Chapter 8
Potential Energy and Conservation of Energy

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Learning Outcomes

After studying this chapter, you will be able to:

- ✓ **8-1 POTENTIAL ENERGY**
- ✓ Calculate the gravitational potential energy of a particle (or, more properly, a particle–Earth system).
- ✓ Calculate the elastic potential energy of a block–spring system.
- ✓ 8-2 **CONSERVATION OF MECHANICAL ENERGY**
- ✓ Identify that the mechanical energy of the system is the sum of the kinetic energies and potential energies of those objects.
- ✓ For an isolated system in which only conservative forces act, apply the conservation of mechanical energy to relate the initial potential and kinetic energies to the potential and kinetic energies at a later instant.

Potential energy

- Potential energy is energy that is associated with the configuration of a system in which a **conservative force** acts.
- When the conservative force does work W on a particle within the system, the change ΔU in the potential energy of the system is

$$\Delta U = -W$$

For a conservative force we can write

$$\boxed{\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.}$$

Gravitational Potential Energy

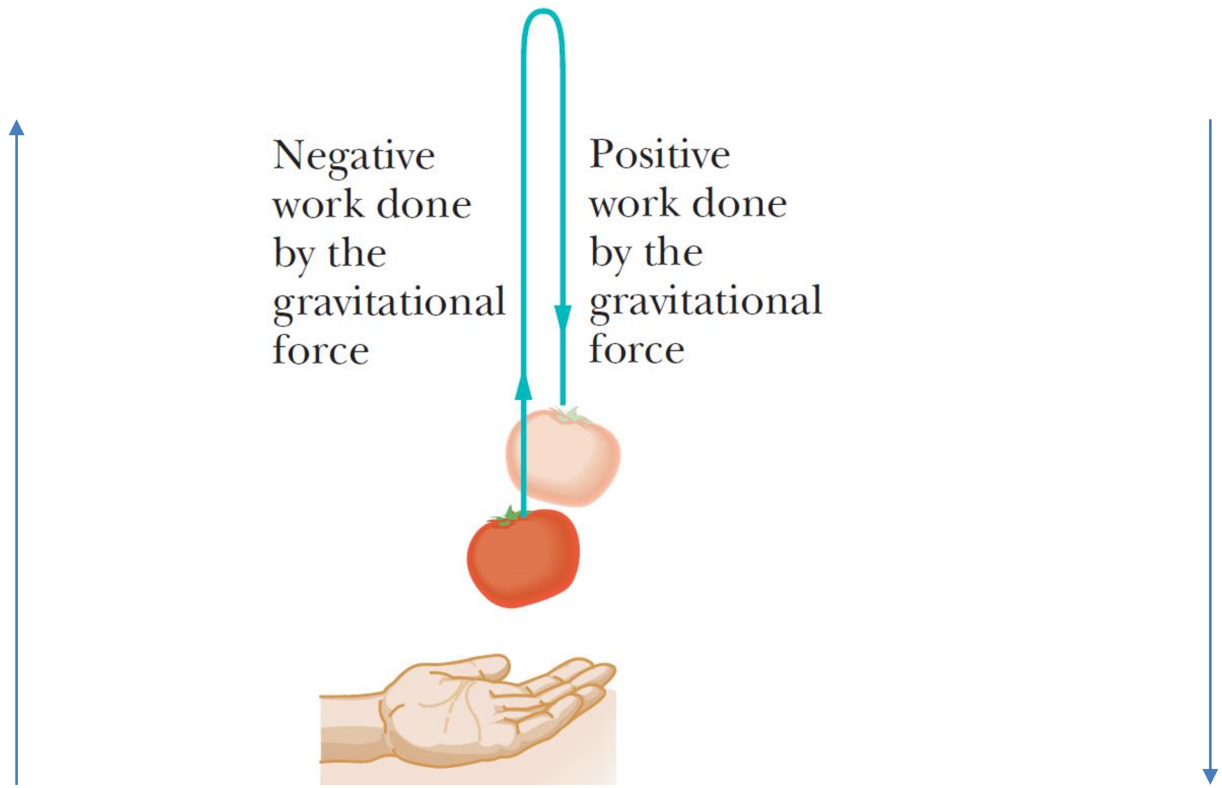
- If the particle moves from height y_i to height y_f , the change in the gravitational potential energy of the particle–Earth system is

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y$$

- If the reference point of the particle is set as $y_i = 0$ And the corresponding gravitational potential energy of the system is set as $U_i = 0$, then the gravitational potential energy U when the particle is at any height y is

$$U(y) = mgy$$

Gravitational Potential Energy



For rising

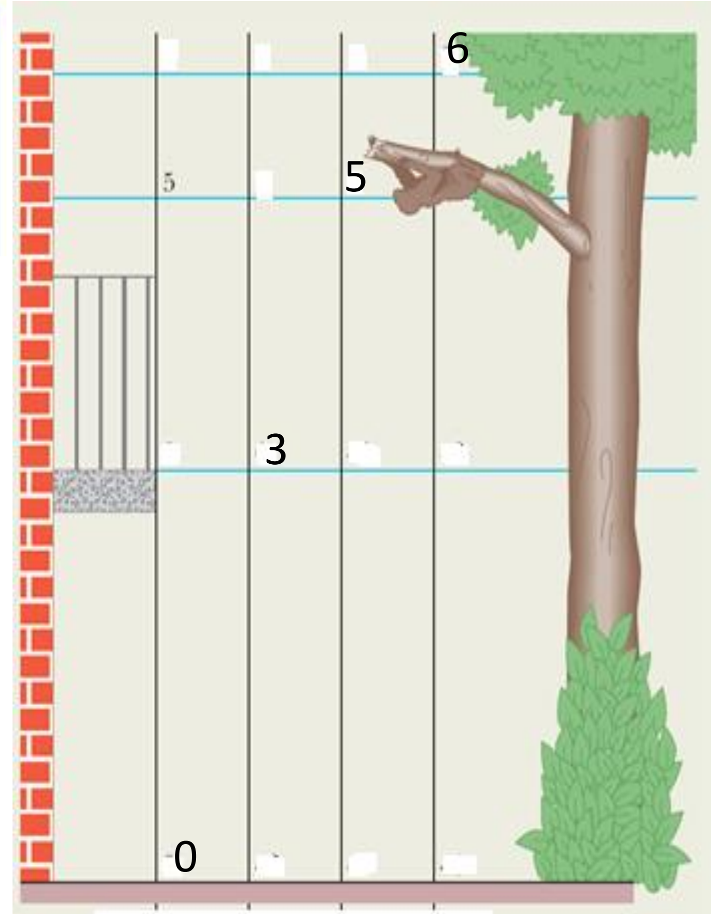
$$\Delta U = -W_g = mgd$$

For Falling

$$\Delta U = -W_g = -mgd$$

Sample Problem

A 2.0 kg sloth hangs 5.0 m above the ground
(a) What is the gravitational potential energy *U* of the sloth - Earth system if *y* = 0 to be
(1) at the ground,



For choice (1) the sloth is at *y* = 5.0 m,

$$U = mgy = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(5.0\ \text{m}) = 98\ \text{J}.$$

(2) at a balcony floor that is 3.0 m above ground,

$$U = mgy = mg(2.0\ \text{m}) = 39\ \text{J},$$

(3) at the limb

$$U = mgy = mg(0) = 0\ \text{J},$$

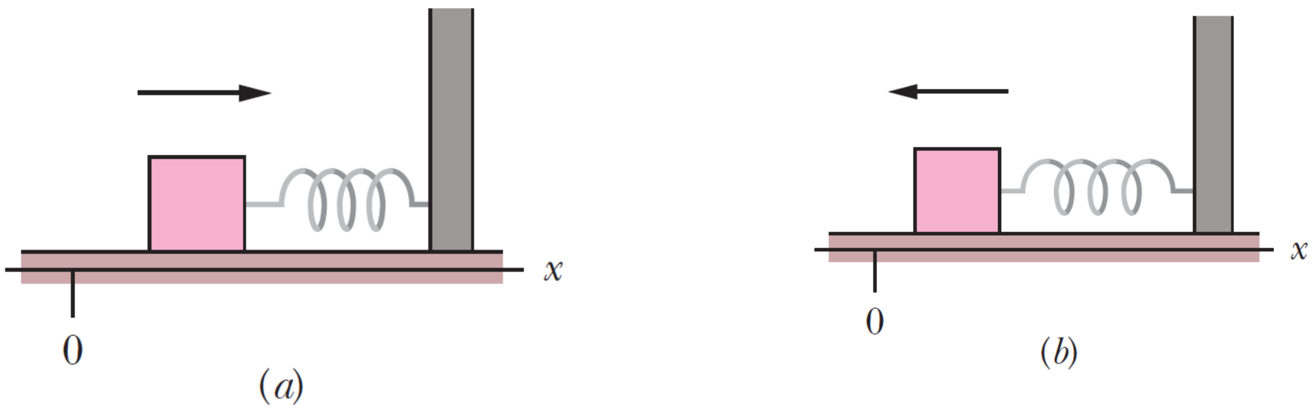
(4) 1.0 m above the limb?

$$U = mgy = mg(-1.0\ \text{m}) = -19.6\ \text{J} \approx -20\ \text{J}.$$

(b) The sloth drops to the ground. For each choice of reference point, what is the change *t:..U* in the potential energy of the sloth - Earth system due to the fall?

$$\Delta U = -W_g = -mgd = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(-5.0\ \text{m}) = -98\ \text{J}.$$

Elastic Potential Energy



- Elastic potential energy is the energy associated with the state of compression or extension of an elastic object.
- For a spring that exerts a spring force $F = -kx$ when its free end has displacement , the elastic potential energy is

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

- The reference configuration has the spring at its relaxed length, at which $x_i = 0$ and $U_i = 0$.

$$U(x) = \frac{1}{2}kx^2$$

Q.36 A spring with spring constant of 40 N/m is compressed by a force a distance of 0.4 m. The potenti energy stored in the spring is:
(A) 0.5 J (B) 2.5 J (C) 3.2 J (D) 10 J (E) 200 J

$$\boxed{U = \frac{1}{2}kx^2 = \frac{1}{2}\text{x}40\text{x}(0.4)^2 = 3.2\text{ J}}$$

Q.28 A spring with spring constant of 20 N/m is compressed by force of 10 N. The potential energy stored in the spring is:
(A) 0.5 J (B) 2.5 J (C) 5 J (D) 10 J (E) 200 J

$$\boxed{x = \frac{F_s}{k} = \frac{10}{20} = 0.5\text{ m} \qquad U_s = \frac{1}{2}kx^2 = \frac{1}{2}\text{x}20\text{x}(0.5)^2 = 2.5\text{ J}}$$

Conservation of Energy

The **mechanical energy** E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it

$$\boxed{E_{\text{mec}} = K + U \qquad \text{(mechanical energy).}}$$

The force transfers energy between K of the object and U of the system

$$\Delta K = W \qquad \Delta U = -W. \qquad \Rightarrow \qquad \Delta K = -\Delta U.$$

$$K_2 - K_1 = -(U_2 - U_1),$$

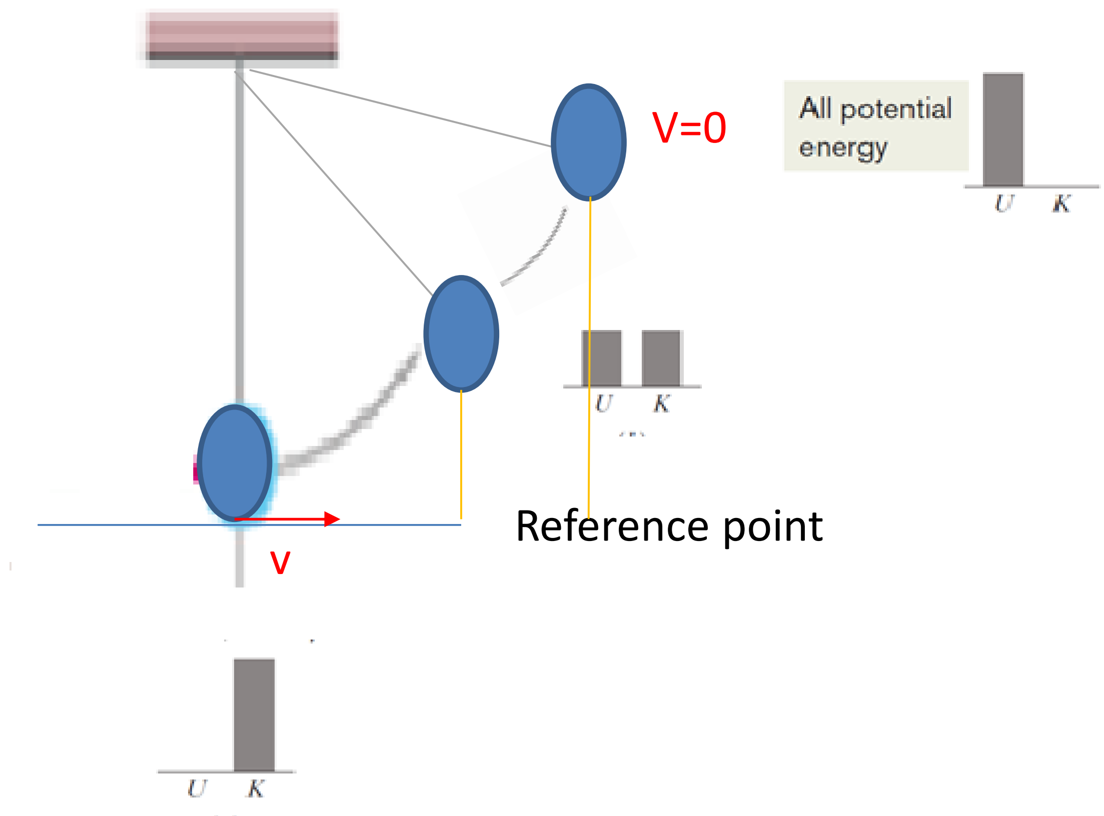
$$K_2 + U_2 = K_1 + U_1 \qquad \text{(conservation of mechanical energy).}$$

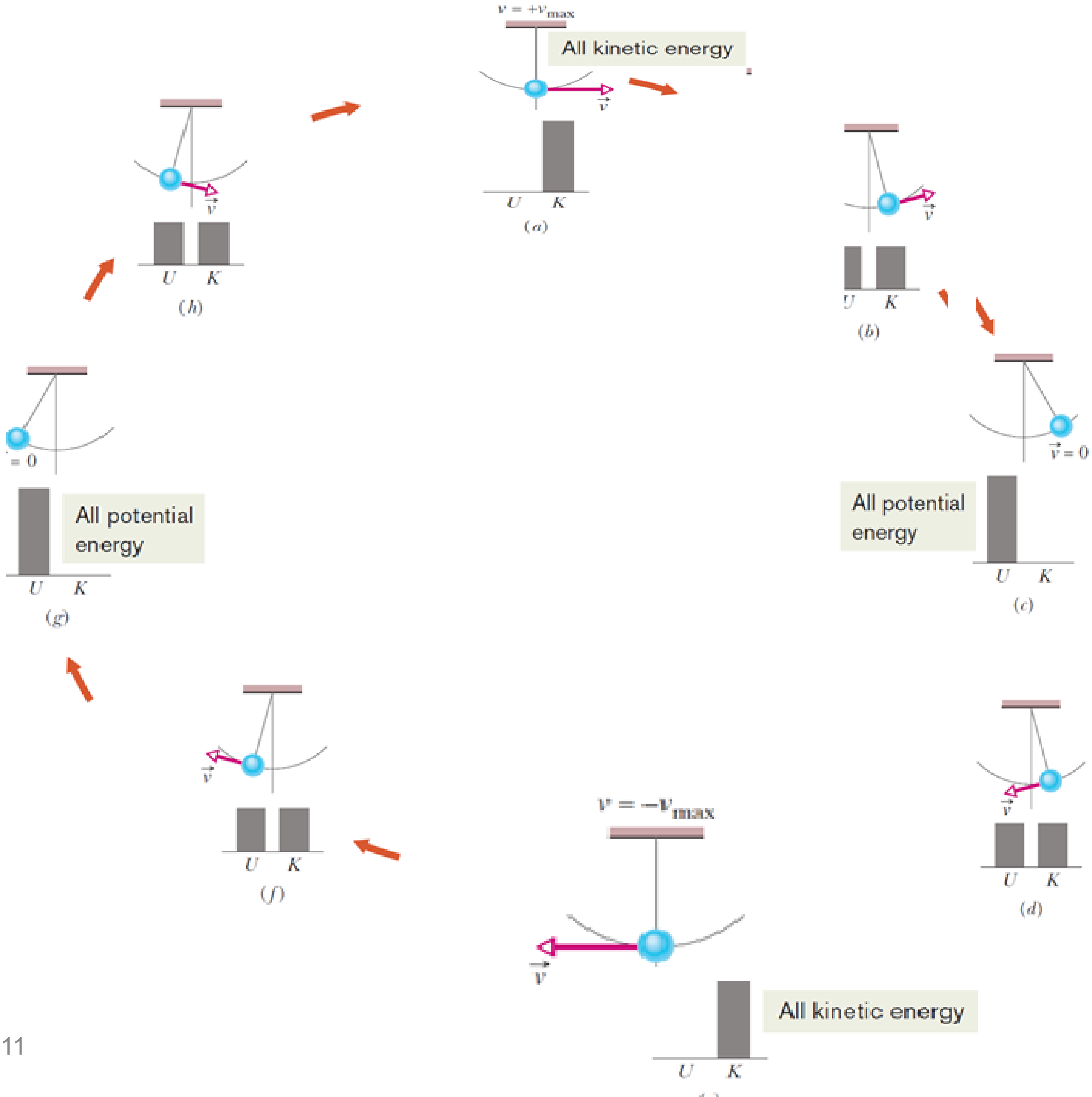
$$\begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system}\end{pmatrix} = \begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system}\end{pmatrix},$$

principle of conservation of mechanical energy

$$\boxed{\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.}$$

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

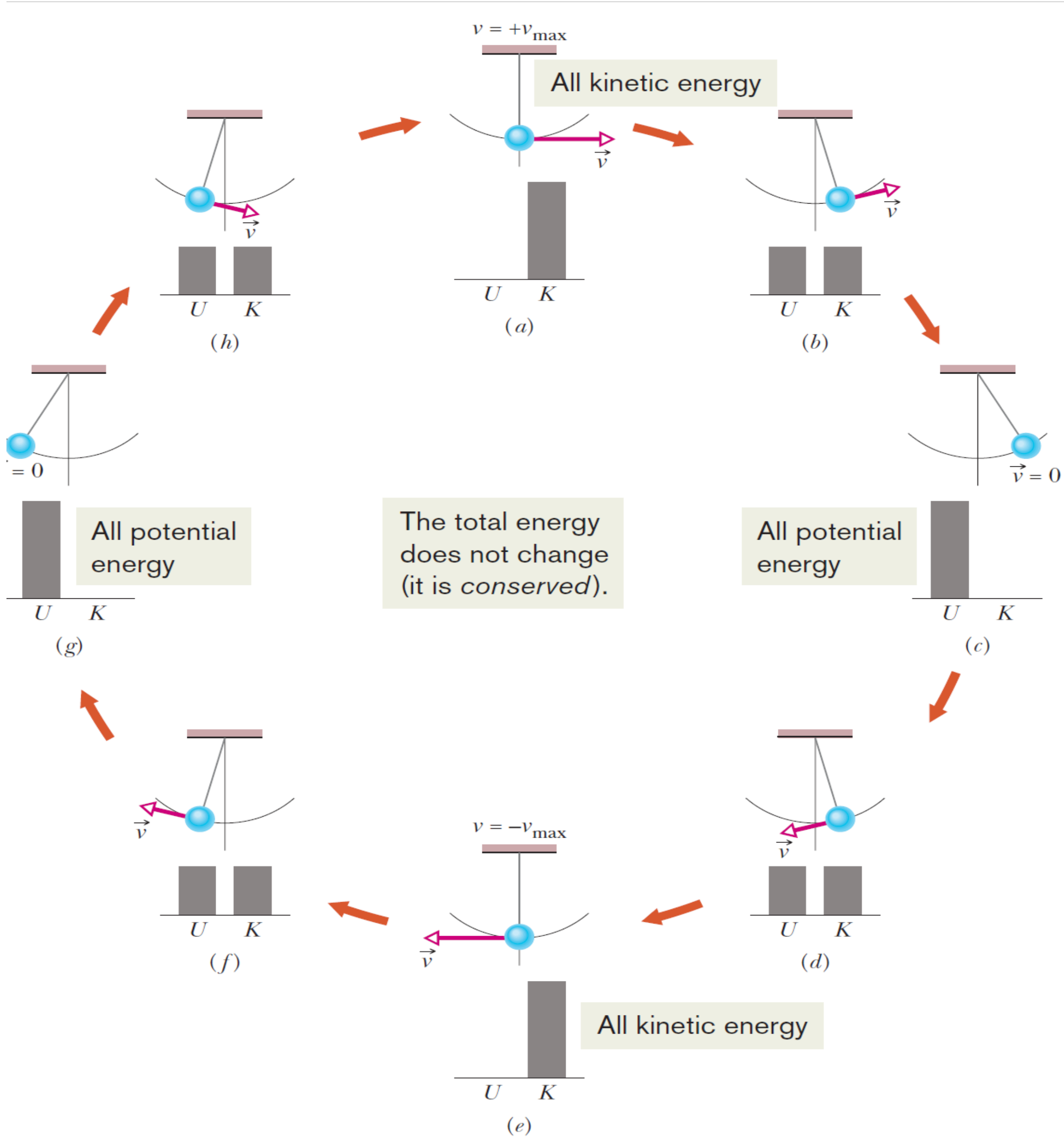

V=0
All potential energy
U
K
U
K
Reference point
V
U
K


$v = +v_{max}$
All kinetic energy
$\vec{v}$
U K
(a)
$\vec{v}$
U K
(b)
$\vec{v} = 0$
All potential energy
U K
(c)
$\vec{v}$
U K
(d)
$v = -v_{max}$
$\vec{v}$
All kinetic energy
U K
$\vec{v}$
U K
(f)
= 0
All potential energy
U K
(g)
$\vec{v}$
U K
(h)

principle of conservation of mechanical energy can be applied:

As a pendulum swings, the energy of the pendulum–Earth system is transferred back and forth between kinetic energy *K* and gravitational potential energy *U*, with the sum *K* + *U* being constant.

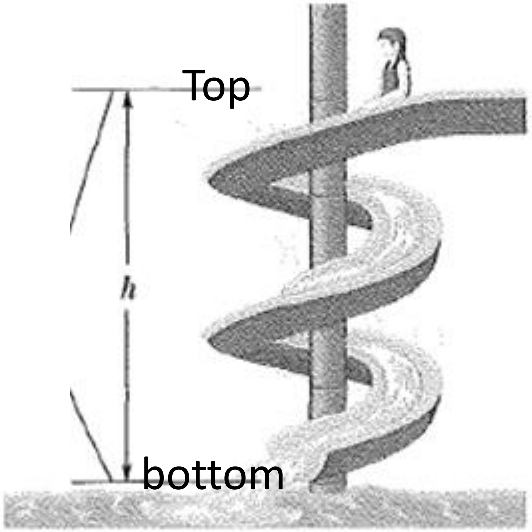
If we know the gravitational potential energy when the pendulum bob is at its highest point (Fig. 8-7*c*), the kinetic energy of the bob at the lowest point (Fig. 8-7*e*).



Sample Problem

A child of mass *m* is released from rest at the top of a water slide, at height *h* = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

The total mechanical energy at the top is equal to the total at the bottom.



Let the mechanical energy be $E_{mec,f}$ When the child is at the top of the slide and $E_{mec,b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{mec,b} = E_{mec,t}.$$

$$K_b + U_b = K_t + U_t,$$

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to $v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \mathrm{m/s^2})(8.5\ \mathrm{m})} = 13\ \mathrm{m/s}.$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