Workshop Solutions to Sections ~~2.1 and 2.2~~(1.1 & 1.2)

1) Find the domain of the function $f(x) = 9 - x^2$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$ **Note:** The domain of any polynomial is $\mathbb{R}$.	2) Find the range of the function $f(x) = 9 - x^2$. Solution: $R_f = (-\infty, 9]$
3) Find the domain of the function $f(x) = 6 - 2x$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	4) Find the range of the function $f(x) = 6 - 2x$. Solution: Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then $R_f = \mathbb{R} = (-\infty, \infty)$
5) Find the domain of the function $f(x) = x^2 - 2x - 3$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
7) Find the domain of the function $f(x) = 5$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	8) Find the range of the function $f(x) = 5$. Solution: $R_f = \{5\}$
9) Find the domain of the function $f(x) = \|x - 1\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$ **Note:** The domain of an absolute value of any polynomial is $\mathbb{R}$.	10) Find the domain of the function $f(x) = \|x + 5\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
11) Find the domain of the function $f(x) = \|x\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	12) Find the range of the function $f(x) = \|x\|$. Solution: $R_f = [0, \infty)$ **Note:** The range of an absolute value of any polynomial is always $[0, \infty)$.
13) Find the domain of the function $f(x) = \|3x - 6\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	14) Find the domain of the function $f(x) = \|9 - 3x\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
15) Find the domain of the function $f(x) = \frac{x+2}{x-3}$ Solution: $f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So, $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$	16) Find the domain of the function $f(x) = \frac{x-2}{x+3}$ Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$

17) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \neq 0 \Longrightarrow x^2 \neq 9 \Longrightarrow x \neq \pm 3$. So,

$$D_f = \mathbb{R} \setminus \{-3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$$

18) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-5x+6}$$

Solution:

$f(x)$ is defined when $x^2 - 5x + 6 \neq 0$
$\Longrightarrow (x-2)(x-3) \neq 0 \Longrightarrow x \neq 2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty,2) \cup (2,3) \cup (3,\infty)$$

19) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-x-6}$$

Solution:

$f(x)$ is defined when $x^2 - x - 6 \neq 0$
$\Longrightarrow (x+2)(x-3) \neq 0 \Longrightarrow x \neq -2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty,-2) \cup (-2,3) \cup (3,\infty)$$

20) Find the domain of the function

$$f(x) = \frac{x+2}{x^2+9}$$

Solution:

$f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the denominator $x^2 + 9$ cannot be 0. So,

$$D_f = \mathbb{R} = (-\infty,\infty)$$

21) Find the domain of the function

$$f(x) = \sqrt[3]{x-3}$$

Solution:

$$D_f = \mathbb{R} = (-\infty,\infty)$$

Note: The domain of an odd root of any polynomial is $\mathbb{R}$.

22) Find the domain of the function

$$f(x) = \sqrt{x-3}$$

Solution:

$f(x)$ is defined when $x - 3 \geq 0 \Longrightarrow x \geq 3$ because $f(x)$ is an even root. So,

$$D_f = [3,\infty)$$

23) Find the domain of the function

$$f(x) = \sqrt{3-x}$$

Solution:

$f(x)$ is defined when $3 - x \geq 0 \Longrightarrow -x \geq -3 \Longrightarrow x \leq 3$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,3]$$

24) Find the domain of the function

$$f(x) = \sqrt{x+3}$$

Solution:

$f(x)$ is defined when $x + 3 \geq 0 \Longrightarrow x \geq -3$ because $f(x)$ is an even root. So,

$$D_f = [-3,\infty)$$

25) Find the domain of the function

$$f(x) = \sqrt{-x}$$

Solution:

$f(x)$ is defined when $-x \geq 0 \Longrightarrow x \leq 0$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,0]$$

26) Find the range of the function

$$f(x) = \sqrt{-x}$$

Solution:

$$R_f = [0,\infty)$$

Note: The range of an even root is always ≥ 0 .

27) Find the domain of the function

$$f(x) = \sqrt{9-x^2}$$

Solution:

$f(x)$ is defined when $9 - x^2 \geq 0 \Longrightarrow -x^2 \geq -9 \Longrightarrow$
$x^2 \leq 9 \Longrightarrow \sqrt{x^2} \leq \sqrt{9} \Longrightarrow |x| \leq 3 \Longrightarrow -3 \leq x \leq 3$. So,

$$D_f = [-3,3]$$

28) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x-3}}$$

Solution:

$f(x)$ is defined when $x - 3 > 0 \Longrightarrow x > 3$. So,

$$D_f = (3,\infty)$$

29) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{9-x^2}}$$

Solution:

$f(x)$ is defined when $9 - x^2 > 0 \Longrightarrow -x^2 > -9$
$\Longrightarrow x^2 < 9 \Longrightarrow \sqrt{x^2} < \sqrt{9} \Longrightarrow |x| < 3 \Longrightarrow -3 < x < 3$. So,

$$D_f = (-3,3)$$

30) Find the domain of the function

$$f(x) = \sqrt{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \geq 0 \Longrightarrow x^2 \geq 9$
$\Longrightarrow \sqrt{x^2} \geq \sqrt{9} \Longrightarrow |x| \geq 3 \Longrightarrow x \geq 3$ or $x \leq -3$. So,

$$D_f = (-\infty,-3] \cup [3,\infty)$$

31) Find the range of the function

$$f(x) = \sqrt{x^2 - 9}$$

Solution:

$$R_f = [0, \infty)$$

32) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x^2 - 9}}$$

Solution:

$f(x)$ is defined when $x^2 - 9 > 0 \Rightarrow x^2 > 9$
$\Rightarrow \sqrt{x^2} > \sqrt{9} \Rightarrow |x| > 3 \Rightarrow x > 3$ or $x < -3$.
So,

$$D_f = (-\infty, -3) \cup (3, \infty)$$

33) Find the domain of the function

$$f(x) = \sqrt{9 + x^2}$$

Solution:

$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value x. So,

$$D_f = \mathbb{R}$$

34) Find the domain of the function

$$f(x) = \sqrt[4]{x^2 - 25}$$

Solution:

$f(x)$ is defined when $x^2 - 25 \geq 0 \Rightarrow x^2 \geq 25$
$\Rightarrow \sqrt{x^2} \geq \sqrt{25} \Rightarrow |x| \geq 5 \Rightarrow x \geq 5$ or $x \leq -5$.
So,

$$D_f = (-\infty, -5] \cup [5, \infty)$$

35) Find the domain of the function

$$f(x) = \sqrt[6]{16 - x^2}$$

Solution:

$f(x)$ is defined when $16 - x^2 \geq 0 \Rightarrow -x^2 \geq -16 \Rightarrow$
$x^2 \leq 16 \Rightarrow \sqrt{x^2} \leq \sqrt{16} \Rightarrow |x| \leq 4 \Rightarrow -4 \leq x \leq 4$.
So,

$$D_f = [-4,4]$$

36) Find the range of the function

$$f(x) = \sqrt{16 - x^2}$$

Solution:

We know that $f(x)$ is defined when $16 - x^2 \geq 0$
$\Rightarrow -x^2 \geq -16 \Rightarrow x^2 \leq 16 \Rightarrow \sqrt{x^2} \leq \sqrt{16}$
$\Rightarrow |x| \leq 4 \Rightarrow -4 \leq x \leq 4$. So,

$$D_f = [-4,4]$$

Using D_f we find the outputs vary from 0 to 4. Hence,

$$R_f = [0,4]$$

37) Find the domain of the function

$$f(x) = \frac{x + |x|}{x}$$

Solution:

$f(x)$ is defined when $x \neq 0$. So,

$$D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$$

38) Find the domain of the function

$$f(x) = \begin{cases} -\frac{1}{x}, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Solution:

It is clear from the definition of the function $f(x)$ that

$$D_f = \mathbb{R} = (-\infty, \infty)$$

39) Find the domain of the function

$$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$$

Solution:

$f(x)$ is defined when

1- $x \geq 0 \quad \Rightarrow \quad D_{\sqrt{x}} = [0, \infty)$

2- $x^2 + 1 > 0$ but this is always true for all x
$\Rightarrow D_{\sqrt{x^2+1}} = \mathbb{R}$.

Hence,

$$D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$$

40) Find the domain of the function

$$f(x) = \sqrt{x - 1} + \sqrt{x + 3}$$

Solution:

$f(x)$ is defined when

1- $x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow D_{\sqrt{x-1}} = [1, \infty)$

2- $x + 3 \geq 0 \Rightarrow x \geq -3 \Rightarrow D_{\sqrt{x+3}} = [-3, \infty)$

Hence,

$$D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$$

41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.

42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.

43) The function $f(x) = -3x^2 + 7$ is a quadratic function.

44) The function $f(x) = 2x + 3$ is a linear function.

45) The function $f(x) = x^7$ is a power function.

46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.	50) The function $f(x) = 3^x$ is a general exponential function.
51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.	52) The function $f(x) = -3$ is a constant function.
53) The function $f(x) = \log_3 x$ is a general logarithmic function.	54) The function $f(x) = \ln x$ is a natural logarithmic function.
55) The function $f(x) = 3x^4 + x^2 + 1$ is Solution: $f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.	56) The function $f(x) = 9 - x^2$ is Solution: $f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$ Hence, $f(x)$ is an even function.
57) The function $f(x) = x^5 - x$ is Solution: $f(-x) = (-x)^5 - (-x) = -x^5 + x$ $= -(x^5 - x) = -f(x)$ Hence, $f(x)$ is an odd function.	58) The function $f(x) = 2 - \sqrt[5]{x}$ is Solution: $f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$ $= -\left(-2 - \sqrt[5]{x}\right)$ Hence, $f(x)$ is neither even nor odd.
59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is Solution: $f(-x) = 3(-x) + \dfrac{2}{\sqrt{(-x)^2+9}} = -3x + \dfrac{2}{\sqrt{x^2+9}}$ $= -\left(3x - \dfrac{2}{\sqrt{x^2+9}}\right)$ Hence, $f(x)$ is neither even nor odd.	60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is Solution: $f(-x) = \dfrac{3}{\sqrt{(-x)^2+9}} = \dfrac{3}{\sqrt{x^2+9}} = f(x)$ Hence, $f(x)$ is an even function.
61) The function $f(x) = \sqrt{4 + x^2}$ is Solution: $f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$ Hence, $f(x)$ is an even function.	62) The function $f(x) = 3$ is Solution: Since the graph of the constant function 3 is symmetric about the $y - axis$, then $f(x)$ is an even function.
63) The function $f(x) = \frac{9-x^2}{x-2}$ is Solution: $f(-x) = \dfrac{9-(-x)^2}{(-x)-2} = \dfrac{9-x^2}{-x-2}$ $= -\left(\dfrac{9-x^2}{x+2}\right)$ Hence, $f(x)$ is neither even nor odd.	64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is Solution: $f(-x) = \dfrac{(-x)^2-4}{(-x)^2+1} = \dfrac{x^2-4}{x^2+1} = f(x)$ Hence, $f(x)$ is an even function.
65) The function $f(x) = 3\lvert x\rvert$ is Solution: $f(-x) = 3\lvert(-x)\rvert = 3\lvert x\rvert = f(x)$ Hence, $f(x)$ is an even function.	66) The function $f(x) = x^{-2}$ is Solution: $f(x) = x^{-2} = \dfrac{1}{x^2}$ $f(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2} = f(x)$ Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is <u>Solution:</u> $f(-x) = (-x)^3 - 2(-x) + 5 \;= -x^3 + 2x + 5$ $= -(x^3 - 2x - 5)$ Hence, $f(x)$ is neither even nor odd.	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is <u>Solution:</u> $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \;= -\sqrt[3]{x^5} + x^3 - x$ $= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$ Hence, $f(x)$ is an odd function.
69) The function $f(x) = 7$ is <u>Solution:</u> Since the graph of the constant function 7 is symmetric about the $y - axis$, then $f(x)$ is an even function.	70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is <u>Solution:</u> $f(-x) = \frac{(-x)^3 - 4}{(-x)^3 + 1} = \frac{-x^3 - 4}{-x^3 + 1} = -\frac{x^3 + 4}{-x^3 + 1}$ Hence, $f(x)$ is neither even nor odd.
71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is <u>Solution:</u> $f(-x) = \frac{(-x)^2 - 1}{(-x)^3 + 3} = \frac{x^2 - 1}{-x^3 + 3} = -\frac{x^2 - 1}{x^3 - 3}$ Hence, $f(x)$ is neither even nor odd.	72) The function $f(x) = x^6 - 4x^2 + 1$ is <u>Solution:</u> $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)\setminus\{0\}$