



CALCULUS II

APPENDIX - A

Numbers, Inequalities, and Absolute values

Dr. Rola Asaad Hijazi

Numbers

Calculus is based on the real number system.
We start with the integers:

$$\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots$$

rational numbers

$$\frac{1}{2}$$

$$-\frac{3}{7}$$

$$65 = \frac{65}{1}$$

$$0.27 = \frac{27}{100}$$

irrational numbers

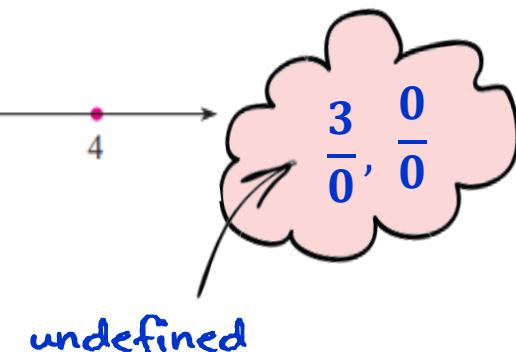
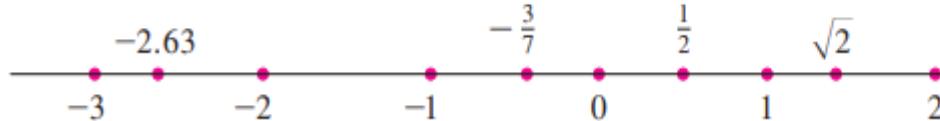
$$\sqrt{5}$$

$$\sqrt{3}$$

$$\pi$$

$$\sin 1^\circ$$

$$\log_{10} 2$$



Intervals

Notation	Set description	Picture
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \geq a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	\mathbb{R} (set of all real numbers)	

Inequalities

Rules for Inequalities:

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and $c < d$, then $a + c < b + d$
3. If $a < b$ and $c > 0$, then $ac < bc$.
4. If $a < b$ and $c < 0$, then $ac > bc$.
5. If $0 < a < b$, then $1/a > 1/b$.



rule 4 says that if we multiply both sides of an inequality by a negative number, then we reverse the direction of the inequality

Example 1:

Solve the Inequality

$$1 + x < 7x + 5$$

Example 2:

Solve the Inequalities

$$4 \leq 3x - 2 < 13$$

Example 3: Solve $x^2 - 5x + 6 \leq 0$

Interval	$x - 2$	$x - 3$	$(x - 2)(x - 3)$
$x < 2$	-	-	+
$2 < x < 3$	+	-	-
$x > 3$	+	+	+

Example 4:

Solve the Inequality

$$x^3 + 3x^2 > 4x$$

Interval	x	$x - 1$	$x + 4$	$x(x - 1)(x + 4)$
$x < -4$	-	-	-	-
$-4 < x < 0$	-	-	+	+
$0 < x < 1$	+	-	+	-
$x > 1$	+	+	+	+

Solve the inequality in terms of intervals and illustrate the solution set on the real number line.

Exercise 25:

$$(x - 1)(x - 2) > 0$$

Exercise 32:

$$x^2 \geq 5$$

Exercise 33:

$$x^3 - x^2 \leq 0$$

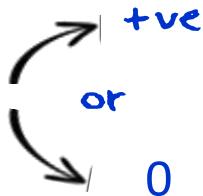
Exercise 35:

$$x^3 > x$$

Absolute Value

The absolute value of a number a , denoted by $|a|$, is the distance from a to 0 on the real number line.

Distances are always



So we have $|a| \geq 0$ for every number a .

examples

$$|-3| = 3, \quad |\sqrt{2} - 1| = \sqrt{2} - 1$$

$$|\sqrt{5} - 5| = 5 - \sqrt{5}$$

$$|\pi - 3| = \pi - 3$$

$$|3 - \pi| = \pi - 3$$

$$|\pi - 4| = 4 - \pi$$



$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$



Remember that if a is negative, then $-a$ is positive

Example 5:

Express $|3x - 2|$ without using the absolute-value symbol.

Exercise 7:

Express $|x - 2|$ if $x < 2$ without using the absolute-value symbol.



$$\sqrt{a^2} = a \text{ if } a \geq 0.$$

$$\sqrt{(-a)^2} = -a \text{ if } a < 0.$$

$$\boxed{\sqrt{a^2} = |a|}$$

Properties of Absolute Values

Suppose a and b are **any real numbers** and n is an integer.

Then

$$1 \quad |ab| = |a||b|$$

$$2 \quad \left| \frac{a}{b} \right| = \frac{|a|}{|b|} \quad b \neq 0$$

$$3 \quad |a^x| = |a|^x$$

Suppose $a > 0$

$$4 \quad |x| = a \iff x = \pm a$$

$$5 \quad |x| < a \iff -a < x < a$$

$$6 \quad |x| > a \iff x > a \text{ or } x < -a$$

Example 6:

$$\text{Solve } |3x - 2| = 3$$

Example 7:

$$\text{Solve } |x - 5| < 2$$

Example 8:

$$\text{Solve } |3x + 2| \geq 2$$

Exercise 55:

$$1 \leq |x| \leq 4$$



Homework

4,8,10 13-22 26,28, 31, 36,44, 47-49 51-52 54