CALCULUS 110

1.1 Four ways to represent a function

Verbally – numerically - visually (graph) - algebraically

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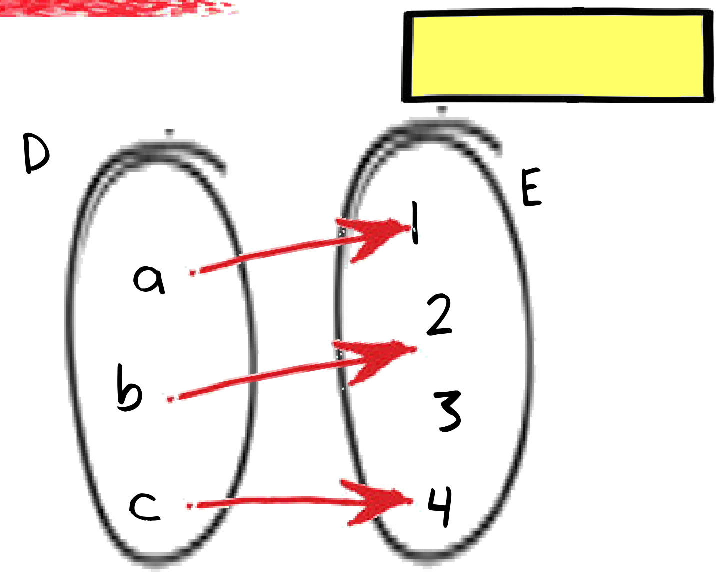
Four ways to represent a function

Function

A Function f is a rule that assigns to each element $x \in D$ exactly one element, called $f(x) \in E$.

We usually consider functions for which the sets D and E are sets of real numbers.
$f: \mathbb{R} \to \mathbb{R}$.

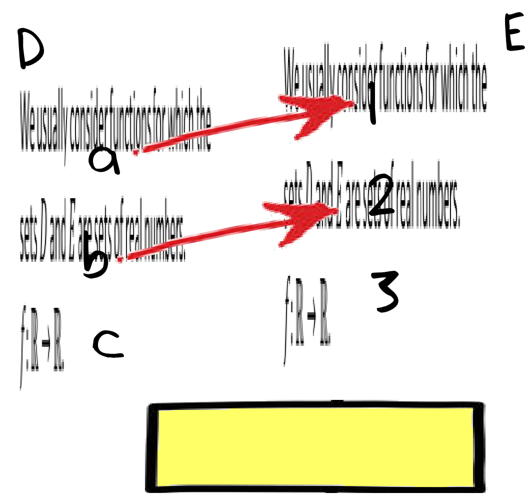


Domain

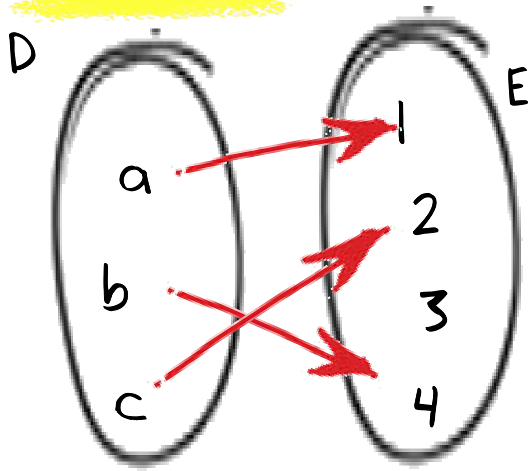
is the value of x

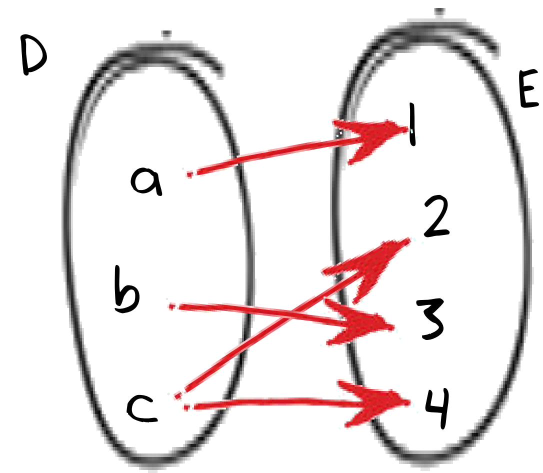
Range

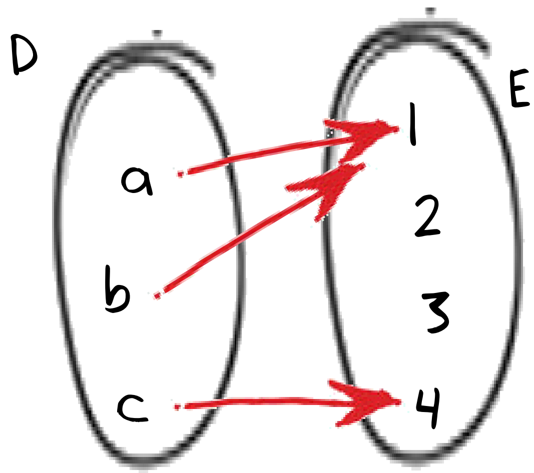
is the set of all possible values of $f(x)$ as x varies throughout the domain.



Four ways to represent a function







Domain = -- --

Range = -- --

Codomain = -- --

Read Example 1

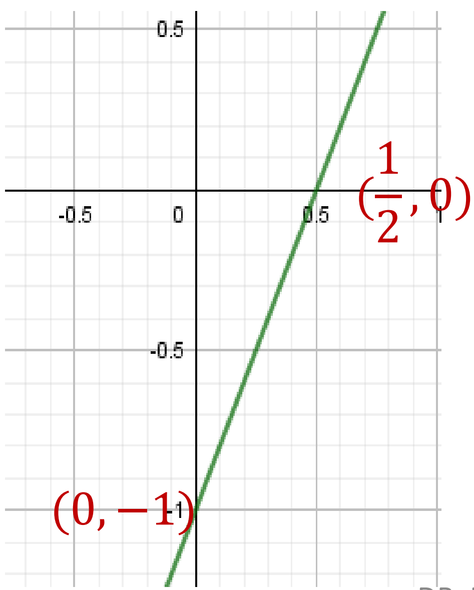
Example 2

Sketch the graph and find the domain and range.

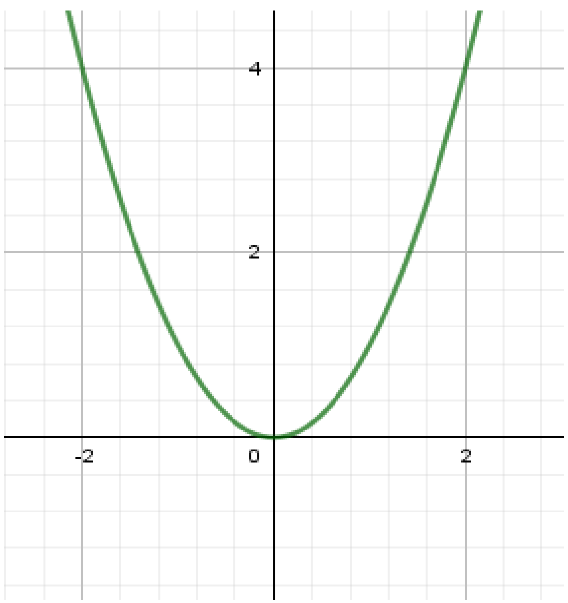
$(a)\ f(x) = 2x - 1$

$(b)\quad g(x) = x^2$

Solution:



Solution:

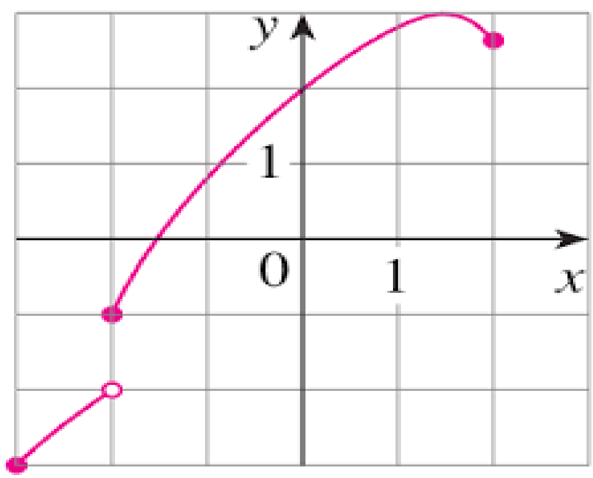


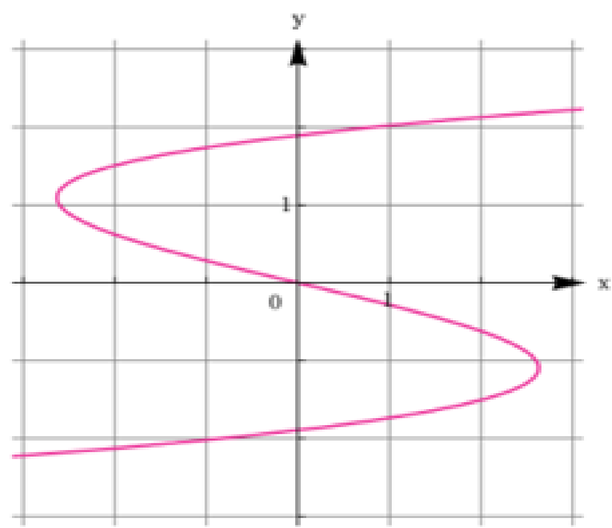
The vertical line Test:

A curve in the xy plane is the graph of a function of x if no vertical line intersects the curve more than once.

Exercises 7,9

Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.





domain=

range=

Example 6 Find the domain of

$(a) \quad f(x) = \sqrt{x+2}$

Solution

$(b) \quad g(x) = \dfrac{1}{x^2 - x}$

Exercise 31 Find the domain of

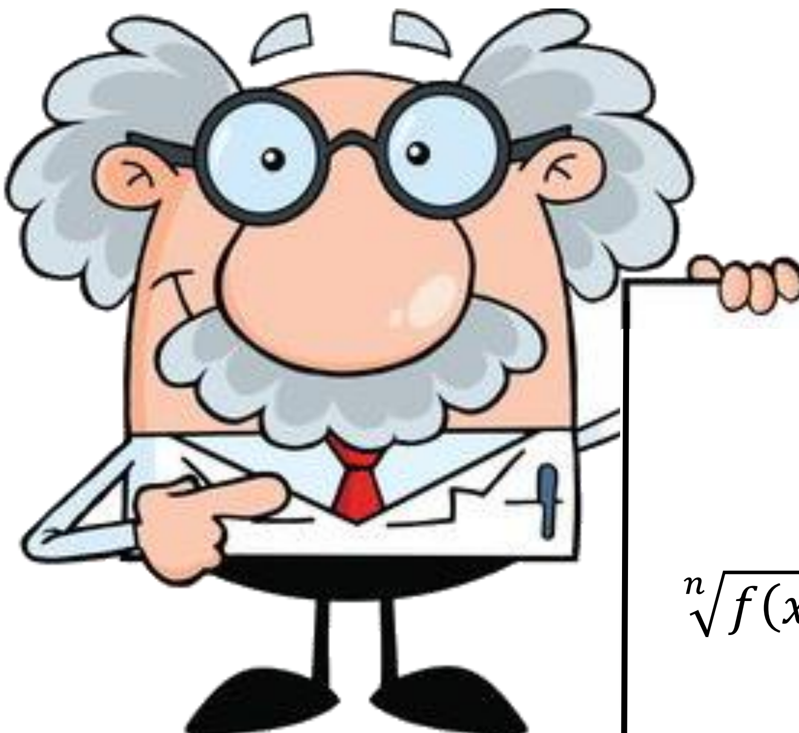
$$f(x) = \frac{x+4}{x^2-9}$$

Solution

Exercise 33 Find the domain of

$$f(t) = \sqrt[3]{2t-1}$$

Solution



$$\sqrt[n]{f(x)}:\begin{cases} domain = \_\_ & if\ \ n = odd \\ & if\ \ n = even \end{cases}$$

Exercise 34

Find the domain of

$$g(t) = \sqrt{3-t} - \sqrt{2+t}$$

Solution:-

Exercise 40

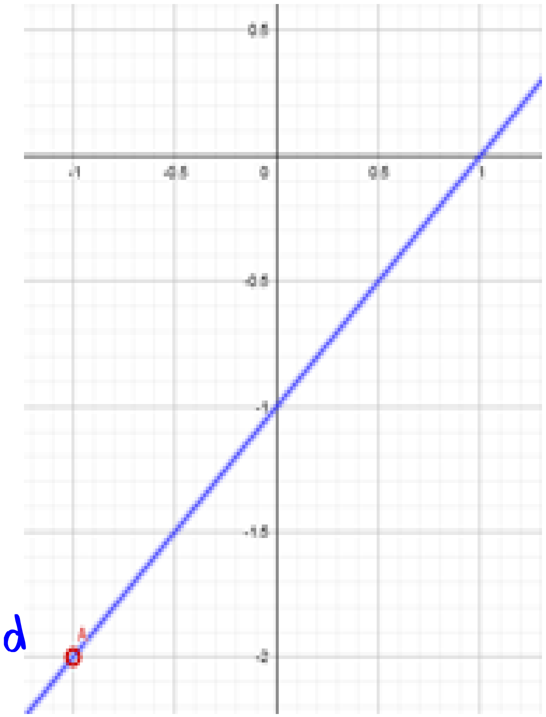
Find the domain and sketch the graph

$$g(t) = \frac{t^2 - 1}{t + 1}$$

Solution:-

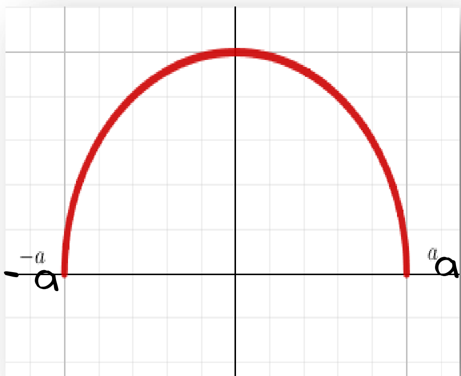
X= -1

undefined



Important Functions

$(1)\ f(x) = \sqrt{a^2 - x^2}$



domain=--- -

range=--- -

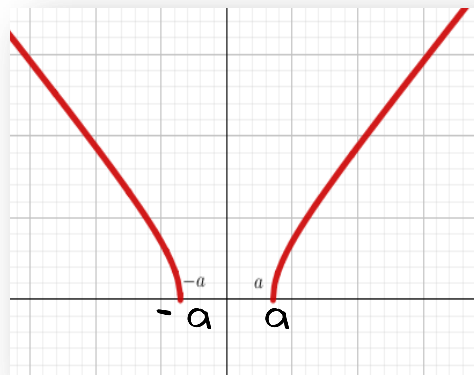
example

$h(x) = \sqrt{4 - x^2}$

domain

Range

$(2) f(x) = \sqrt{x^2 - a^2}$



domain=----- -

range=--- -

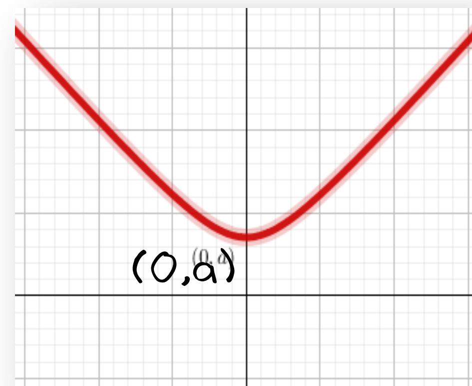
example

$f(x) = \sqrt{x^2 - 9}$

domain

Range

$(3) f(x) = \sqrt{x^2 + a^2}$



domain = --- -

range = --- -

example

$f(x) = \sqrt{x^2 + 5}$

domain

Range

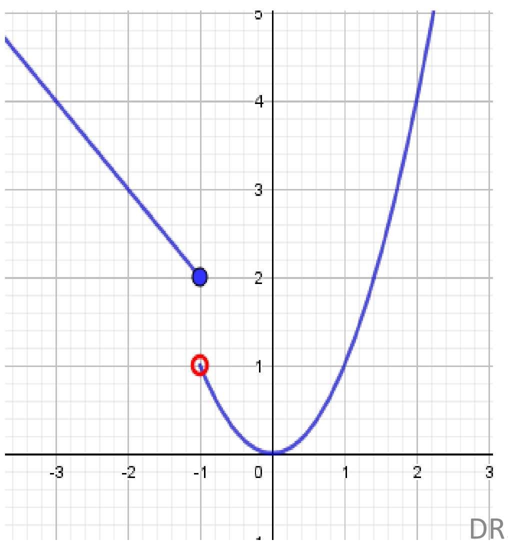
Piecewise function is a function that is defined by different formulas in different parts of their domains.

Example 7

$$f(x) = \begin{cases} 1 - x, & x \le -1 \\ x^2, & x > -1 \end{cases}$$

Evaluate $f(-2)$, $f(-1)$ and $f(0)$ and sketch the graph.

Solution:-



Example 8

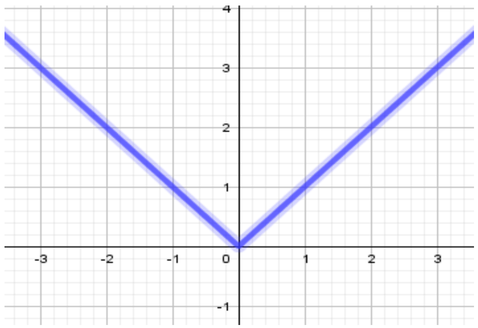
(a) Sketch the graph of

$$f(x) = |x| = \begin{cases} x & if \;\; x \ge 0 \\ -x & if \;\; x < 0 \end{cases}$$

Solution:-

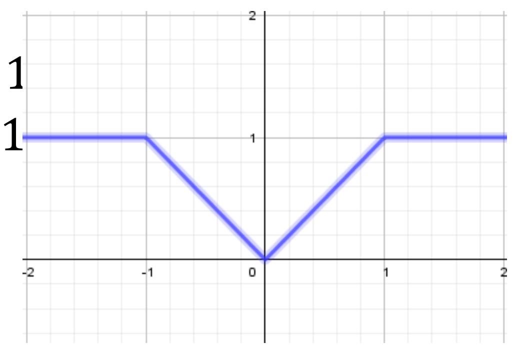
Domain

Range



(b) Sketch the graph of

$$f(x) = \begin{cases} |x| & if \;\; |x| \le 1 \\ 1 & if \;\; |x| > 1 \end{cases}$$

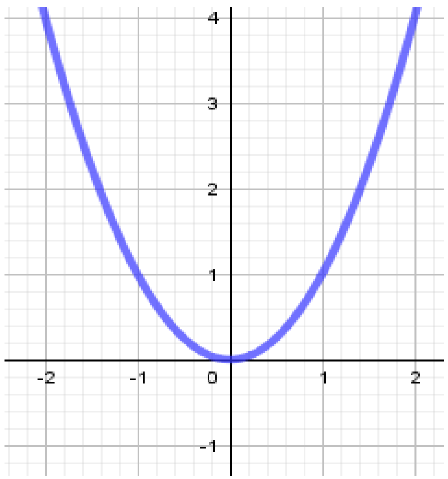


Summetry

Even function

if $f(-x) = f(x)\ \forall\ x \in D$, then
f is called an even function.

The graph is symmetric with
respect to the _ _ _ _ _ _ _ .

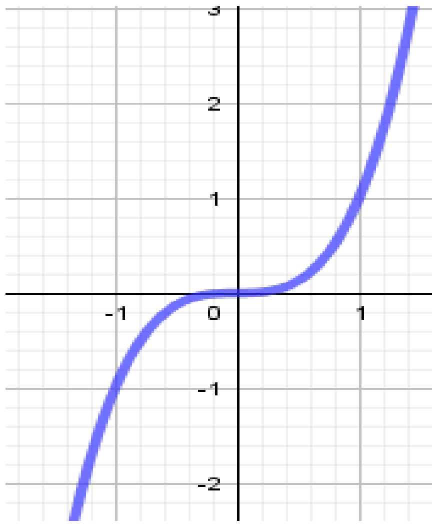


$f(x) = x^2$ is even

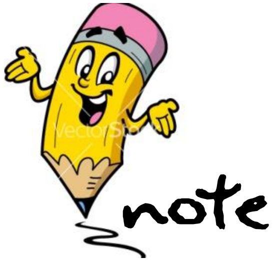
Odd function

If $f(-x) = -f(x)\ \forall\ x \in D$,
then f is called an odd function.

The graph is **symmetric with**
respect to the _ _ _ _ _ _ _ .



$f(x) = x^3$ is odd



Special Properties

Adding:

The sum of two even functions is even
The sum of two odd functions is odd
The sum of an even and odd function is neither even nor odd (unless one function is zero).

Multiplying:

The product of two even functions is an even function.
The product of two odd functions is an even function.
The product of an even function and an odd function is an odd function.

Example 11

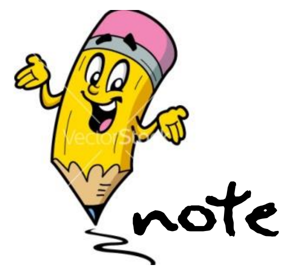
Determine whether each of the following functions is even, odd or neither even nor odd.

(a) $f(x) = x^5 + x.$

Solution:

(b) $f(x) = 1 - x^4$

Exercise 76 $f(x) = x|x|$



(a) $f(x) = c$ is _ _ _ _ _

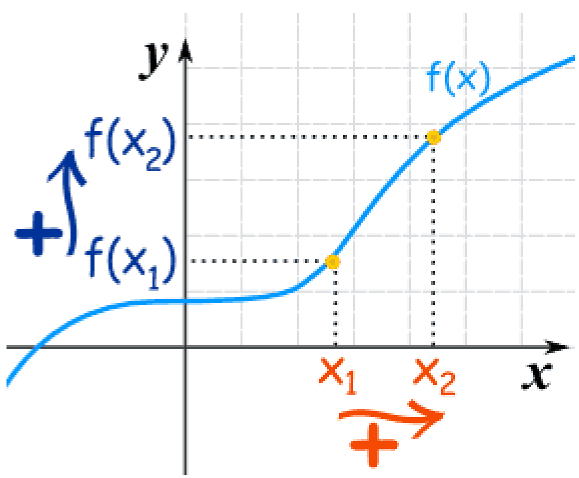
(b) $f(x) = |x|$ is _ _ _ _ _

(c) $f(x) = x^n$ is $\begin{cases} \text{— — —} & if\ n \text{ is even} \\ \text{\_ \_ \_ \_ \_,} & if\ \ n \text{ is odd} \end{cases}$

Increasing and Decreasing

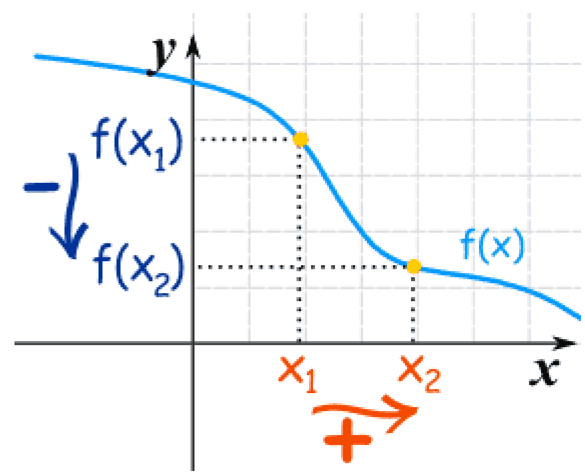
A function f is called **increasing** on an interval I

if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.



A function f is called **decreasing** on an interval I

$if\ f(x_1) > f(x_2)\ ,\ x_1 < x_2$ in I .

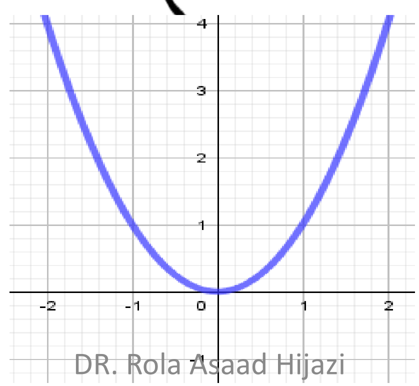


Example :

$f(x) = x^2$

$f(x)$ is decreasing $(-\infty, 0]$.

$f(x)$ is increasing $[0, \infty)$.



7 - 10, 32 - 34, 41, 46, 73 - 78