(5.3) The Fundamental Theorem of Calculus

Home work 7-43 (odd),
59 – 63 (odd)

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students

Introduction

The Fundamental Theorem of Calculus is appropriately named because it establishes a Connection between the two branches of calculus:

Differential Calculus

Differential calculus arose from the tangent problem

Integral Calculus

Integral calculus arose the area problem.

Newton's mentor at Cambridge, Isaac Barrow (1630 –1677), discovered that these two problems are actually closely related. In fact, **he realized that differentiation and integration are inverse processes.**

The Fundamental Theorem of Calculus Part (1)

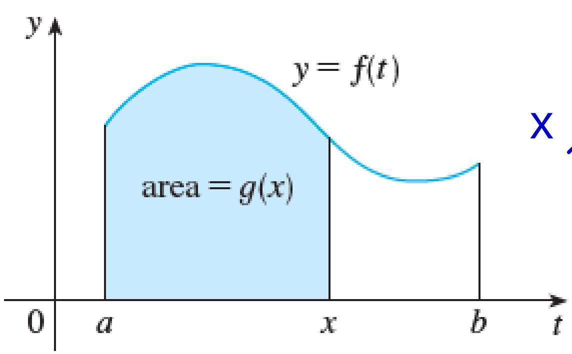
If f is continuous on $[a, b]$, and g is defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

التكامل يعني المساحة
تحت المنحنى

then

- g(x) is continuous on [a,b]
- and differentiable on (a,b)
- $$g'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$



تفاضل التكامل =
الدالة الأصلية بالمتغير X

- We abbreviate this theorem as **FTC1**.
- If x is a function of $x : u(x)$, then we have

$$g'(x) = \frac{d}{dx}\int_a^{u(x)} f(t)dt = f\big(u(x)\big).u'(x)$$

تفاضل التكامل . تفاضل الحد الأعلى

See example 4

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Example (2):

Find the derivative of the function

$$g(x) = \int_0^x \sqrt{1+t^2}\,dt$$

Example (4):

Find $\dfrac{d}{dx}\displaystyle\int_1^{x^4} \sec t \; dt$

FTC 1 $$\frac{d}{dx}\int_a^{u(x)} f(t)dt = f\big(u(x)\big).u'(x)$$

تفاضل التكامل . تفاضل الحد الأعلى

The Fundamental Theorem of Calculus Part (2)

In Section 5.2 we computed integrals from the definition as a limit of Riemann sums and we saw that this procedure long and difficult. The second part of the Fundamental Theorem, which follows easily from the first part, provides us with a much simpler method for the evaluation of integrals.

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

antiderivative

Where $F' = f$

- We abbreviate this theorem as **FTC2**.
- هذه النظرية تحل التكامل المحدود وإنه عبار عن إيجاد antiderivative
- $$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

النظرية الأساسية تربط بين التفاضل والتكامل

The Fundamental Theorem of Calculus Part (1)

If f is continuous on $[a, b]$, and g is defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

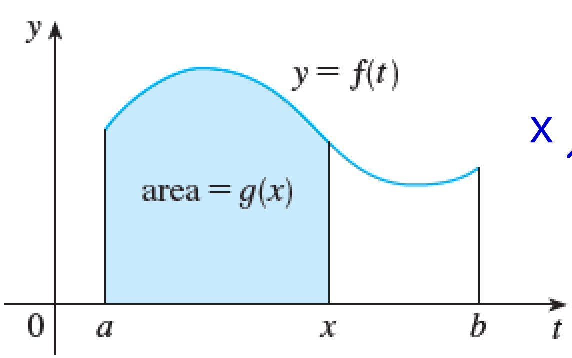
التكامل يعني المساحة
تحت المنحنى

then

- $g(x)$ is continuous on $[a,b]$
- and differentiable on (a,b)
- $$g'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

FTC1

تفاضل التكامل =
الدالة الأصلية بالمتغير X



The Fundamental Theorem of Calculus Part (2)

If f is continuous on $[a, b]$, then

FTC2

$$\int_a^b f(x)dx = F(b) - F(a)$$

antiderivative

Where $F' = f$

In Section 5.2 we computed integrals from the definition as a limit of Riemann sums and we saw that this procedure long and difficult. The second part of the FTC 2, which follows easily from the first part, provides us with a much simpler method for the evaluation of integrals.

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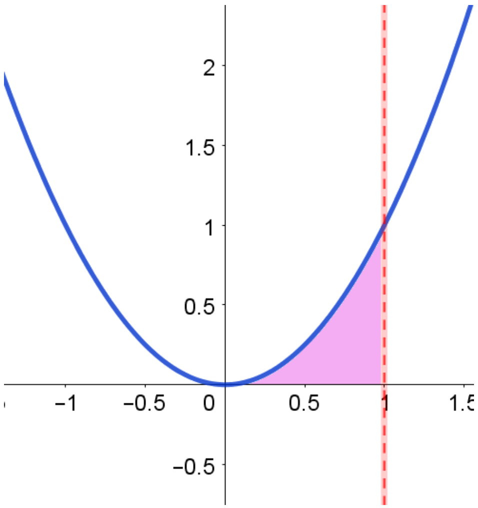
Example (5): Evaluate the integral

$$\int_1^3 e^x dx$$

Example (6):

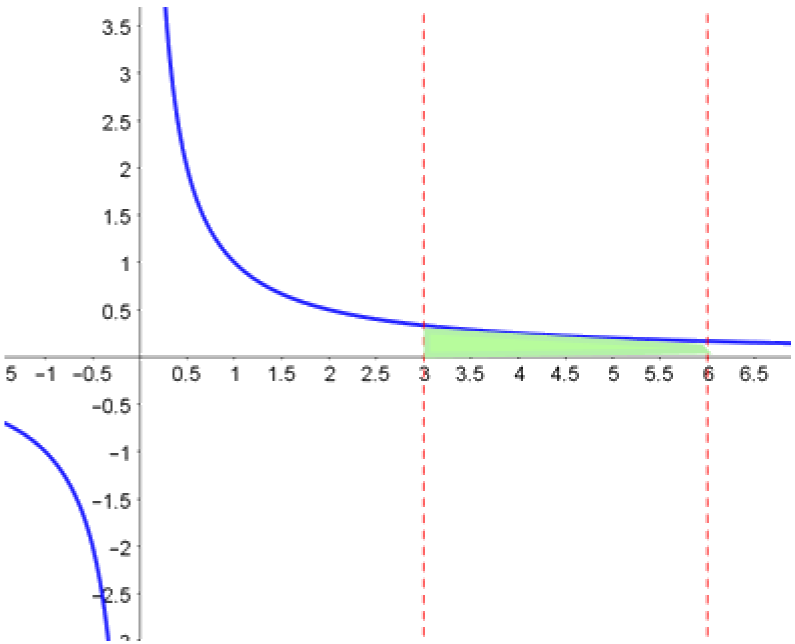
Find the area under the parabola $y = x^2$ From $0\ to\ 1$.



If you compare the calculation in Example 6 with the one in Example 5.1.2, you will see that the Fundamental Theorem gives a much shorter method.

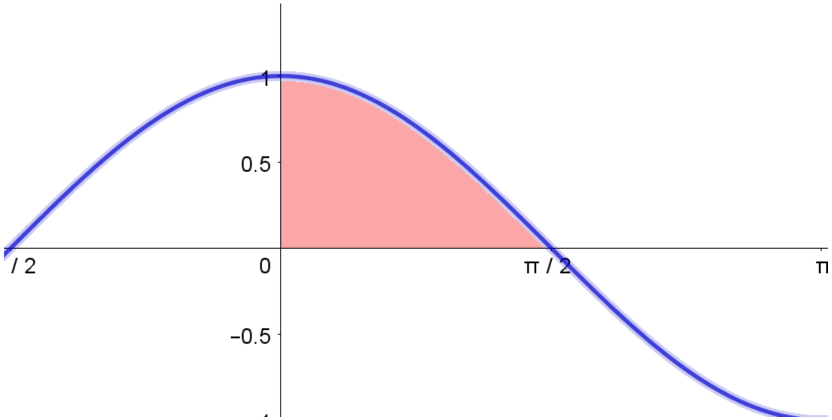
Example (7):

Evaluate $\int_3^6 \frac{1}{x}\,dx$



Example (8):

Find the area under the cosine curve from 0 to b, where $0 \le b \le \pi/2$.



Example (9):

What is wrong with the following calculation?

$$\int_{-1}^{3} \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\Bigg|_{-1}^{3} = -\frac{4}{3}$$

1

2

3

Exersice (35):

Evaluate $\int_1^2 \frac{v^3 + 3v^6}{v^4} \; dv$

Exercise (63):

Find the derivative of the function.

$$y = \int_{\cos x}^{\sin x} \ln(1 + 2v)\, dv$$