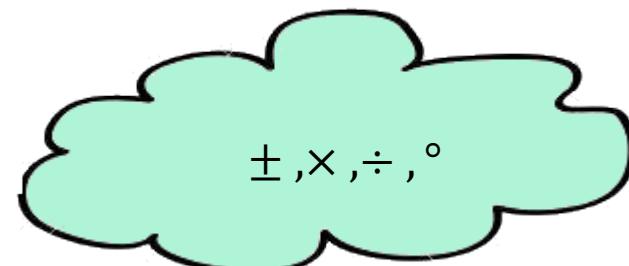


# CALCULUS II

## (1.3) New functions from old functions

Dr. Rola Asaad Hijazi

In this section we start with the basic functions we discussed in Section 1.2 and obtain new functions by



We also show how to combine pairs functions by standard arithmetic operations + composition.

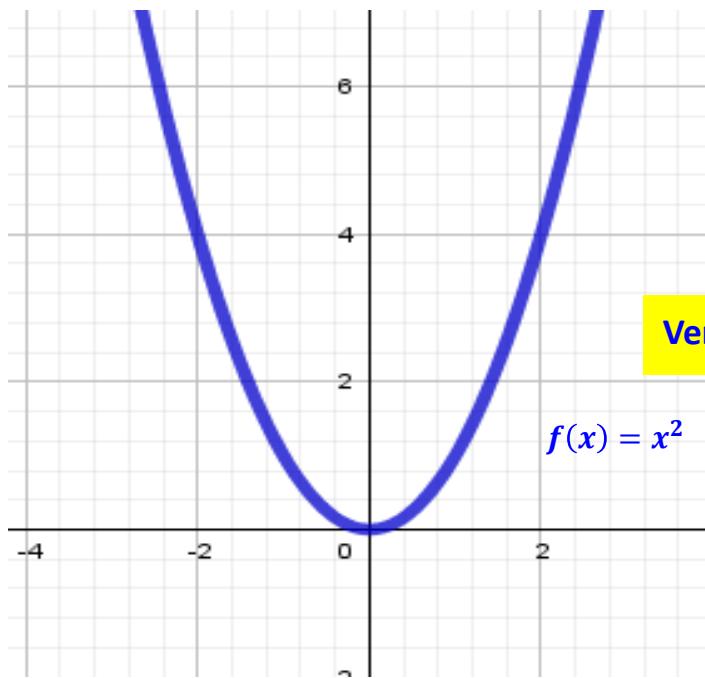
## 1.3 - TRANSFORMATIONS OF FUNCTIONS

### (A) VERTICAL SHIFT (UPWARD)

Suppose  $D_f = [a, b]$ ,  $R = [d, e]$  and  $c > 0$ . To obtain the graph of:

(i)  $g(x) = f(x) + c$  shift the graph of  $y = f(x)$  a distance  $c$  units **upward**.

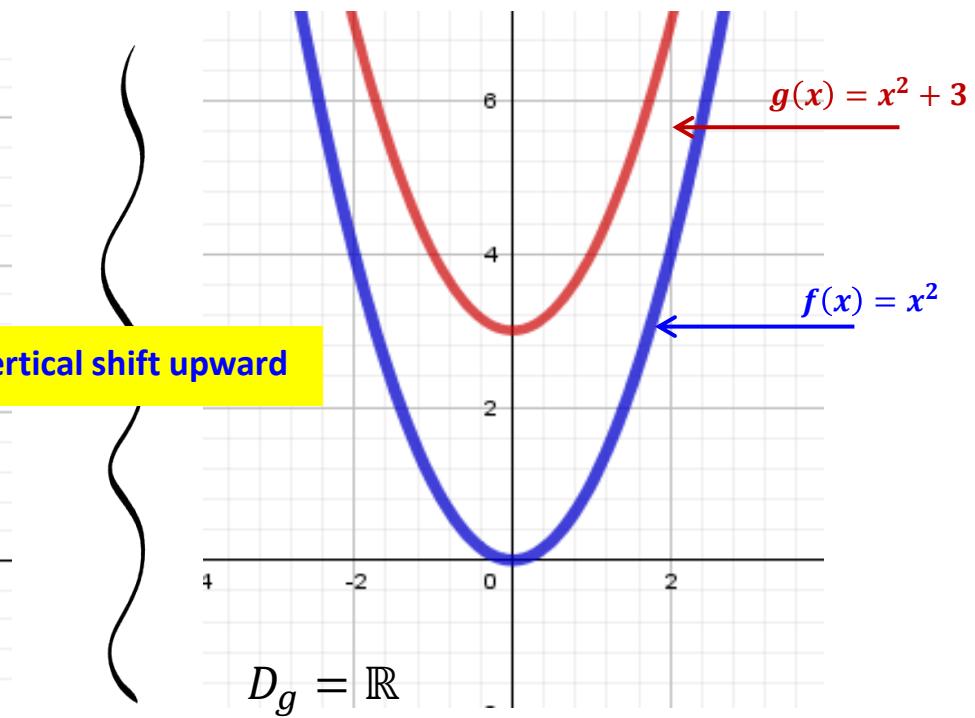
$$D_g = \underline{\hspace{2cm}}, \quad R_g = \underline{\hspace{2cm}}$$



Vertical shift upward

$$f(x) = x^2$$

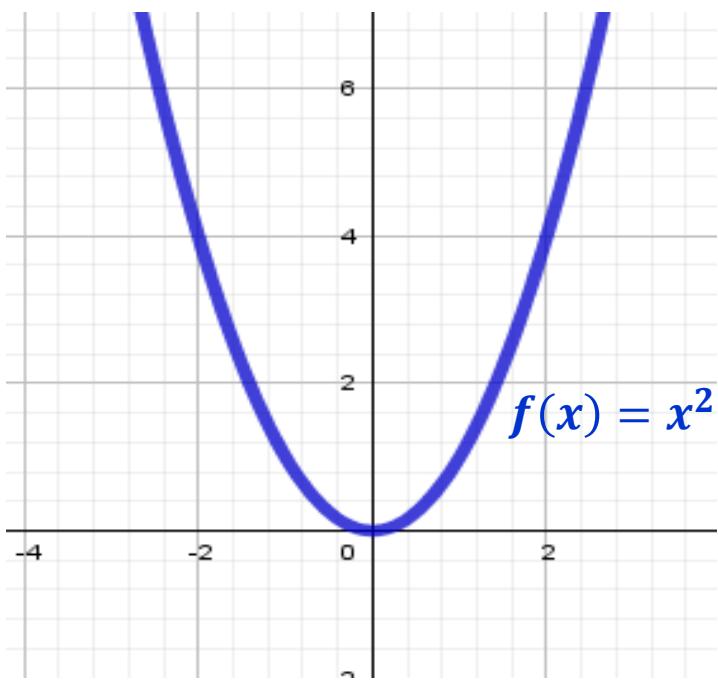
$$D_f = \mathbb{R} \quad R_f = [0, \infty)$$



$$D_g = \mathbb{R}$$

$$R_g = [d + c, e + c] = \underline{\hspace{2cm}}$$

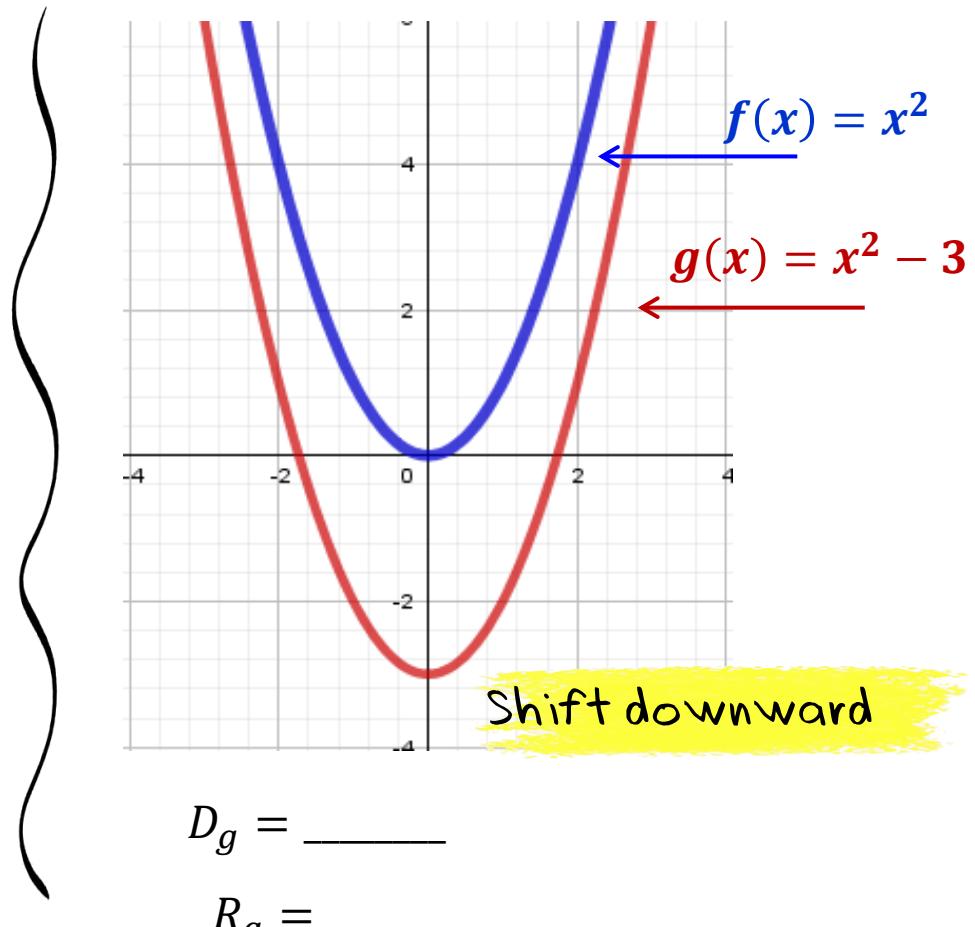
(A) VERTICAL SHIFT (downward)



$$D_f = \underline{\hspace{2cm}}$$

$$R_f = \underline{\hspace{2cm}}$$

(ii)  $g(x) = f(x) - c$  shift the graph of  $y = f(x)$  a distance  $c$  units **downward**  
 $D_g = \underline{\hspace{2cm}}$ ,  $R_g = \underline{\hspace{2cm}}$

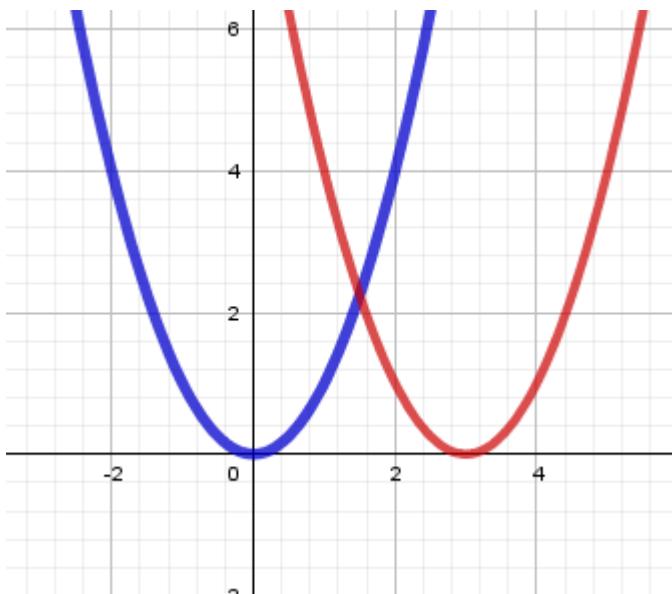


$$D_g = \underline{\hspace{2cm}}$$

$$R_g = \underline{\hspace{2cm}}$$

## (B) HORIZONTAL SHIFT

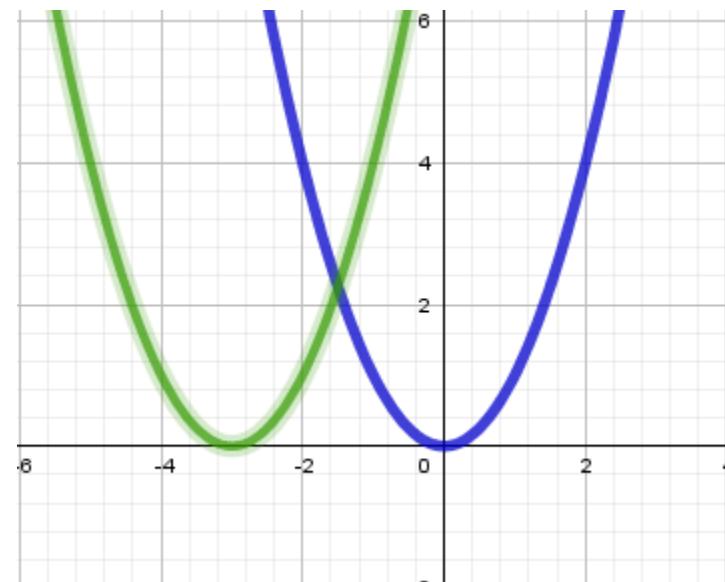
(i)  $y = f(x - c)$  shift the graph of  $y = f(x)$  a distance  $c$  units to the **right**.



$$f(x) = (x - 3)^2$$

$$D_f = \underline{\hspace{2cm}} \quad R_f = \underline{\hspace{2cm}}$$

(ii)  $y = f(x + c)$  shift the graph of  $y = f(x)$  a distance  $c$  units to the **left**



$$f(x) = (x + 3)^2$$

$$D_f = \underline{\hspace{2cm}} \quad R_f = \underline{\hspace{2cm}}$$

# VERTICAL AND HORIZONTAL REFLECTING

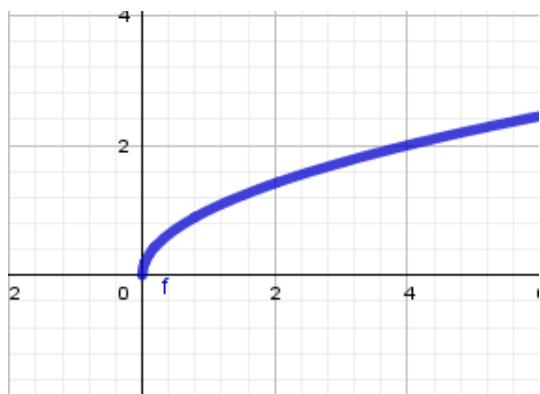
to obtain the graph of:

$y = -f(x)$  reflect the graph  $f(x)$   
about the  $x$ -axis (vertical)

$$D = \underline{\hspace{2cm}} R = \underline{\hspace{2cm}}$$

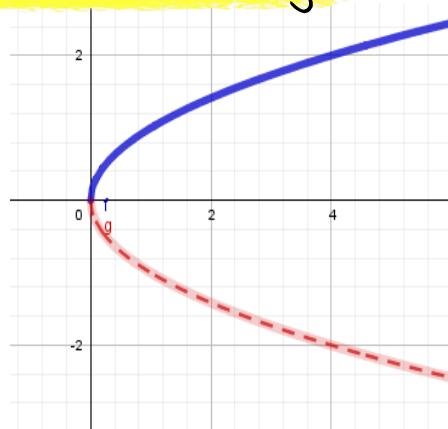
$y = f(-x)$  reflect the graph  $f(x)$   
about the  $y$ -axis (horizontal)

$$D = \underline{\hspace{2cm}} R = \underline{\hspace{2cm}}$$



$$\begin{aligned} f(x) &= \sqrt{x} \\ &= [0, \infty) \\ R &= [0, \infty) \end{aligned}$$

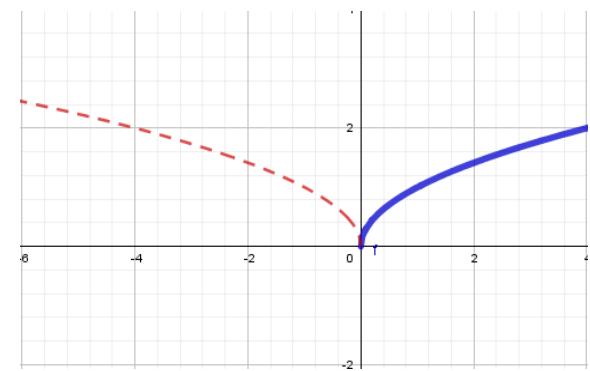
Vertical reflecting



$$f(x) = -\sqrt{x}$$

$$D = \underline{\hspace{2cm}}$$

$$R = \underline{\hspace{2cm}}$$



$$f(x) = \sqrt{-x}$$

$$D = \underline{\hspace{2cm}}$$

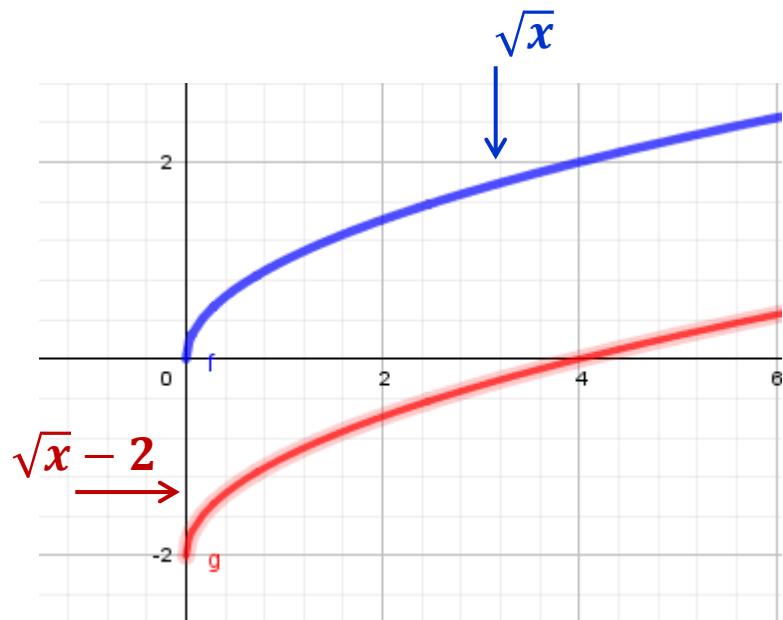
$$R = \underline{\hspace{2cm}}$$

## Example 1

Sketch the graphs of the following :

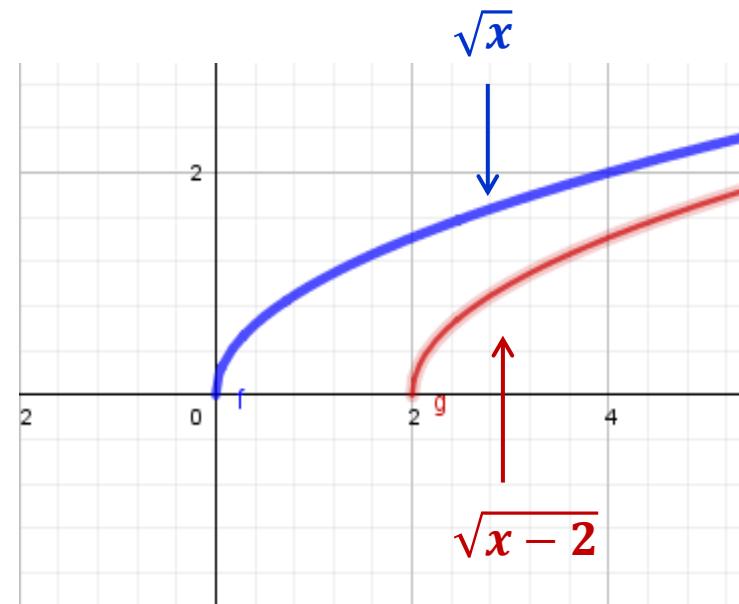
$$y = \sqrt{x} - 2$$

Shift up or down?



$$y = \sqrt{x - 2}$$

Shift left or right?



$$D_f = \dots, R_f = \dots$$

$$D_f = \dots, R_f = \dots$$

## Example 2

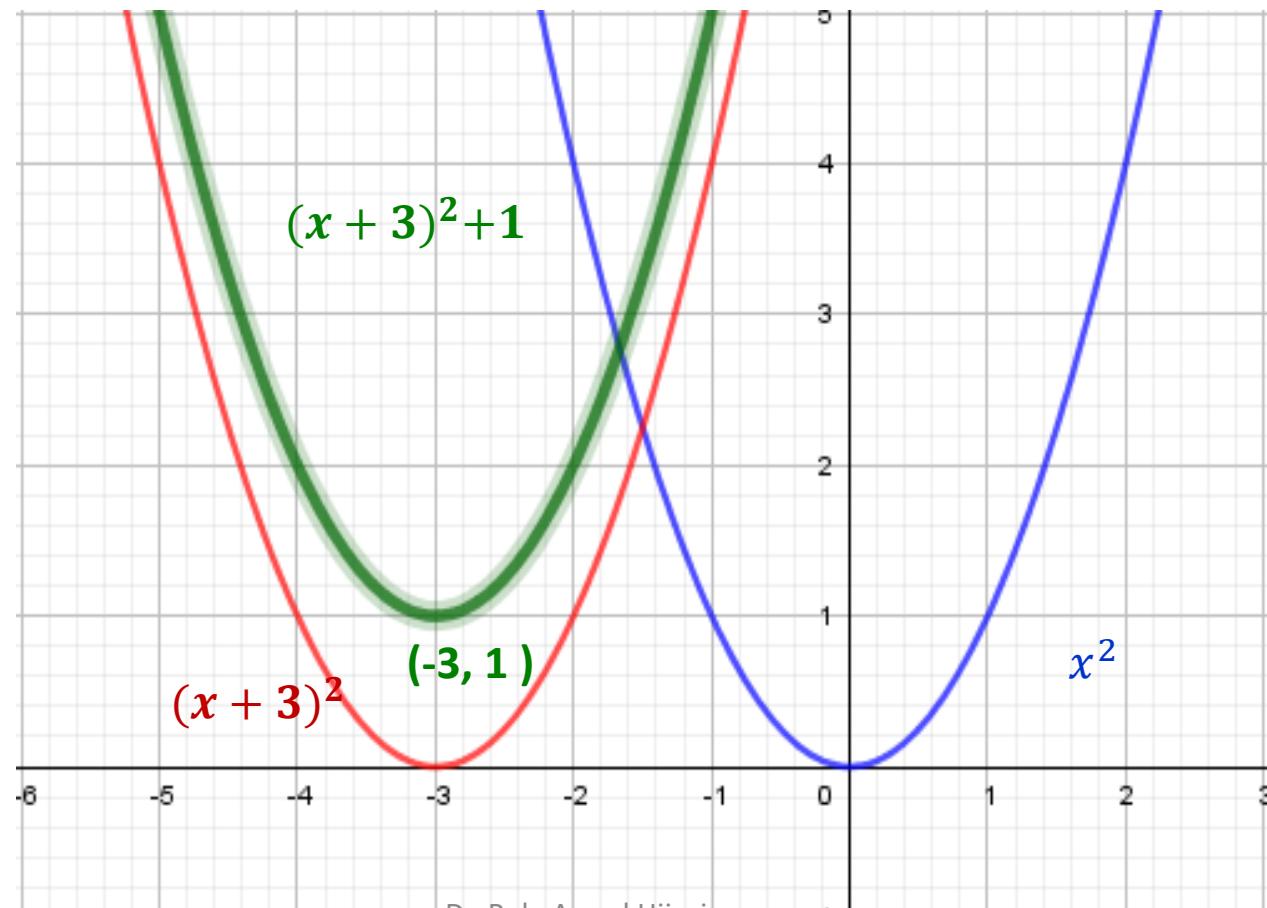
Sketch the graph of  $x^2 + 6x + 10$ .

Add and Abstract  $(\frac{1}{2}6)^2$ ,  
we get  $f(x) = (x + 3)^2 + 1$

Shift  $x^2$  : -----

Shift  $(x + 3)^2$  : -----

$D = \text{-----}$ , Range = -----

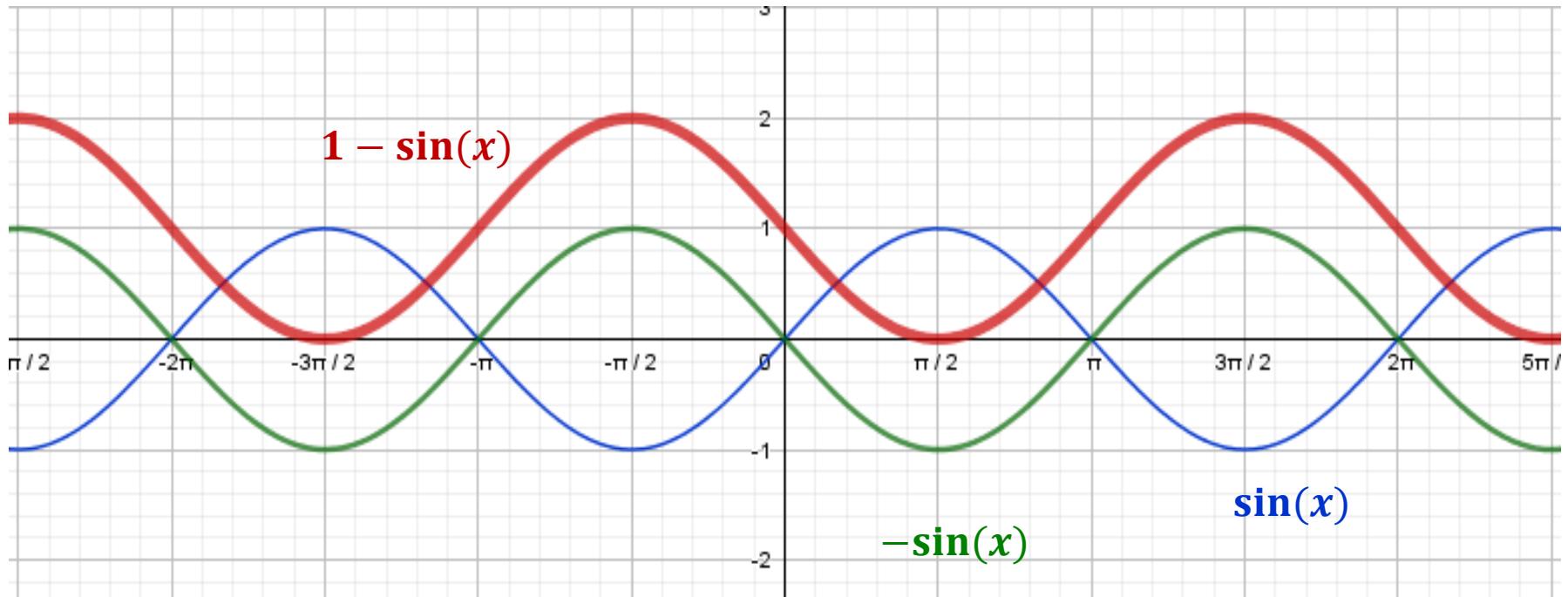


## Example 3(a)

Sketch the graph of (a)  $1 - \sin(x)$ .

Reflect  $\sin(x)$  -----

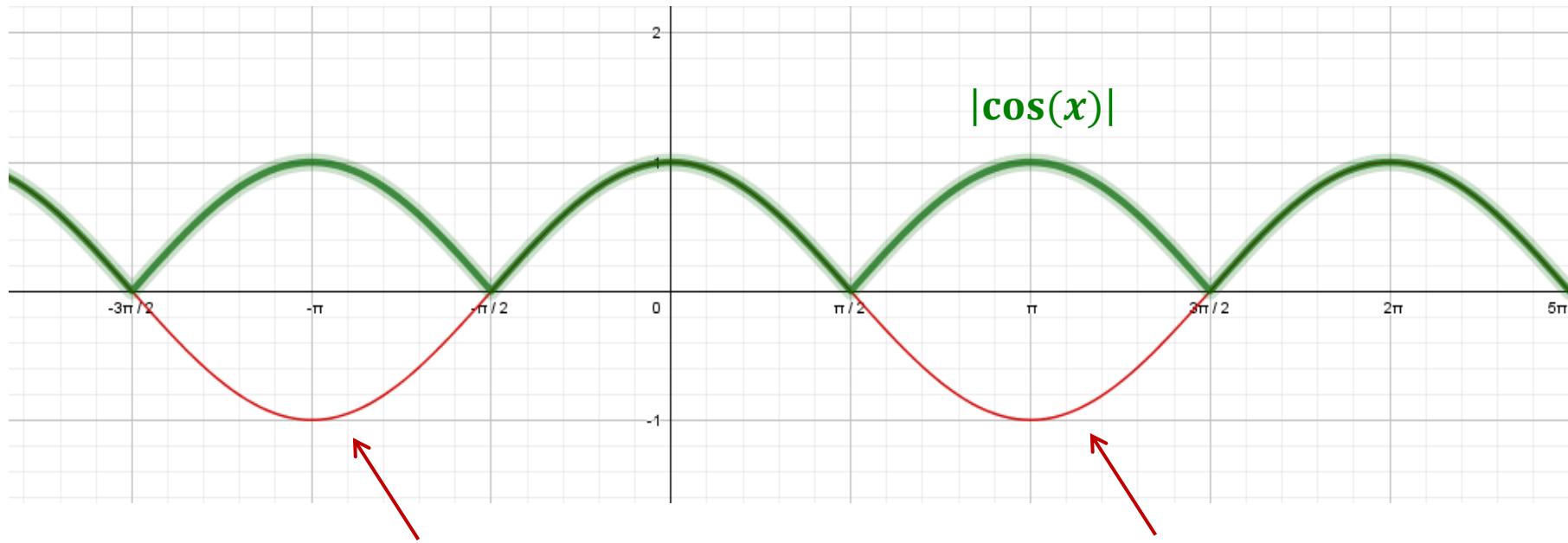
Shift  $(-\sin(x))$ : -----



$$D = \mathbb{R}, \quad Range = \text{-----}$$

## Example 3(b)

Sketch the graph of  $|\cos(x)| = \begin{cases} \cos(x) & \text{if } \cos x \geq 0 \\ -\cos(x) & \text{if } \cos(x) < 0 \end{cases}$



$\cos(x) < 0$ , reflect about x-axis

$\cos(x) < 0$ , reflect about x-axis

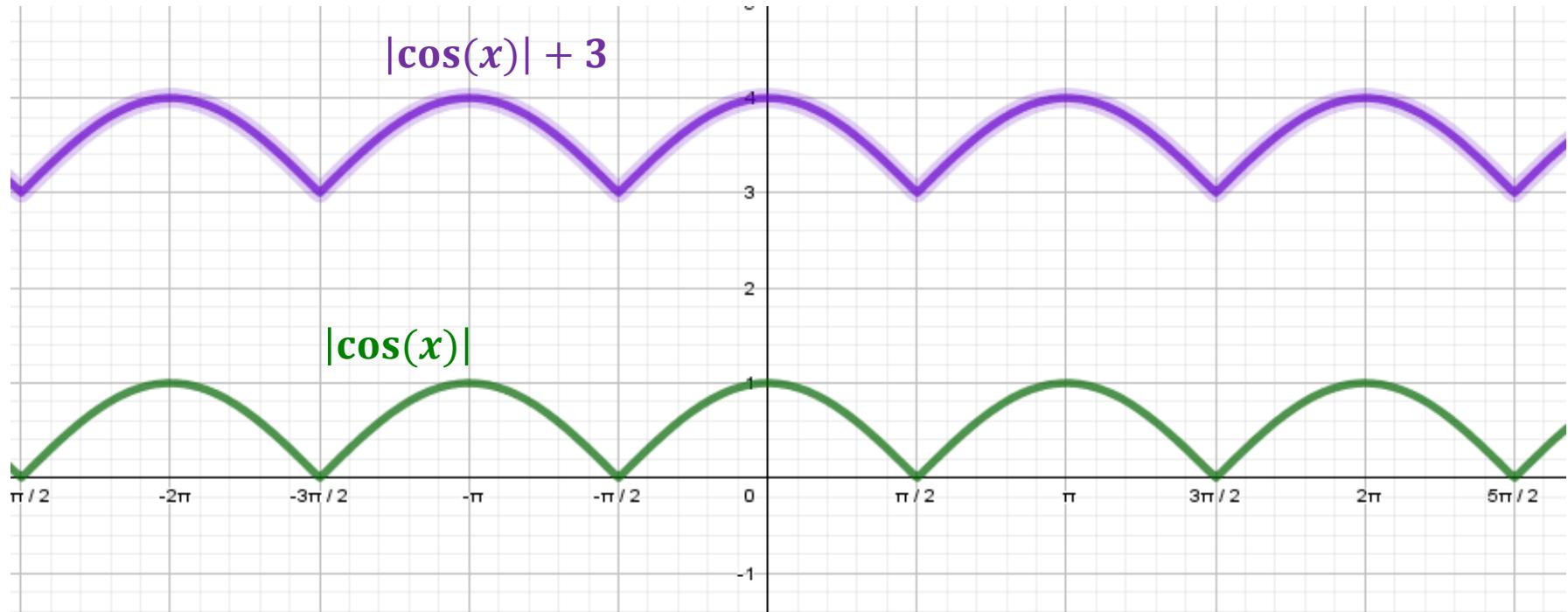
Domain =-----,

Range =-----

### Example 3(c)

Sketch the graph of  $|\cos(x)| + 3$

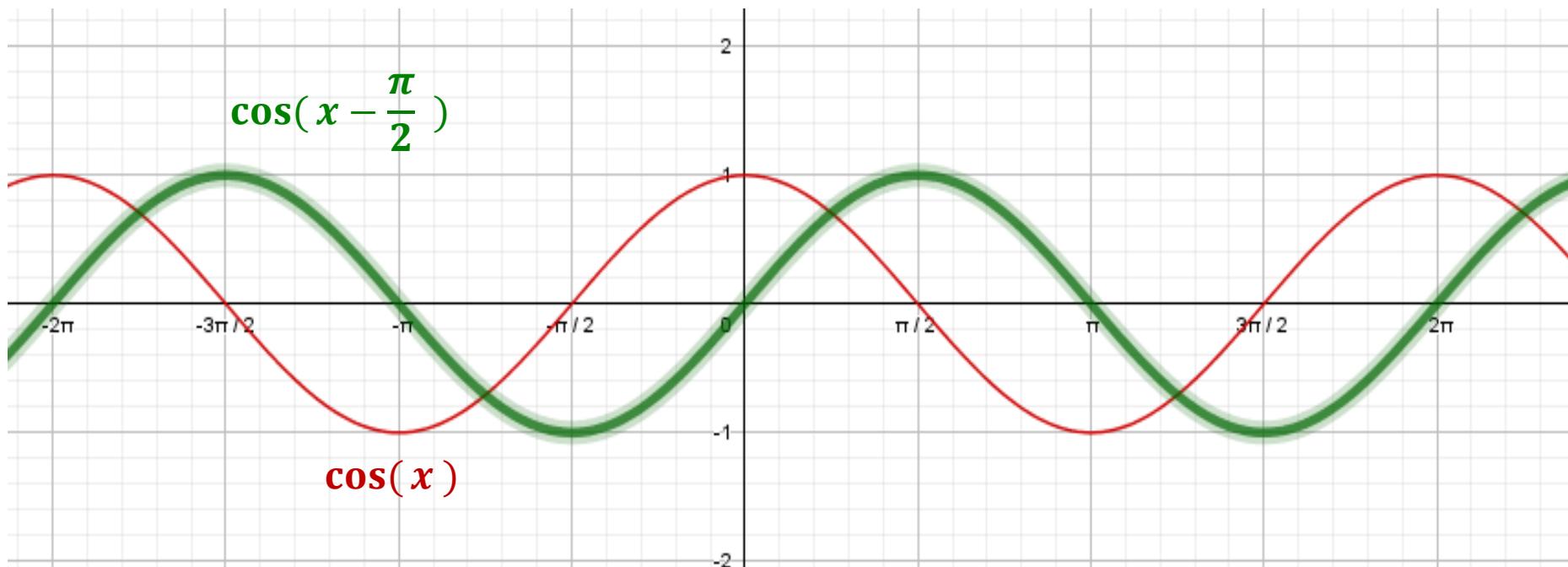
Shift  $|\cos(x)|$  -----



Domain =  $\mathbb{R}$ , Range =-----].

### Example 3(d)

Sketch the graph of  $(d) \cos(x - \frac{\pi}{2})$



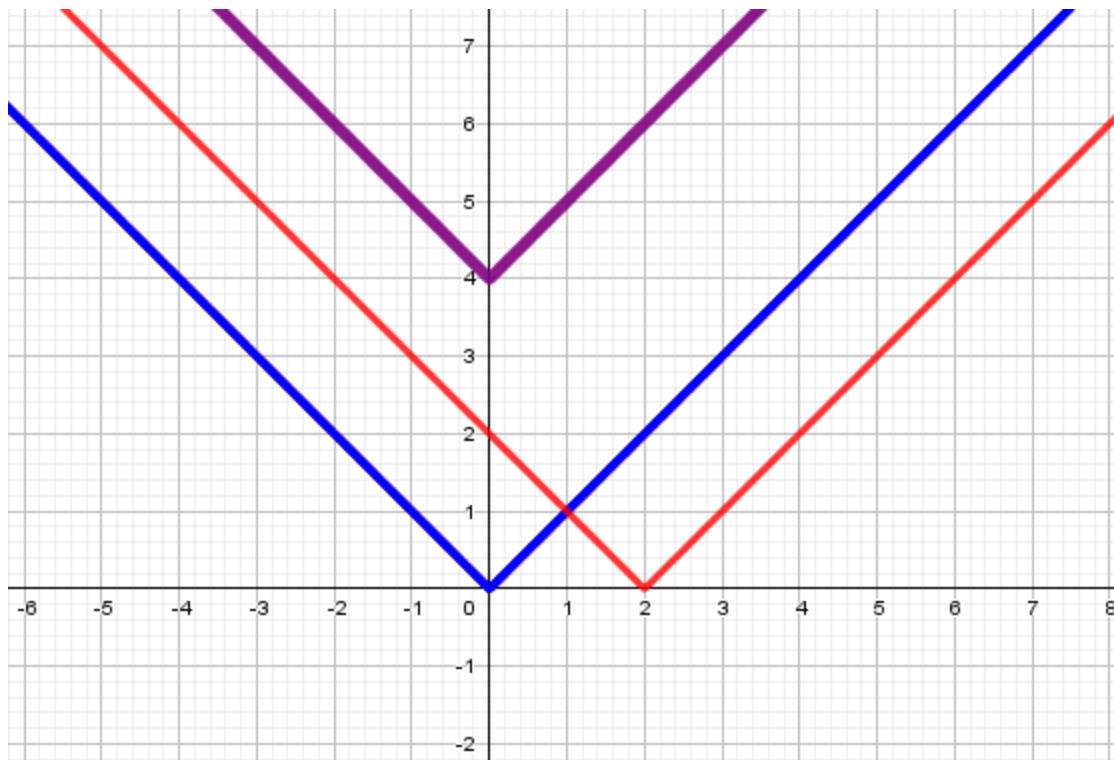
Domain =  $\mathbb{R}$ , Range =-----

## Extra examples

$$y = |x|$$

$$y = \text{-----}$$

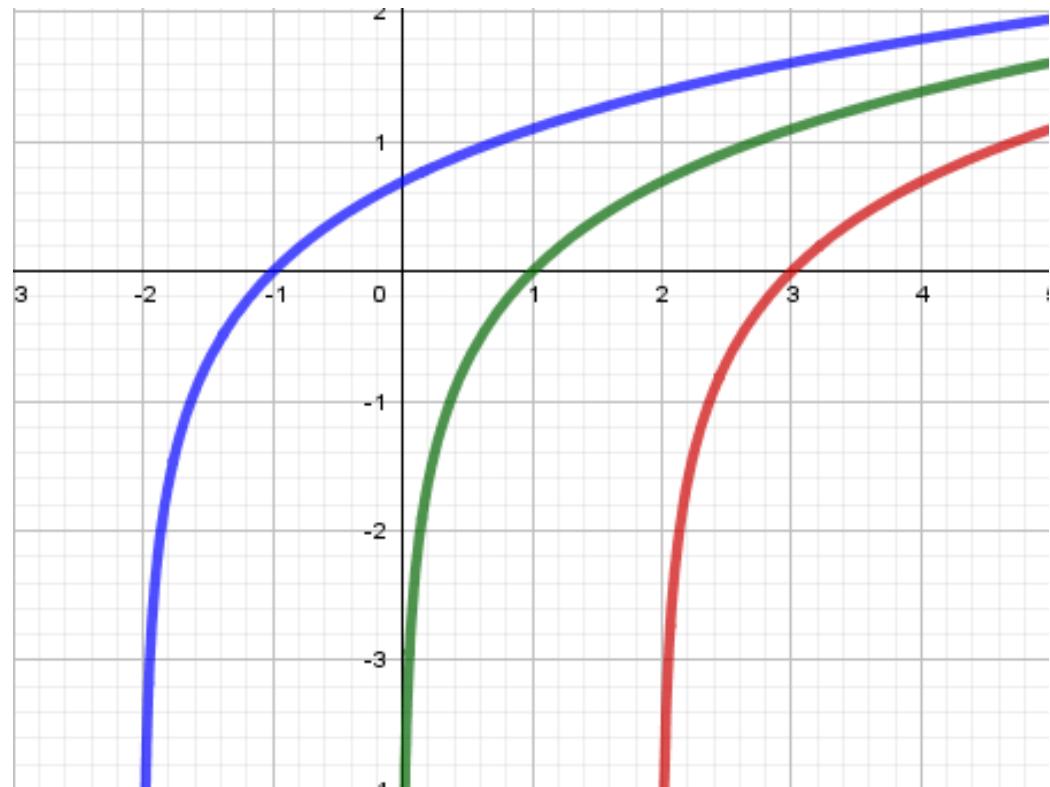
$$y = \text{_____}$$



$$y = \ln(x)$$

$$y = \text{-----}$$

$$y = \text{-----}$$



# Combinations of Functions

$\pm, \times, \div, \circ$

Two functions  $f$  and  $g$  can be combined to form new functions  $f \pm g$ ,  $fg$  and  $f/g$ ,  $f \circ g$  as follows:

1  $(f \pm g)(x) = f(x) \pm g(x)$

2  $(f \cdot g)(x) = f(x) \cdot g(x)$

3  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

4  $(f \circ g)(x) = (f(g(x)))$



remark

If  $D_f = A$ ,  $D_g = B$  then:

1  $D_{f+g} = A \cap B$

2  $D_{f \cdot g} = A \cap B$

3  $D_{f/g} = A \cap B - \{x: g(x) = 0\}$

4  $D_{f \circ g} = D_{f \circ g} \cap D_g$

5  $D_{g \circ f} = D_{g \circ f} \cap D_f$

## Example 7

Let  $f(x) = \sqrt{x}$ ,  $g(x) = \sqrt{2 - x}$ .

Find each function and its domain:

a)  $f(x) + g(x)$

b)  $\frac{f(x)}{g(x)}$  =

c)  $f \circ g(x) =$

d)  $g \circ f(x) =$

e)  $f \circ f(x) =$

f)  $g \circ g(x) =$

### Example 6

if  $f(x) = x^2$  and  $g(x) = x - 3$ .  
find  $f \circ g, g \circ f$ .

### Solution

### Example

$f(x) = x^2 - 1, g(x) = x - 1$

Evaluate  $\left(\frac{f}{g}\right)(x)$ , and its domain

### Solution



Don't simplify the function  $\left(\frac{f}{g}\right)$  before calculating the domain.

In general  $\frac{x^2-1}{x-1} \neq x + 1$

## Example 8

Find  $f \circ g \circ h$  of  $f(x) = x/(x + 1)$ ,  
 $g(x) = x^{10}$  and  $h(x) = x + 3$

Solution

## Example 9

Given  $F(x) = \cos^2(x + 9)$ .  
Find  $f, g$  and  $h$  s.t.  $f \circ g \circ h = F(2)$

Solution

## Exercise

(1) Write the equation for the graphs that are obtained from the graph of  $f$  as follows:

a Shift 3 units upward



b Shift 3 units downward



c 3 units to the right



d

3 units to the left



e

Reflect about the  $x - axis$



f

Reflect about the  $y - axis$



## Exercise (30)

$$f(x) = \sqrt{3 - x}, g(x) = \sqrt{x^2 - 1}$$

Find  $f \pm g, fg, f/g$  and their domains.

b

$$\frac{f}{g}(x) =$$

Solution

\_\_\_\_\_

b



homework

29 – 39 (odd), 41, 47