



CALCULUS II

(1.4) Exponential Function $f(x) = a^x$

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Exponential Function

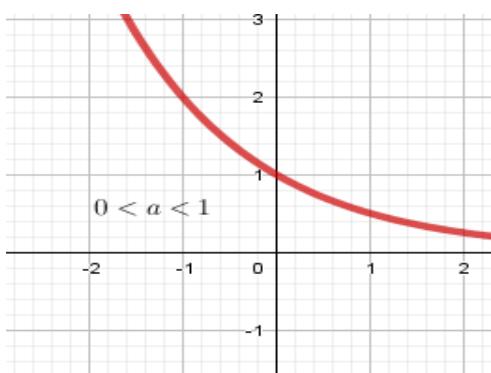
is of the form $f(x) = a^x$ 

If $x = \begin{cases} n(+ve) & \Rightarrow a^n = a \dots a \\ 0 & \Rightarrow a^0 = 1 \\ -n & \Rightarrow a^{-n} = \frac{1}{a^n} \\ \frac{p}{q} & \Rightarrow a^{\frac{p}{q}} = \sqrt[q]{a^p} \\ irrational, \sqrt{3}, \pi, \dots & \Rightarrow a^\pi, a^{\sqrt{3}}, \dots \end{cases}$

We have 3 kinds of exponential functions:

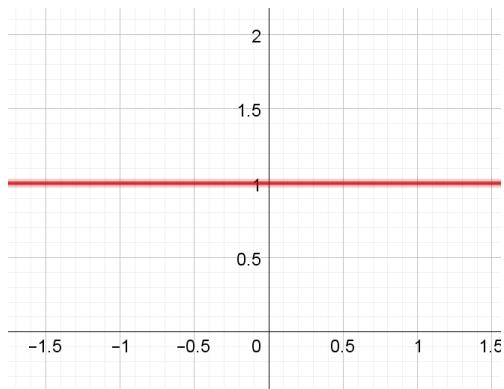
1

$$0 < a < 1, \quad y = \left(\frac{1}{2}\right)^x$$



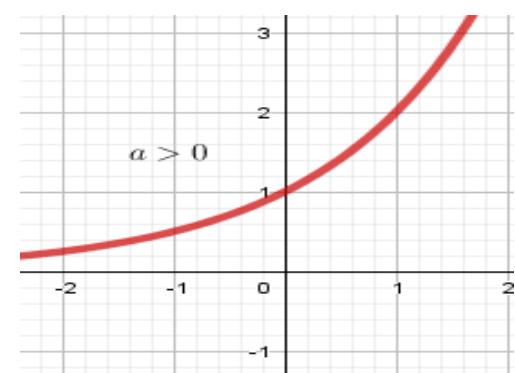
2

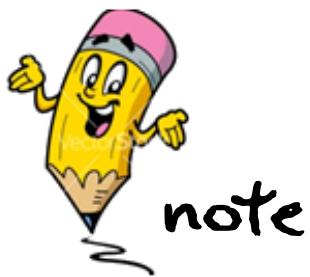
$$a = 1, \quad y = (1)^x = 1$$



3

$$a > 1, \quad y = 2^x, \quad y = e^x$$

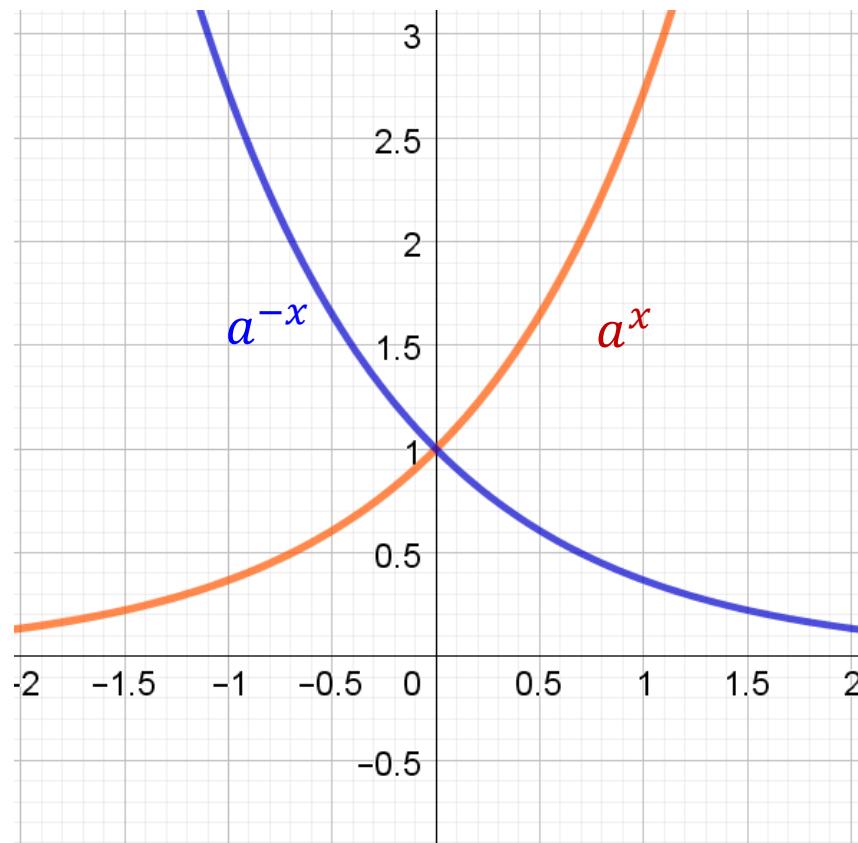




from (1) and (2) it is clear that

$$f_1(x) = \left(\frac{1}{a}\right)^x = a^{-x} \text{ and } f_2(x) = a^x$$

that f_1 is the reflection of $f_2(x) = a^x$ about $y - axis$



Law of Exponents

If a, b are positive numbers and x, y are any real numbers, then:

1

$$a^{x+y} = a^x a^y$$

2

$$a^{x-y} = \frac{a^x}{a^y}$$

3

$$(a^x)^y = a^{xy}$$

4

$$(ab)^x = a^x b^x$$

note

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

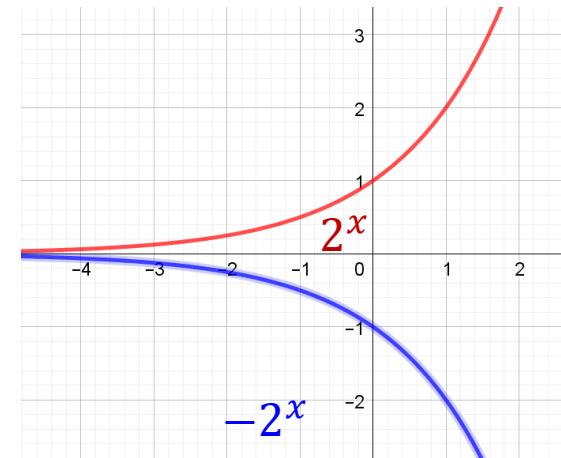
Example

Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.

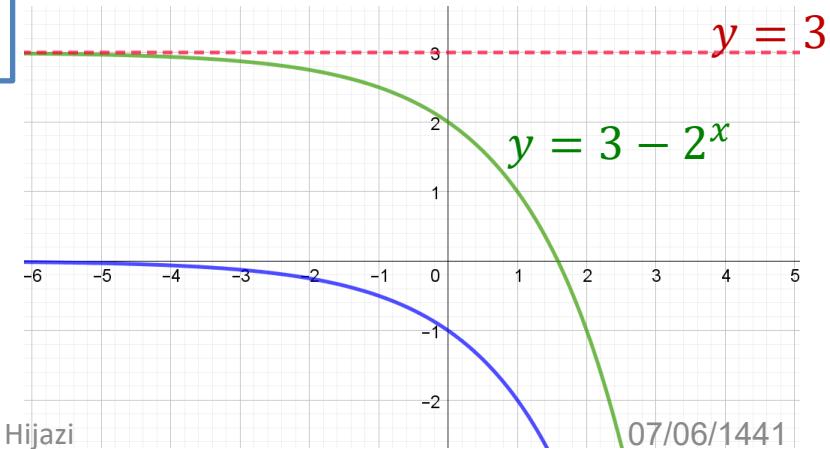
Solution:

(1)

(2)



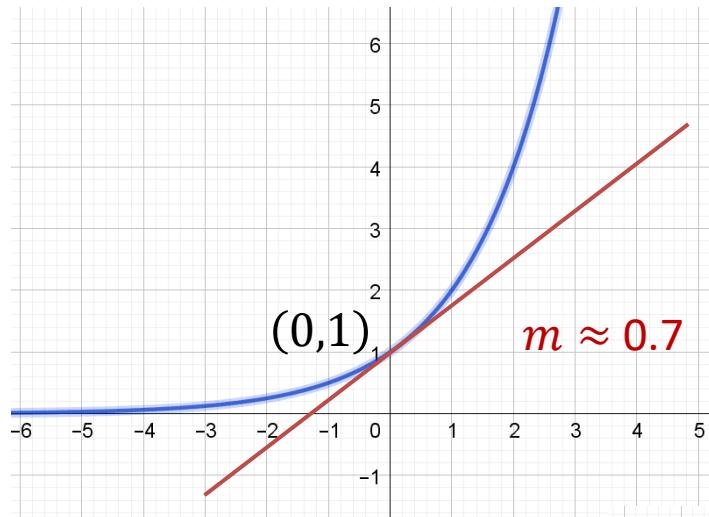
Domain =
Range =



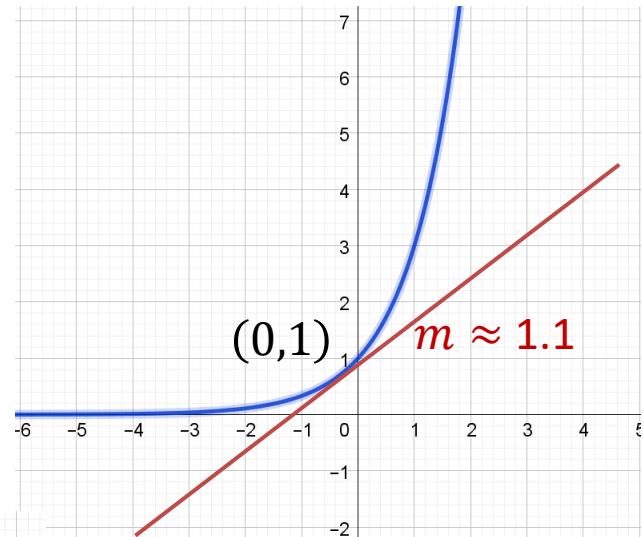
The irrational number $e \approx 2.71828$

The following figures show that the slope of the tangent lines of the graph $y = 2^x$ and $y = 3^x$ at the point $(0,1) \approx 0.7$ and 1.1 respectively.

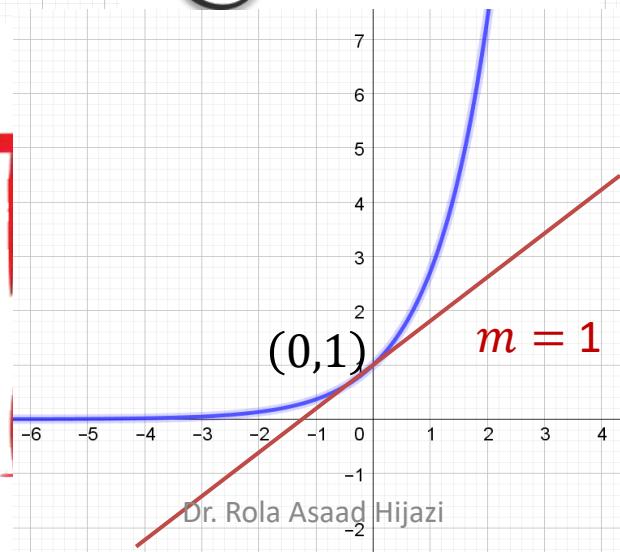
1 $y = 2^x$



2 $y = 3^x$



3 $y = e^x$



e : is an irrational number such that the slope of the tangent line of $y = a^x$ at $(0,1) = 1$.

$$e \approx 2.71828$$

Natural Exponential Function

$$f(x) = e^x$$

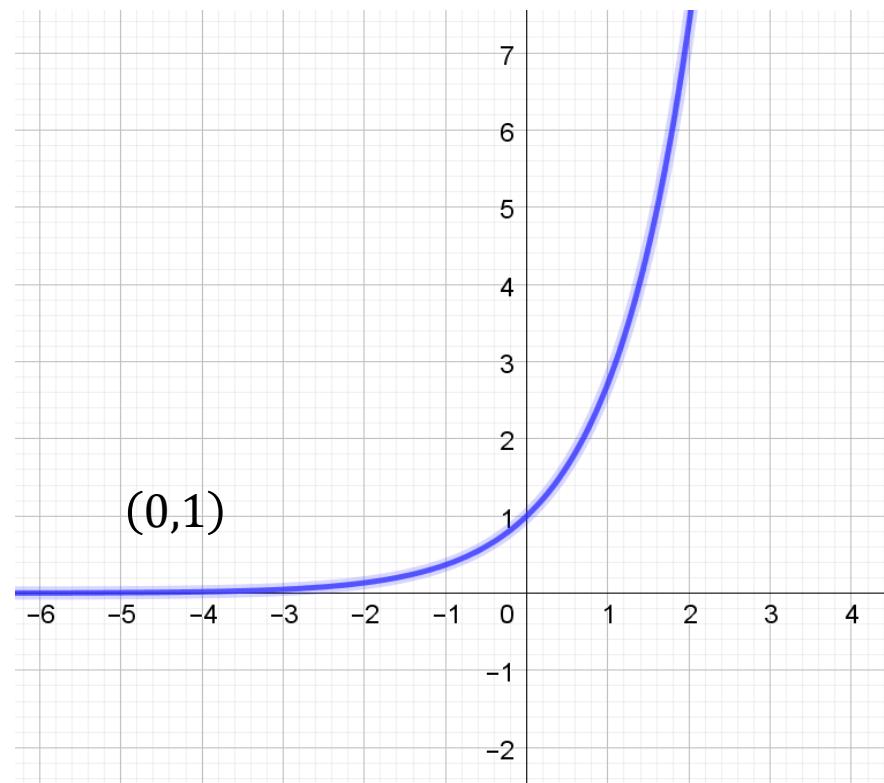
$$D_f = \text{---}$$

$$R_f = \text{---}$$



note

e^x is always positive



Exercise 2

Use the law of exponents to rewrite and simplify the expression:

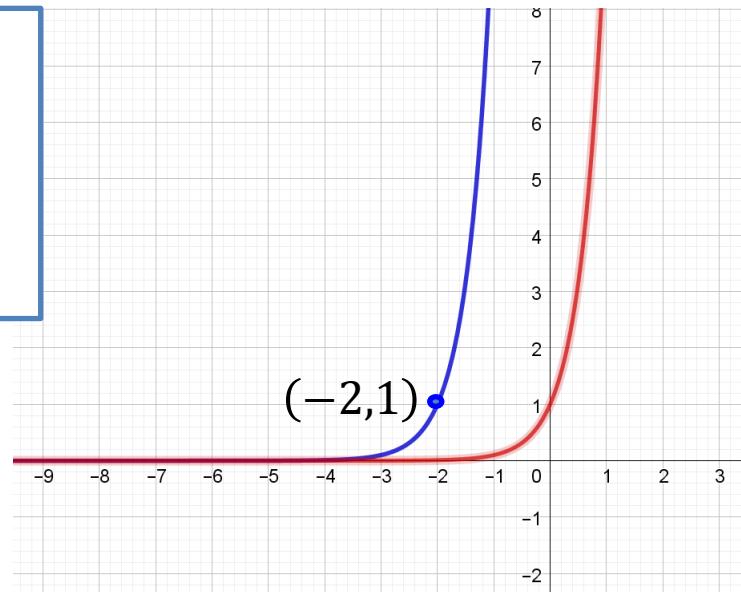
a) $8^{4/3} =$

b) $x(3x^2)^3 =$

Exercise 13

Sketch $y = 10^{x+2}$

Solution



$D_f =$
 $R_f =$

Exercise 19

Find the domain of the following functions

a) $f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$

solution

b) $f(x) = \frac{1+x}{e^{\cos x}}$

solution

Exercise 20

a) $g(t) = \sqrt{10^t - 100}$

solution

b) $g(t) = \sin(e^t - 1)$

solution



homework

1, 3, 17