CALCULUS 110

(3.6) Derivatives of Logarithmic Functions

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Derivatives of Logarithmic Functions

In this section we use implicit differentiation to find the derivatives of the logarithmic functions $y = \log_a x$ and, in particular, the natural logarithmic function $y = \ln(x)$. [It can be proved that logarithmic functions are differentiable]

1. $y = \log_a x \qquad y' = \dfrac{1}{x \ln a}$
2. $y = \log_a g(x) \qquad y' = \dfrac{1}{g(x)} \cdot g'(x)$
3. $y = \ln(x) \qquad y' = \dfrac{1}{x}$
4. $y = \ln(g(x)) \qquad y' = \dfrac{1}{g(x)} \cdot g'(x)$
5. $y = ln\lvert x \rvert \qquad y' = \dfrac{1}{x} \forall x \neq 0$

There are 4 cases for exponents and bases

- $y = a^n \; a, n = constants \Rightarrow y' = 0$
- $y = (f(x)^n \Rightarrow y' = n(f(x)^{n-1} \cdot f'(x)$
- $y = a^{g(x)} \Rightarrow y' = a^{g(x)}(ln a) g'(x)$
- $y = (f(x))^{g(x)}$

  Apply ln on both sides

Example 1 Find y', $y = \ln(x^3 + 1)$

Example 2 Find y', $y = \ln(sinx)$

Example 6 Find y', $y = ln|x|$

$$y = \ln|x| = \begin{cases} \ln(-x), & x < 0 \\ \ln(x), & x > 0 \end{cases}$$

Example 4

Find y', $y = \log_{10}(2 + sinx)$

Example 5 Find y', $y = \ln\left(\frac{x+1}{\sqrt{x-2}}\right)$

Example 3 Find y', $y = \sqrt{lnx}$

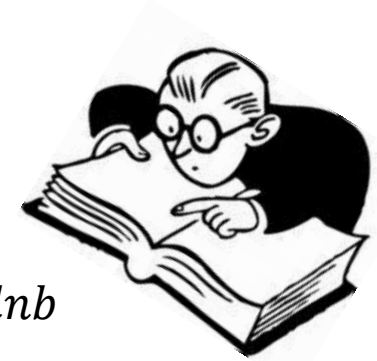
Logarithmic Differentiation

The calculation of derivatives of complicated functions involving products, quotients or powers, can often be simplified by taking logarithms. The method used in the following example is called **Logarithmic Differentiation**.

Example 7 Find y', $y = \dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

solution

Recall that:



- $ln\dfrac{a}{b} = lna - lnb$
- $lnab = lna + lnb$
- $lna^r = rlna$

Example 8

Find y', $y = x^{\sqrt{x}}$

solution

Steps in Logarithmic Differentiation

- Take natural logarithms of both sides, and use the laws of logarithms to simplify.
- Differentiate implicitly with respect to x.
- Solve the resulting equation for y'.

Exercise 52 Find y', $if\ x^y = y^x$

Exercise 19 $y = ln(e^{-x} + xe^{-x})$

homework

3 - 15 (odd), 21, 31, 43, 47