



CALCULUS II

(3.4) The Chain Rule

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3.4 The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f \circ g(x) = f(g(x))$ is differentiable at x and:

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Rules for Derivative using chain rule:

$$(1) \quad F(x) = (f(x))^n \Rightarrow F' = n(f(x))^{n-1} \cdot f'(x)$$

$$(2) \quad F(x) = \sqrt{f(x)} \Rightarrow F' = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$(3) \quad F(x) = \sin(f(x)) \Rightarrow \cos(f(x)) \cdot f'(x)$$

$$(4) \quad F(x) = e^{f(x)} \Rightarrow F'(x) = e^{f(x)} \cdot f'(x)$$

$$(5) \quad y = a^{g(x)} \Rightarrow y' = a^{g(x)} \ln a \cdot g'(x)$$

Example 1

Find $F'(x)$, $F(x) = \sqrt{x^2 + 1}$

Example 2

Find y' and y''

- (a) $y = \sin(x^2)$ (b) $y = \sin^2(x)$

Example 3

$$y = (x^3 - 1)^{100}$$

Example 4

Find $f'(x)$ where

$$f(x) = \frac{1}{\sqrt[3]{x^2 + x + 1}}$$

solution

Example 5

Find $g'(t)$ where $g(t) = \left(\frac{t-2}{2t+1}\right)^9$

solution

Example 6

Find y' where $y = (2x + 1)^5(x^3 - x + 1)^4$

Example 7

Differentiate

$$(a) y = e^{\sin x}$$

$$(b) y = e^{\sec 3\theta}$$

Example 8

Differentiate

$$f(x) = \sin(\cos(\tan x))$$

Exercise 23

$$y = e^{\tan a}$$

Exercise 53

Find an equation of the tangent line to the curve at the given point:

$$y = \sin(\sin x), (\pi, 0)$$

solution

Exercise

$$y = 2^{\cos x^2}$$

solution



1–15 (odd), 44, 47, 48