CALCULUS 110

(4.3) How Derivatives Affect the shape of a Graph

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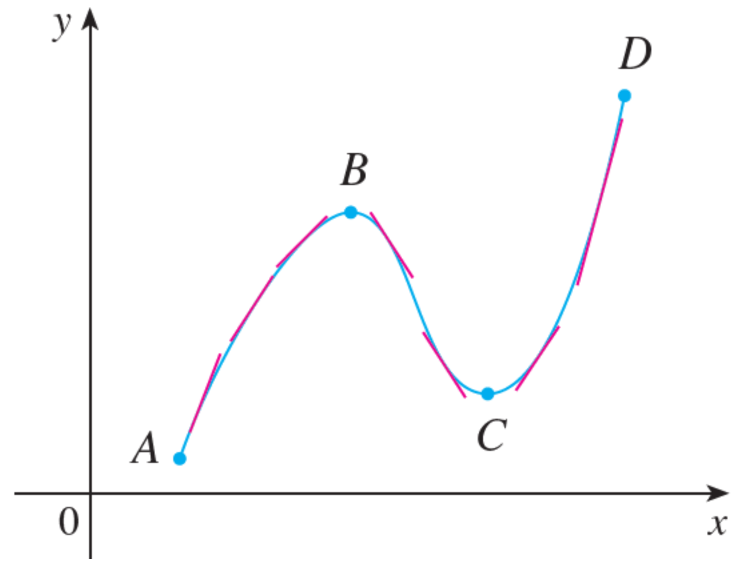
How Derivatives Affect the shape of a Graph

A What Does f' Say About f ?

Increasing and Decreasing Test.

- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval.



Recall that:

We can check the increasing and decreasing by :

- Definition in section 1.1
- graph
- $f'(x)$

Example Find the intervals on which f is increasing or decreasing.

(a) $f(x) = e^{x^3+2x}$

solution

Since $f'(x) = e^{3x^2+2} > 0$
$\forall x \in \mathbb{R}$

Thus f is increasing on $\mathbb{R}$.

(b) $f(x) = \dfrac{1}{e^x + 1}$

a. $f(x)$ is increasing on $\mathbb{R}$
b. $f(x)$ is decreasing on $\mathbb{R}$
c. $f(x)$ is not monotonic
d. $f(x)$ is increasing on (−∞,−1)

solution

Since $f'(x) = \dfrac{-e^x}{(e^x+1)^2} < 0$

Thus f is decreasing on $\mathbb{R}$.

Monotonic means increasing or decreasing

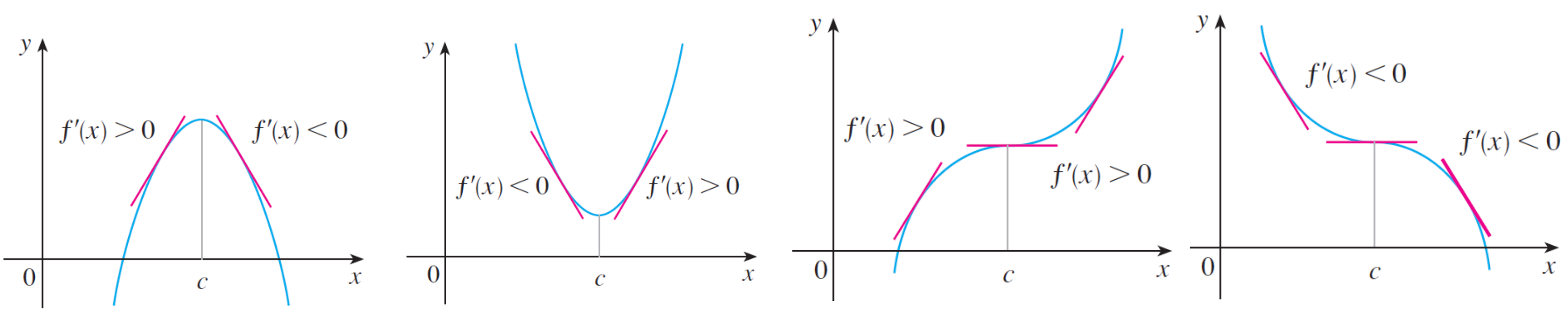
The 1st Derivative Test:

Suppose that c is a critical number of a continuous function f:

1. If f' changes from +ve to - ve at c, then f has a local maximum at c.

2. If f' changes from - ve to +ve at c, then f has a local minimum at c.

3. If f' doesn't change sign then f has no local maximum or minimum at c.



(a) Local maximum (b) Local minimum (c) No max. or min. (d) No max. or min.

Example 1+2

(a) Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.

(b) Find the local minimum and maximum

solution

(a) $f'(x) = 12x^3 - 12x^2 - 24x$

First find the critical numbers when $f'(x) = 0$

$$\Rightarrow 12x(x^2 - x - 2) = 0$$

$$\Rightarrow 12x(x-2)(x+1) = 0$$

The critical numbers are:

$$\Rightarrow x = 0 \qquad x = 2 \qquad x = -1$$

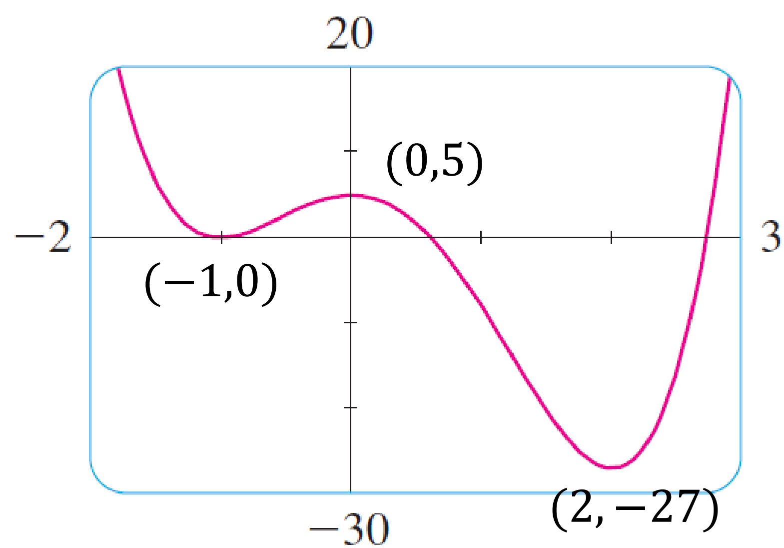
		-1		0		2	
$12x$	-		-		+		+
$(x-2)$	-		-		-		+
$(x+1)$	-		+		+		+
$f'(x)$	-		+		-		+
	dec.		Inc.		dec.		Inc.

(b) By the 1st derivative test:

$f(-1) = 0$ is a local minimum.

$f(0) = 5$ is a local maximum.

$f(2) = -27$ is a local minimum



Use $f'(x)$ *to find*

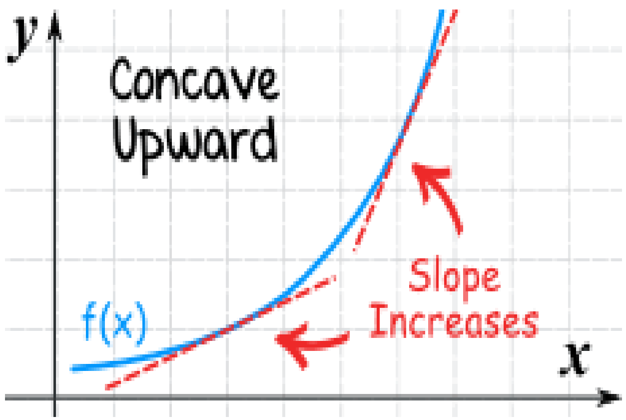
(1) Critical numbers

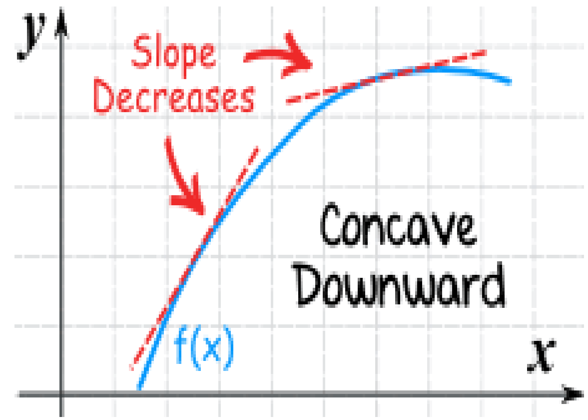
(2) Increasing and decreasing intervals

(3) Local maximum and minimum.

(4) Absolute maximum and minimum

What does f'' Say About f?





- The above Figures shows that the graphs of two functions are increasing on (a, b), But the look different because they bend in different directions.

- How can we distinguish between these two types of behavior?

Concave upward

If the graph of f lies above all of its tangents on interval , then its called **concave upward on I**.

Concave downward

If the graph of f lies below all of its tangents on I, it is called **concave downward on I.**

Concavity Test

- If $f''(x) > 0$ for all $x \in I$ then the graph of f is concave upward on I.
- If $f''(x) < 0$ for all $x \in I$ then the graph of f is concave downward on I.

Inflection point

A point P on a curve $y = f(x)$ is called an inflection point if f:

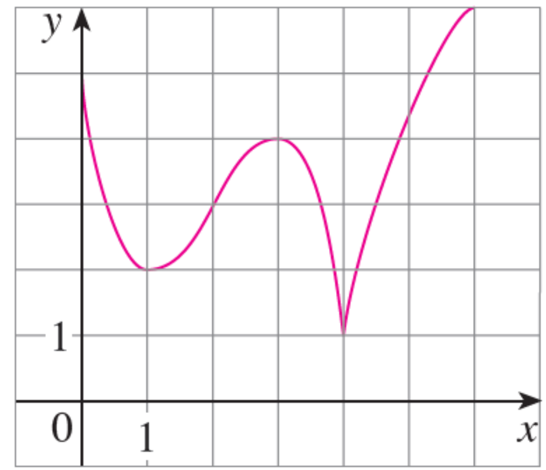
- is continuous there
- The curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Exercise 1

Use the given graph of f to find the following.
(a) The open intervals on which f is increasing.
(b) The open intervals on which f is decreasing.
(c) The open intervals on which f is CU.
(d) The open intervals on which f is CD.
(e) The coordinates of the points of inflection.

Solution

1



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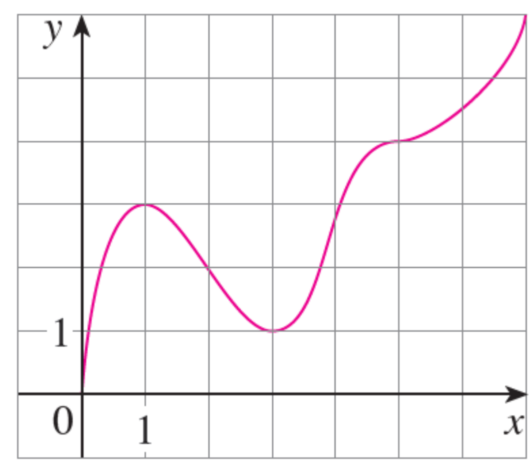


Example 6

Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection, and local maximum and minimum. Use this information to sketch the curve.

solution

(A) Use the 1st derivative test to find local minima and maxima.

Step 1: Find the critical points:

$f'(x) = 4x^3 - 12x^2 = 0$

$\Rightarrow 4x^2(x-3) = 0$

$\Rightarrow x = 0,\ x = 3$ are critical points $\in D_f$

		0		3	
$4x^2$	+		+		+
$(x-3)$	-		-		+
$f'(x)$	-		-		+
	Dec.		Dec.		Inc.

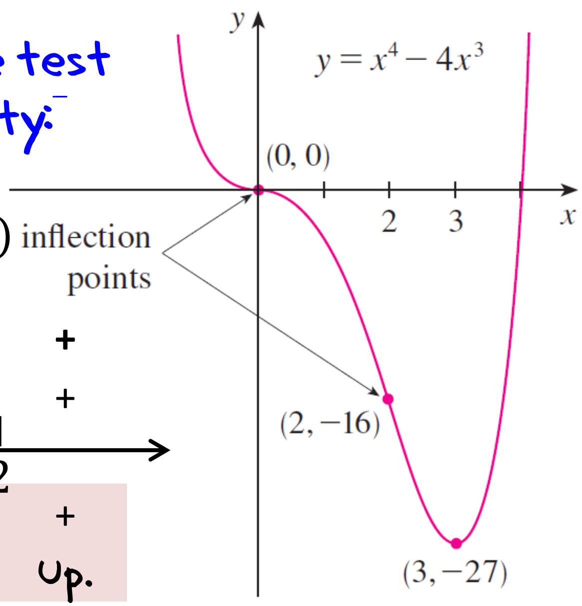
No Local max. or mini. at $x = 0$.

$f'(x)$ changes from $-$ve to $+$ve at $(3, f(3))$.

$\Rightarrow f(3) = -27$ is a local minimum.

(B) Use 2nd derivative test to check the concavity:

$f''(x) = 12x(x-2)$



		0		2	
$12x$	-		+		+
$(x-2)$	-		-		+
$f'(x)$	+		-		+
	up.		D.		up.

(C) Inflection points

(1) $y = x^4 - 4x^3$ is continuous

(2) Since the curve changes from CU to CD at 0 and from CD to CU at 2 thus $(0,0)$ and $(2, f(2))$ are inflection points.

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