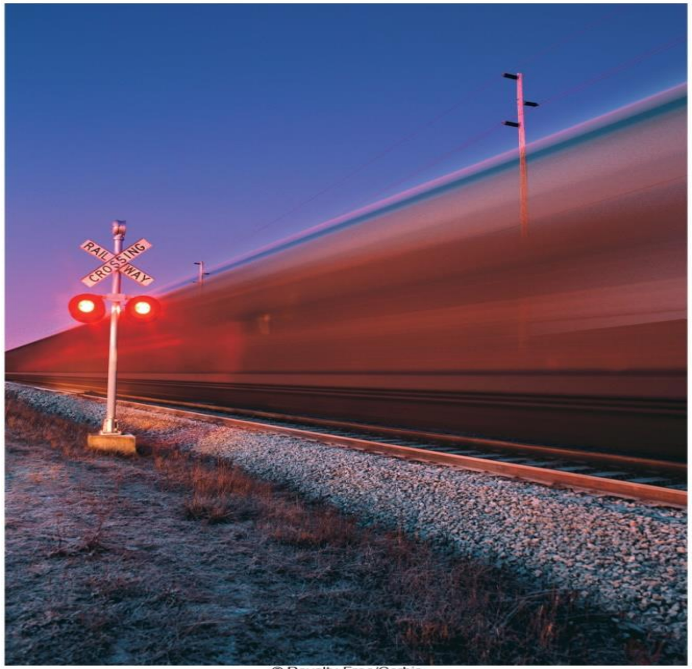
Chapter (2) Motion in a straight line



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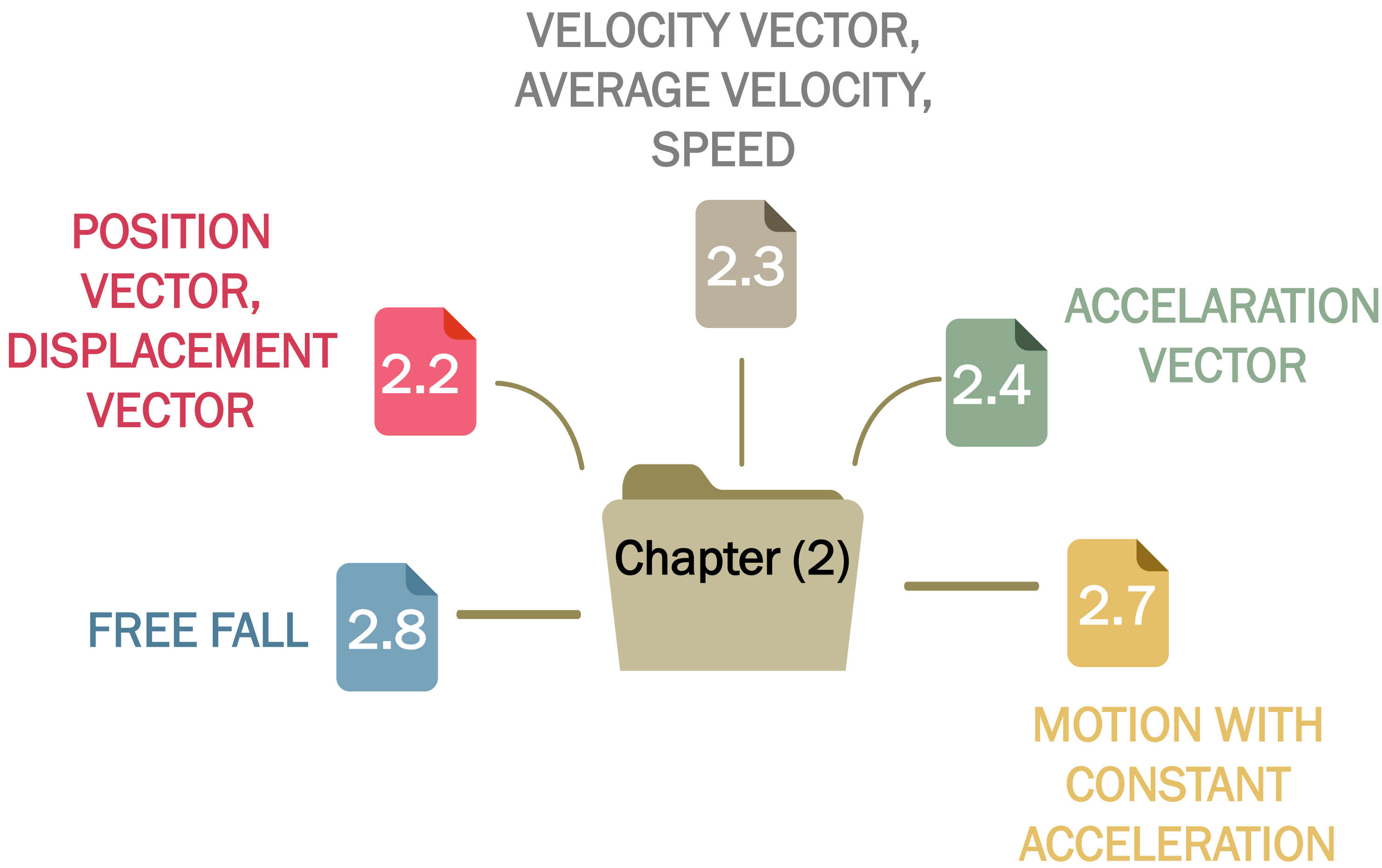
Outline


VELOCITY VECTOR, AVERAGE VELOCITY, SPEED
2.3
POSITION VECTOR, DISPLACEMENT VECTOR
2.2
ACCELARATION VECTOR
2.4
Chapter (2)
FREE FALL
2.8
2.7
MOTION WITH CONSTANT ACCELERATION

Learning outcomes

After studying this chapter, you will be able:

1. Understand SI units, scientific notation, prefix, unit conversion
2. Determine the direction and magnitude of a particle's position vector from its components, and vice versa.
3. Apply the relationship between particle's displacement vector and its initial and final position vectors.
4. Indicate velocity vector in unit vector notation.
5. Determine the direction and magnitude of a particle's velocity vector from its components, and vice versa.
6. Given a particle's position vector as a function of time, determine its instantaneous velocity vector.
7. Determine the direction and magnitude of a particle's acceleration vector from its components, and vice versa.
8. Determination of average acceleration vector in unit-vector notations
9. Given a particle's velocity vector as a function of time, determine its instantaneous acceleration vector.
10. Apply the constant acceleration equations to find acceleration, velocity, position, and time.
11. understand the free fall equations.

Background (SI System of Units)

- The international system of units is abbreviated SI:
  - *Used for scientific work around the world.*
- The seven base units are:

Table 1.1 Unit Names and Abbreviations for the Base Units of the SI System of Units

Unit	Abbreviation	Base Unit for
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	current
kelvin	K	temperature
mole	mol	amount of a substance
candela	cd	luminous intensity

Background (Scientific Notation)

- Dealing with really big numbers or really small numbers can be difficult. To deal with big and small numbers we use

scientific notation:

$$\text{Number} = \text{mantissa}.\ 10^{\text{exponent}}$$

- The mantissa is usually chosen so that it has one digit preceding the decimal point, but not always.

e.g. 3.00×10^8.

- Multiplication and division are simplified using scientific notation:

e.g. $(7 \times 10^{27})(7 \times 10^{9}) = 49 \times 10^{36} = 4.9 \times 10^{37}$

Background (Prefix)

Table 1.3 SI Standard Prefixes

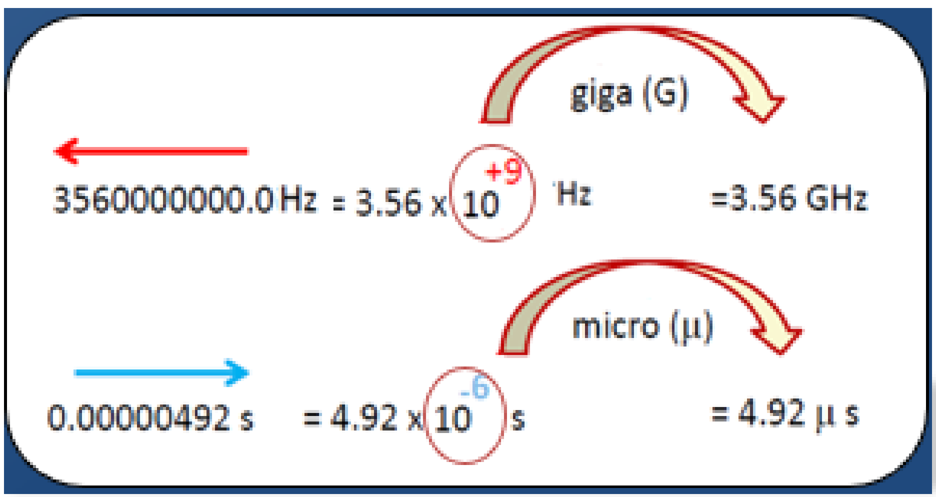
Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta-	Y	10^{-24}	yocto-	y
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{18}	exa-	E	10^{-18}	atto-	a
10^{15}	peta-	P	10^{-15}	femto-	f
10^{12}	tera-	T	10^{-12}	pico-	p
10^{9}	giga-	G	10^{-9}	nano-	
10^{6}	mega-	M	10^{-6}	micro-	
10^{3}	kilo-	k	10^{-3}	milli-	
10^{2}	hecto-	h	10^{-2}	centi-	
10^{1}	deka-	da	10^{-1}	deci-	

- ❑ Prefix represents a certain power of 10, to be used as a multiplication factor.
- ❑ Attaching a prefix to an (SI) unit has the effect of multiplying by the associated factor

Prefix	Symbol	Factor
Tera-	T	10^{12}
Giga-	G	10^{9}
Mega-	M	10^{6}
Kilo-	K	10^{3}
Centi-	c	10^{-2}
Milli-	m	10^{-3}
Micro-	μ	10^{-6}
Nano-	n	10^{-9}
Pico-	p	10^{-12}

Background

Examples:



Remember!

Prefix	Symbol	Factor
Tera-	T	10^{12}
Giga-	**G**	$\mathbf{10^{9}}$
Mega-	M	10^{6}
Kilo-	K	10^{3}
Centi-	c	10^{-2}
Milli-	m	10^{-3}
Micro-	**μ**	$\mathbf{10^{-6}}$
Nano-	n	10^{-9}
Pico-	p	10^{-12}

Background (Unit conversion)

$5 \text{ cm} \times 10^{-2} = 5 \times 10^{-2} \text{ m}$

Remember!

Prefix	Symbol	Factor
Tera-	T	10^{12}
Giga-	G	10^{9}
Mega-	M	10^{6}
Kilo-	K	10^{3}
Centi-	**c**	10^{-2}
Milli-	m	10^{-3}
Micro-	μ	10^{-6}
Nano-	n	10^{-9}
Pico-	p	10^{-12}

2.2 Position vector

- To locate an object means to find it’s position relative to a reference point origin (or zero point) of an axis.
- All position vectors are measured relative to the origin of the coordinate system.
- SI-unit “m” (it’s a length unit)
- Position is a vector quantity: has a magnitude and a direction.

Direction	Positive ⇒ if it is right to the reference point Negative ⇒ if it is left to the reference point

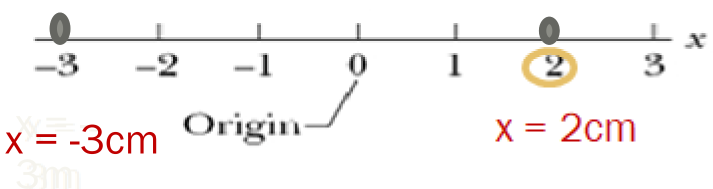
2.2 Position vector

Position vector in one dimension

On X-axis

Left to Ref. Point
(Negative direction)

Right to Ref. Point
(Positive direction)



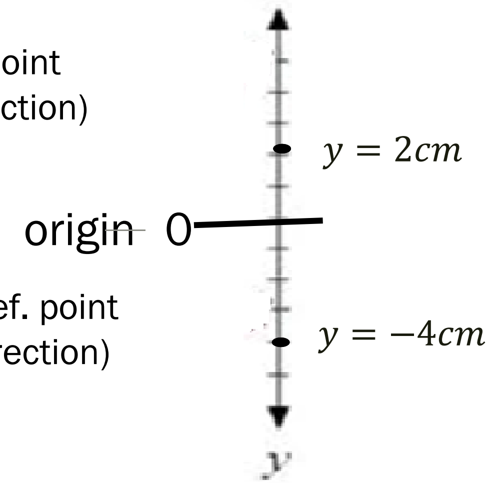
Position left to the reference point (x=-3cm)

Position right to the reference point (x=2cm)

On y-axis

Up the Ref. point
(Positive direction)

Below the Ref. point
(Negative direction)



Position up the reference point (y=2cm)

Position below the reference point (y=-4cm)

2.2 Position vector

Position vector in three dimensions

- Position vector is denoted with $\vec{r}$
- Written in a unit vector notation in 3D as:

$$\vec{r} = (x)\hat{\imath} + (y)\hat{\jmath} + (z)\hat{k}$$

Write a position vector

In three dimension

$$\vec{r} = -3\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$$

Write a position vectors in x-direction (1D)

Left to the reference point $\vec{r} = -3\hat{\imath}$ OR x=-3cm

Right to the reference point $\vec{r} = 2\hat{\imath}$ OR x=2cm

Up the reference point $\vec{r} = 2\hat{\jmath}$ OR y=2cm

Below the reference point $\vec{r} = -4\hat{\jmath}$ OR y=-4cm

Note: we can leave the vector arrows for one dimension

2.2 Position vector

Position vector in one dimension as a function in time

- If the object move➔ the position of an object can change as a function of time
- If the motion in one dimension, this mean that the x-component of the vector is a function of time➔ $x(t)$
- If we want to specify the position at some specific time , we use the notation At t_1➔ $x(t_1)=x_1$

Example:

x-component of a position vector as a function of time $x(t) = (3t + 5)$

Where x is the position and t is the time

Position vector in three dimensions as a function in time

$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z\,(t)\hat{k}$
$\vec{r}(t) = (3t + 1)\hat{\imath} + (2t)\hat{\jmath} + (t^2 + 5)\hat{k}$

However, it's a valid way of expressing a position by only x-component if it is in 1 D only

$Position\ vector\ in\ 1D$: $\vec{r}(\mathrm{t}) = (3\mathrm{t} + 5)\hat{\imath}$ OR x(t)=3t+5

2.2 Displacement vector

- Displacement refers to ***"how far out of place an object is***" from its origin or starting point
- It is the object's ***overall change in position*** ➔ Is the ***difference between final position and initial position***
- SI-unit "m"

- **The displacement vector on one dimension** has only one component for example x-component , which is the difference between the x-component of the final and initial position vectors

$$\Delta x = x_f - x_i$$

- Displacement is a vector quantity: has magnitude and direction.
- Direction: if Δx is positive $\Rightarrow$ moving to the right

  if Δx is negative $\Rightarrow$ moving to the left

Displacement vector in three dimensions

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

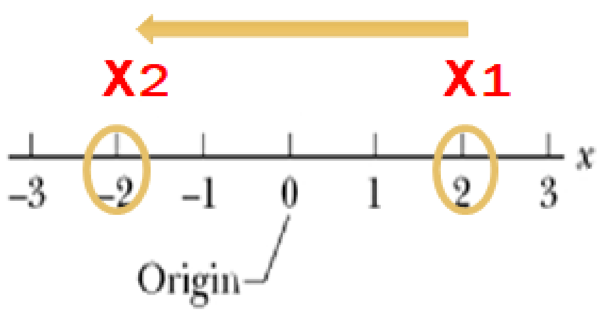
$$\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$$

2.2 Displacement vector

Displacement vector in one dimension

On X-axis

Moving to the left

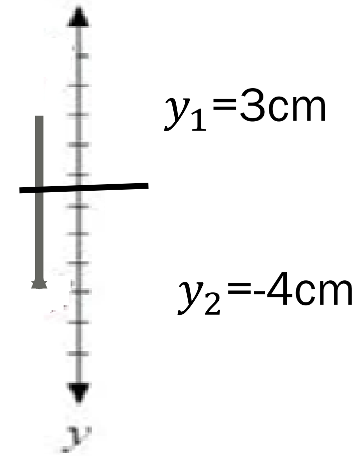


$\Delta \boldsymbol{x} = \boldsymbol{x_f}\text{-}\boldsymbol{x_i}$

$\Delta x = (-2) - (2)$= - 4cm

On y-axis

Moving down



$\Delta \boldsymbol{y} = \boldsymbol{y_f}\text{-}\boldsymbol{y_i}$

$\Delta y = (-4) - (3)$= -7cm

<u>Write a displacement vector in x-direction (1D)</u>

Moving to the left $\overrightarrow{\Delta r} = -4\,\hat{\imath}$

However, it's a valid way of expressing a displacement as; $\Delta x = -4cm$; its understood that it refers to the x-component of the displacement vector

<u>Write a displacement vector in y-direction(1D)</u>

Moving down $\overrightarrow{\Delta r} = -7\hat{\jmath}$

However, it's a valid way of expressing a displacement as; $\Delta y = -7cm$; its understood that it refers to the y-component of the displacement vector

2.2 Velocity Vector

velocity vector in one dimension

Average velocity

- The average velocity ***is The ratio of displacement per time interval***

In x-direction $\overline{v_x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}\ (m/s)$

- Average velocity vector in only x-direction $\vec{\bar{v}} = (\frac{\Delta x}{\Delta t})\hat{\imath}$

OR
in 1D (x-axis) its enough to write the vector as x-component only
$\overline{v_x} = \frac{\Delta x}{\Delta t}$

- Velocity is a vector quantity: has magnitude and direction.
- Direction: if velocity is positive ⇒ moving to the right

  if velocity is negative ⇒ moving to the left

Average velocity vector in three dimensions

$$\vec{\bar{v}} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{k} = \overline{v_x}\hat{\imath} + \overline{v_y}\hat{\jmath} + \overline{v_z}\hat{k}$$

the operation applies to each of the components of the vector

2.3 Velocity Vector

velocity vector in one dimension

Instantaneous velocity

- The instantaneous velocity It describes the velocity at a very specific time
- Is obtained in the limit that the time interval for the averaging procedure approaches 0 (time derivate of the displacement)

In x-direction $$v_x = \lim_{\Delta t \to 0} \bar{v}_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt} \quad (m/s)$$

instantanouse velocity vector $\vec{v} = (\frac{dx}{dt})\hat{\imath}$

OR in 1D (x-axis) its enough to write the vector as x-component only $v_x = \frac{dx}{dt}$

Instantons velocity vector in three dimensions

$$\vec{v} = \frac{\overrightarrow{dr}}{dt} = \frac{dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} = v_x\,\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

the derivative operation applies to each of the components of the vector

2.3 Speed

Instantaneous Speeds

- Is The magnitude of instantaneous velocity.
- Is the absolute value of the velocity vector.

$$\text{Instantaneous speed} = |Instantaneous\ velocity|$$

- A **scalar** quantity (**has no direction**)
- Always positive

Speed limits are always posted as positive numbers, and the radar monitors that measure the speed of passing cars also always display positive numbers

Example 2.1 (Page 44)

During the time interval 0 to 10 s, the position vector of a car on the road is given by:

$$x(t) = 17.2 - 10.1t + 1.1t^2$$

(a) What is the position vector at t=0s?

(b)What is its instantaneous velocity vector at t= 6s?

(c) What is the car's average velocity during this interval (from 0s to 10s)?

SOLUTION:

(a) $\vec{r}\,(t)$ =$(17.2 - 10.1t + 1.1t^2)\,\hat{\imath}$

At t=0 ➔ $\vec{r}(0) = (17.2 - 10.1(0) + 1.1(0))\,\hat{\imath}$

$\vec{r}\,(0)$ =17.2 m $\hat{\imath}$

(b) Take derivative to get the instantaneous velocity vector:

$$v_x(t) = \frac{dx}{dt} = \frac{d}{dt}(17.2 + 10.1t + 1.1t^2) = -10.1 + 2.2t$$

$$\vec{v}(t) = (-10.1 + 2.2t)\,\hat{\imath}$$

At t=6➔ $\vec{v}\,(t) = (-10.1 + 2.2t)\,\hat{\imath}$➔ $\vec{v}(6) = (-10.1 + (2.2x6))\,\hat{\imath} = 3.1\,{}^{m}/_{s}\,\hat{\imath}$

Example 2.1 (Page 44)

SOLUTION

(c) WE know that:

- $x(t) = 17.2 - 10.1t + 1.1t^2$
- $\vec{v}(t) = (-10.1 + 2.2t)\,\hat{\imath}$

We need to calculate x_2 and x_1 to find the displacement

The average velocity $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

- At t_2=10s ➔ x_2=? $x(t) = 17.2 - 10.1t + 1.1t^2$

$$x(10) = 17.2 - 10.1(10) + 1.1(10)^2 = 26.2\text{m}$$

- At t_1=0s ➔ x_1=? $x(0) = 17.2$m (calculated from (a))

The average velocity $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{26.2 - 17.2}{10 - 0} = \frac{9}{10} = 0.9\text{m/s}$

2.4 Acceleration Vector

acceleration vector in one dimension

Average acceleration

- The average acceleration is defined as the velocity change per time interval

In x-direction $$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{x2} - v_{x1}}{t_2 - t_1} \quad (m/s^2)$$

- The velocity changes could be in (**magnitude, or direction**),to be able to say the particle undergoes an acceleration

- Average acceleration vector $\vec{\bar{a}} = (\frac{\Delta v_x}{\Delta t})\hat{\imath}$

OR
in 1D (x-axis) its enough to write the vector as x-component only
$\overline{a_x} = \frac{\Delta v_x}{\Delta t}$

Average acceleration vector in three dimensions

$$\vec{\bar{a}} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\Delta v_x \hat{\imath} + \Delta v_y \hat{\jmath} + \Delta v_z \hat{k}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{\imath} + \frac{\Delta v_y}{\Delta t}\hat{\jmath} + \frac{\Delta v_z}{\Delta t}\hat{k} = \overline{a_x}\hat{\imath} + \overline{a_y}\hat{\jmath} + \overline{a_z}\hat{k}$$

the operation applies to each of the components of the vector

2.4 Acceleration Vector

acceleration vector in one dimension

Instantaneous acceleration

- The instantaneous acceleration describes the acceleration at a very specific time
- Is defined as the limit of the average acceleration as the time interval approaches 0 (time derivate of the velocity)

x-component $$a_x = \lim_{\Delta t \to 0} \bar{a}_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt}$$

instantaneous acceleration vector $\vec{a} = (\frac{dv_x}{dt})\hat{\imath}$

OR
in 1D (x-axis) its enough to write the vector as
x-component only
$a_x = \frac{dv_x}{dt}$

Instantons acceleration vector in three dimensions

$$\vec{a} = \frac{\overrightarrow{dv}}{dt} = \frac{dv_x\hat{\imath} + dv_y\hat{\jmath} + dv_z\hat{k}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} = a_x\,\hat{\imath} + a\hat{\jmath} + a\hat{k}$$

the derivative operation applies to each of the components of the vector

2.4 Acceleration Concepts

- Deceleration of an object: is a decrease in the speed of the object over time.
- If the velocity and acceleration are in the same direction, the object speeds up.
- If the velocity and acceleration are in opposite directions, the object slows down.

Velocity	Acceleration	Motion
+	+	Speeding up in the positive direction
-	-	Speeding up in the negative direction
+	-	Slowing down in the positive direction
-	+	Slowing down in the negative direction

Concept Check

- When you're driving a car along a straight road, you could be traveling in the positive or negative direction and you could have a positive acceleration or a negative acceleration.

  If you have negative velocity and positive acceleration you are:

A. slowing down in the positive direction.
B. speeding up in the negative direction.
C. speeding up in the positive direction.
D. slowing down in the negative direction.

Exercise 2.37 (page 62)

•2.37 The position of an object as a function of time is given as $x = At^3 + Bt^2 + Ct + D$. The constants are $A = 2.10\ \text{m/s}^3$, $B = 1.00\ \text{m/s}^2$, $C = -4.10\ \text{m/s}$, and $D = 3.00\ \text{m}$.

a) What is the velocity of the object at $t = 10.0$ s?

b) At what time(s) is the object at rest?

c) What is the acceleration of the object at $t = 0.50$ s?

SOLUTION:

(a) The velocity is given by the time derivative of the position function

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\left(2.1t^3 + (1)t^2 + (-4.1)t + (3)\right) = 3(2.1)t^2 + 2(1)t + (-4.1)$$

$$v(t) = 6.3t^2 + 2t - 4.1$$

At t=10s ➔ $v(10)=(6.3)(10^2)+(2)(10)-4.1$

$= 645.9$ m/s

Exercise 2.37 (page 62)

SOLUTION:

(b) To find the time when the object is at rest, set the velocity to zero, and solve for time; t.

$$v(t) = 6.3t^2 + 2t + -4.1$$

$$0 = 6.3t^2 + 2t - 4.1$$

$$6.3t^2 + 2t - 4.1 = 0$$

This is a quadratic equation of the form $ax^2 + \mathrm{bx} + \mathrm{c} = 0,$ whose solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where, a=6.3

b=2

c=-4.1

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2(2) \pm \sqrt{(2x2)^2 - 4(6.3)(-4.1)}}{2(6.3)} =$$

$$t = 0.664s$$

OR, $t = -0.981s$

Exercise 2.37 (page 62)

SOLUTION:

(c) The acceleration is given by the time derivative of the velocity

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(6.3t^2 + 2t - 4.1) = 12.6t + 2$$

At t=0.5s ➔ $a(0.5)$ =(12.6)(0.5)+2

= 8.3 m/s^2

2.7 Constant Acceleration

Many physical situations involve constant acceleration .

- **Constant acceleration** does not mean the velocity is constant.
- **Constant acceleration** means the velocity changes with constant rate.
- If v = constant ➔ a=0.
- If v changes with constant rate ➔ a= constant

2.7 Constant Acceleration

- We can derive useful equations for the case of constant acceleration.
- The Five Kinematic Equations

Equation	Missing Quantity
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2}at^2$	v_0

x_0 → Initial position
x → final position
$x - x_0$ → displacement
v_0 → Initial velocity
v → final velocity
t → time
a → Constant acceleration

2.7 Constant Acceleration

Remarks

- When the object starts from rest $v_0 = 0$
- When the object stops $v = 0$
- $x_0 = 0$ unless something else mentioned

Solved problem 2.2 (page 46)

Assuming a constant acceleration of a = 4.3 m/s^2, starting from rest:

(a) what is the speed of the airplane reached after 18 seconds?

(b) How far down the runway has this airplane moved by the time it takes off?

SOLUTION:

(a)

Given

$a = 4.3\, m/s^2$

$v_{o=}0$

$t = 18s$

Remember!

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$x - x_0 = vt - \frac{1}{2}at^2$$

$\boldsymbol{At\ t = 18\ s}$

$\boldsymbol{v = v_o + at = 0 + (4.3)(18) = 77.4\ m/s}$

$\boldsymbol{S = |77.4| = 77.4\ m/s}$

Solved problem 2.2 (page 46)

Assuming a constant acceleration of a = 4.3 m/s^2, starting from rest:

(a) what is the speed of the airplane reached after 18 seconds?

(b) How far down the runway has this airplane moved by the time it takes off?

SOLUTION:

(b)

Given

$$\begin{cases} a = 4.3\, m/s^2 \\ v_\circ = 0 \\ t = 18s \\ x_\circ = 0 \end{cases}$$

Remember!

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$

$$x - x_0 = vt - \tfrac{1}{2}at^2$$

$$\boldsymbol{At\ t = 18\ s}$$

$$\boldsymbol{x = x_\circ + v_\circ t + {}^1/_2\, at^2 = 0 + (0)(18) + \frac{1}{2}(4.3)(18^2) = 697m}$$

2.7 Constant Acceleration (Free Fall)

- Free fall is the motion of an object under influence of gravity and ignoring any other effects such as air resistance
- Free fall is an example of motion in one dimension with constant acceleration.
- All objects in free fall accelerate downward at the same rate and is independent of the object's mass, density or shape
- Near the surface of the Earth, the acceleration due to the force of gravity is constant and is always in the downward direction.

$$a = a_y = -g$$

$$g = 9.8\, m/s^2 \quad downward$$

2.7 Constant Acceleration (Free Fall)

Motion along x-axis	motion is along y axis➔ x=y (Free Fall)
$v = v_o + at$ $x - x_o = v_o t + {}^1\!/_2\, at^2$ $v^2 = {v_o}^2 + 2a(x - x_o)$ $x - x_o = {}^1\!/_2\,(v_o + v)t$ $x - x_o = vt - {}^1\!/_2\, at^2$	$v = v_o + at$ $y - y_o = v_o t + {}^1\!/_2\, at^2$ $v^2 = {v_o}^2 + 2a(y - y_o)$ $y - y_o = {}^1\!/_2\,(v_o + v)t$ $y - y_o = vt - {}^1\!/_2\, at^2$ a= -g = -(9.8)

Extra Exercise

- At a construction site a pipe struck the ground with a velocity of -24 m/s.

How long was it falling ? ($v_\circ = 0$)

SOLUTION:

Given

$$v = -24 m/s$$
$$v_\circ = 0$$
$$t = ?$$

From

$$v = v_o + at$$

$$t = \frac{v - v_0}{a} = \frac{-24 - 0}{-9.8} = 2.45 \text{ seconds}$$

Remember!

$$v = v_o + at$$
$$y - y_o = v_o t + {}^1\!/_2\, at^2$$
$$v^2 = {v_o}^2 + 2a(y - y_o)$$
$$y - y_o = {}^1\!/_2\,(v_o + v)t$$
$$y - y_o = vt - {}^1\!/_2\, at^2$$

Equation summary

Position vector

$\vec{r} = (x)\hat{\imath} + (y)\hat{\jmath} + (z)\hat{k}$

Displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Displacement (1D) $\Delta x = x_2 - x_1 \quad (x_1 \equiv x(t_1),\ x_2 \equiv x(t_2))$

$\text{speed} = v = |\vec{v}| = |v_x|$

Average velocity (1D) $\bar{v}_x = \dfrac{\Delta x}{\Delta t}$

Instantaneous velocity (1D) $v_x = \lim_{\Delta t \to 0} \bar{v}_x = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} \equiv \dfrac{dx}{dt}$

Velocity vector (3D) $\vec{v} = \dfrac{d\vec{r}}{dt}$

Equations of motion in x-direction

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$
$$x = x_0 + v_x t$$
$$v_x = v_{x0} + a_x t$$
$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$
$$x = x_{0+}v_{xo}t - \frac{1}{2}a_x t^2$$

Average acceleration (1D) $\bar{a}_x = \dfrac{\Delta v_x}{\Delta t}$

Instantaneous acceleration (1D) $a_x = \lim_{\Delta t \to 0} \bar{a}_x = \lim_{\Delta t \to 0} \dfrac{\Delta v_x}{\Delta t} \equiv \dfrac{dv_x}{dt}$

Acceleration vector (3D) $\vec{a} = \dfrac{d\vec{v}}{dt}$

Free fall equations

$$v = v_o + at$$
$$y - y_o = v_o t + {}^1\!/_2\, at^2$$
$$v^2 = v_o{}^2 + 2a(y - y_o)$$
$$y - y_o = {}^1\!/_2\,(v_o + v)t$$
$$y - y_o = vt - {}^1\!/_2\, at^2$$

The END OF CHAPTER (2)