the variable  $x$  after integrating. We simply evaluate the expression in  $u$  between the appropriate values of  $u$ .

Example (8):  $\int_1^e \frac{dx}{(3 - 5x)^2}$

Example (9):  $\int_1^e \frac{\ln x}{x} dx$



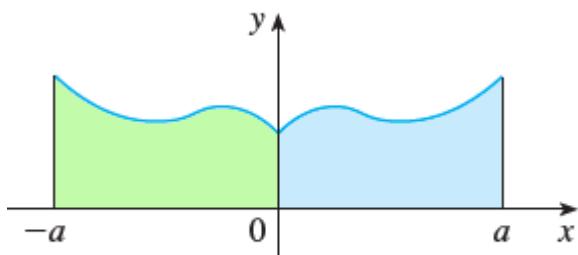
# Symmetry

## Integrals of Symmetric Functions

Suppose  $f$  is continuous on  $[-a, a]$

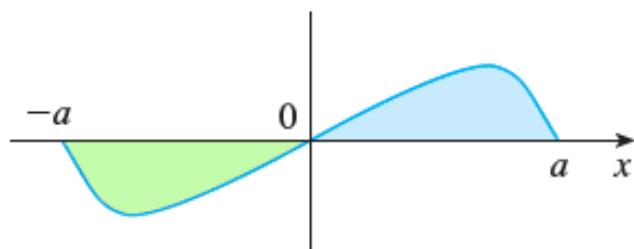
1 If  $f$  is even ( $f(-x) = f(x)$ ) then

$$\int_{-a}^a f(x)dx =$$



2 If  $f$  is odd ( $f(-x) = -f(x)$ ) then

$$\int_{-a}^a f(x)dx =$$



Example (I):

$$\text{Evaluate } \int_{-2}^2 (x^4 + 1) dx$$

Example (II):

$$\text{Evaluate } \int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx$$