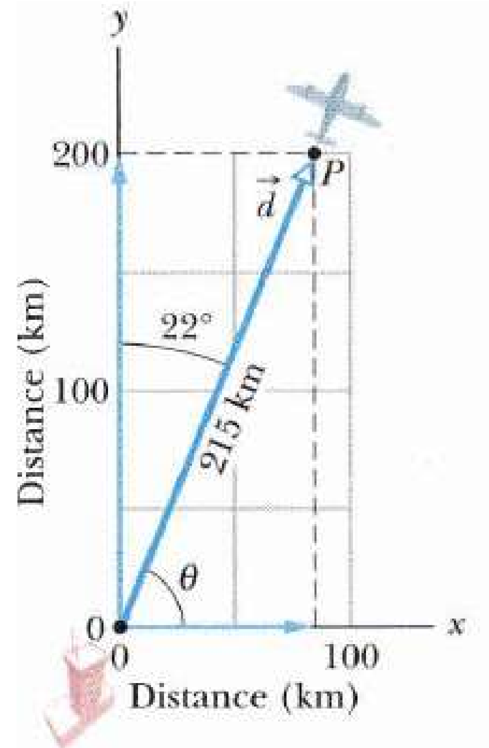
Sample Problem 3-2

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



$$d_x = d\cos\theta = (215\text{ km})(\cos 68°) = 81\text{ km} \qquad \text{(Answer)}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin 68°) = 199\text{ km} \approx 2.0 \times 10^2\text{ km.} \qquad \text{(Answer)}$$

Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.

Sample Problem 3-4

Figure 3-16*a* shows the following three vectors:

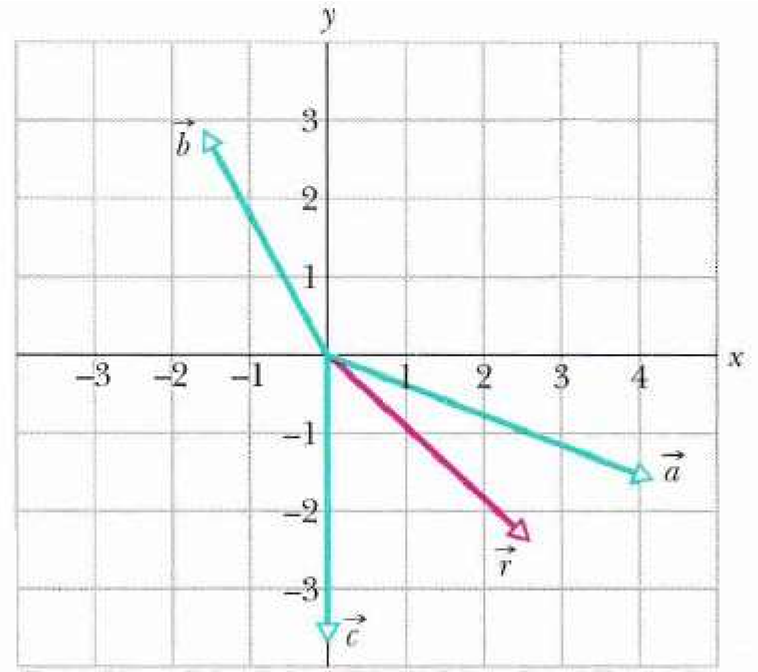
$$\vec{a} = (4.2\text{ m})\hat{\text{i}} - (1.5\text{ m})\hat{\text{j}},$$

$$\vec{b} = (-1.6\text{ m})\hat{\text{i}} + (2.9\text{ m})\hat{\text{j}},$$

and

$$\vec{c} = (-3.7\text{ m})\hat{\text{j}}.$$

What is their vector sum $\vec{r}$ which is also shown?



(*a*)

Calculations: For the x axis, we add the x components of $\vec{a}$, $\vec{b}$, and $\vec{c}$, to get the x component of the vector sum $\vec{r}$:

$$r_x = a_x + b_x + c_x$$
$$= 4.2\text{ m} - 1.6\text{ m} + 0 = 2.6\text{ m}.$$

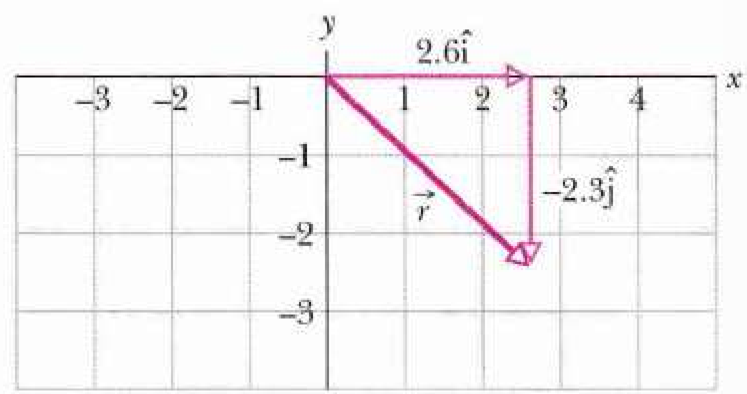
Similarly, for the y axis,

$$r_y = a_y + b_y + c_y$$
$$= -1.5\text{ m} + 2.9\text{ m} - 3.7\text{ m} = -2.3\text{ m}.$$

$$\vec{r} = (2.6\text{ m})\hat{\text{i}} - (2.3\text{ m})\hat{\text{j}}, \qquad \text{(Answer)}$$

$$r = \sqrt{(2.6\text{ m})^2 + (-2.3\text{ m})^2} \approx 3.5\text{ m} \qquad \text{(Answer)}$$

$$\theta = \tan^{-1}\left(\frac{-2.3\text{ m}}{2.6\text{ m}}\right) = -41°, \qquad \text{(Answer)}$$



(*b*)

Sample Problem

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

Calculations

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\phi}$$

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00 \qquad b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61$$

$$\begin{aligned}\vec{a}\cdot\vec{b} &= (3.0\hat{i} - 4.0\hat{j})\cdot(-2.0\hat{i} + 3.0\hat{k})\\ &= (3.0\hat{i})\cdot(-2.0\hat{i}) + (3.0\hat{i})\cdot(3.0\hat{k})\\ &\quad + (-4.0\hat{j})\cdot(-2.0\hat{i}) + (-4.0\hat{j})\cdot(3.0\hat{k})\end{aligned}$$

$$\begin{aligned}\vec{a}\cdot\vec{b} &= -(6.0)(1) + (9.0)(0) + (8.0)(0) - (12)(0)\\ &= -6.0.\end{aligned}$$

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\phi}$$

$$-6.0 = (5.00)(3.61)\cos\phi,$$

$$\phi = \cos^{-1}\frac{-6.0}{(5.00)(3.61)} = 109^\circ \approx 110^\circ$$

Sample Problem 3-9

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

Calculations: Here we write

$$\begin{aligned}\vec{c} &= (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k}) \\ &= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i}) \\ &\quad + (-4\hat{j}) \times 3\hat{k}.\end{aligned}$$

$$\begin{aligned}\vec{c} &= -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i} \\ &= -12\hat{i} - 9\hat{j} - 8\hat{k}. \qquad \text{(Answer)}\end{aligned}$$

Samples of Exam Questions

Logic Questions

Q.21 The scalar product $\hat{i} \cdot \hat{j}$ is equal to:

(A) $\hat{k}$ (B) $2\hat{i}$ (C) $2\hat{j}$ (D) zero (E) $\hat{i}\hat{j}$

$$\boxed{\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1} \; \boxed{\hat{i} \bullet \hat{j} = \hat{j} \bullet \hat{k} = \hat{i} \bullet \hat{k} = 0}$$

Q.13 The result of $\hat{j} \bullet \hat{j}$ is:

(A) $\hat{i}$ (B) $\hat{k}$ (C) $\hat{j}$ (D) Zero (E) 1

Logic Questions

Q.27 The vector product $\hat{j} \times \hat{i}$ is equal to:

(A) $\hat{j}$ (B) $-\hat{i}$ (C) $\hat{k}$ (D) 1 $\boxed{\text{(E) } -\hat{k}}$

$$\boxed{\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0}$$

$$\boxed{\begin{array}{lll} \hat{i} \times \hat{j} = \hat{k} & \hat{j} \times \hat{k} = \hat{i} & \hat{k} \times \hat{i} = \hat{j} \\ \hat{j} \times \hat{i} = -\hat{k} & \hat{k} \times \hat{j} = -\hat{i} & \hat{i} \times \hat{k} = -\hat{j} \end{array}}$$

Logic Questions

Q.28 The value of $\hat{i}\cdot(\hat{k}\times\hat{j})$ is:

(A) $\hat{j}$ (B) zero (C) $\hat{k}$ (D) -1 (E) 1

$$\boxed{\hat{i}\bullet(\hat{k}\times\hat{j})=\hat{i}\bullet(-\hat{i})=-\hat{i}\bullet\hat{i}=-1}$$

Q.15 The result of $(\hat{i}\times\hat{k})\bullet\hat{j}$ is:

(A) $\hat{i}$ (B) 1 (C) $\hat{j}$ (D) -1 (E) Zero

$$\boxed{(\hat{i}\times\hat{k})\bullet\hat{j}=-\hat{j}\bullet\hat{j}=-1}$$

Q.20 The result of $(\hat{k}\times\hat{i})\bullet\hat{j}$ is:

(A) $\hat{i}$ (B) 1 (C) $\hat{j}$ (D) $\hat{k}$ (E) Zero

$$\boxed{(\hat{k}\times\hat{i})\bullet\hat{j}=\hat{j}\bullet\hat{j}=1}$$

Q.15 The result of $(\hat{k}\times\hat{j})\times\hat{i}$ is:

(A) $\hat{i}$ (B) 1 (C) Zero (D) $\hat{k}$ (E) $\hat{j}$

Logic Questions

Q.15 The result of $(\hat{i} \times \hat{j}) \times \hat{i}$ is:

(A) $\hat{i}$ (B) 1 (C) Zero (D) $\hat{k}$ (E) $\hat{j}$

$$\boxed{(\hat{i} \times \hat{j}) \times \hat{i} = \hat{k} \times \hat{i} = \hat{j}}$$

Q.26 If $\bar{A} \bullet \bar{B} = 0$,the angle between the vectors $\bar{A}$ and $\bar{B}$ is: (Hint: $\bar{A}$ and $\bar{B}$ are non-zero vectors)

(A) 180° (B) Zero (C) 90° (D) 315° (E) 45°

If scalar product is zero, the vectors are perpendicular (متعامدين) and the angle between them is 90^0

Q.26 If $\bar{A} \times \bar{B} = 0$, the angle between the vectors $\bar{A}$ and $\bar{B}$ should be: (Hint: $\bar{A}$ and $\bar{B}$ are n

(A) 315° (B) 30 (C) 90° (D) 180° (E) 4

If cross product is zero, the vectors are parallel (متوازيين) and the angle between them is zero

Vector Components

Q.18 A vector $\vec{A}$ has x-component of 10 m and y-component of 15 m. The magnitude of this vector is:

(A) 14.14 m (B) 18 m (C) 22.36 m (D) 35.12 m (E) 11.18 m

$$A_x = 10\text{ m} \qquad A_y = 15\text{ m}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{100 + 225} = \sqrt{325} = 18.02 \approx 18$$

Q.28 The components of vector $\vec{A}$ are given as A_x=5.5 m and A_y=-5.3 m. The magnitude of vector $\vec{A}$ is:

(A) 9.2 m (B) 8.4 m (C) 6.9 m (D) 6.1 m (E) 7.6 m

$$A_x = 5.5\text{ m} \qquad A_y = -5.3\text{ m}$$

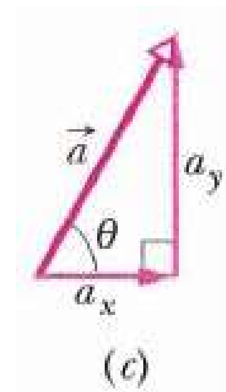
$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(5.5)^2 + (-5.3)^2} = \sqrt{30.25 + 28.09} = \sqrt{58.34} = 7.64 \approx 7.6$$

Vector Components

Q.19 A vector has a magnitude of 14 units makes an angle of 30° with the x axis. Its y component is:
A) 8 units (B) 9 units (C) 5 units (D) 6 units (E) 7 units

$$A = 14 \text{ units} \qquad \theta = 30^0$$

$$A_y = A\sin\theta = 14\text{x}\sin 30^0 = 14\text{x}\frac{1}{2} = 7 \text{ units}$$



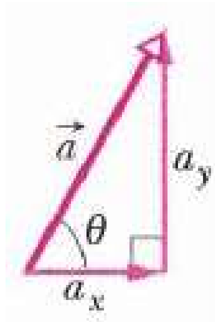
Vector Components

Q.24 If the magnitude of a vector is 18m and its x-component of 10m. The angle it makes with the positive x-axis is:

(A) 48.2° (B) 63.4° (C) 66.4° (D) 60° (E) 56.25°

$$A = 18\ \text{m} \qquad A_x = 10\ \text{m}$$

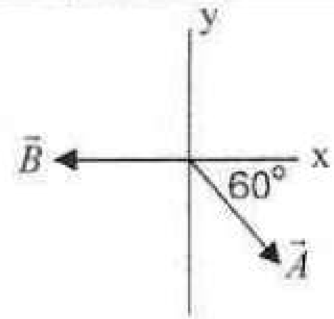
$$A_x = A\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{A_x}{A} \Rightarrow \quad \theta = \cos^{-1}\left(\frac{A_x}{A}\right) = \cos^{-1}\left(\frac{10}{18}\right) = \cos^{-1}(0.555) = 56.25^0$$



(c)

Vector Addition

Q.20 As shown in the figure, if the magnitudes of $\vec{A}$ *and* $\vec{B}$ are 10 units and 15 units respectively then the x-component of the resultant of $\vec{A}$ *and* $\vec{B}$ is:



(A) -10 units (B) -15 units (C) -20units (D) zero (E) -5 units

$A = 10$ units $\quad B = 15$ units

$$\vec{A} + \vec{B} = \boxed{(A_x + B_x)}\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$$A_x = A\cos\theta = 10x\cos(60) = 10x\frac{1}{2} = 5 \text{ units}$$

$$B_x = -15 \text{ units} \quad \Rightarrow A_x + B_x = 5 - 15 = -10 \text{ units}$$

Vector Addition

Q.22 if $\vec{A} = 4\hat{i} - 6\hat{j}$ then the vector $\frac{1}{2}\vec{A}$ is:

A) $2\hat{i} - \hat{j}$ (B) $2\hat{i} - 5\hat{j}$ (C $2\hat{i} - 4\hat{j}$ (D) $2\hat{i} - 3\hat{j}$ (E) $2\hat{i} - 2\hat{j}$

$$\boxed{A = 4i - 6j \quad ====> \quad ½ A = 2i - 3j}$$

Vector Addition

Q.23 Two vectors are given as $\vec{A} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$. The result of $\vec{A} - \vec{B}$ is:

(A) $5\hat{i} - 3\hat{j}$ (B) $4\hat{i} - 3\hat{j}$ (C) $3\hat{i} - 3\hat{j}$ (D) $2\hat{i} - 3\hat{j}$ (E) $\hat{i} - 3\hat{j}$

$$A_x = 2 \quad A_y = -2 \quad A_z = 4$$

$$B_x = -1 \quad B_y = 1 \quad B_z = 4$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}$$

$$= (2 - (-1))\hat{i} + (-2 - 1)\hat{j} + (4 - 4)\hat{k}$$

$$= 3\hat{i} - 3\hat{j}$$

Vector Addition

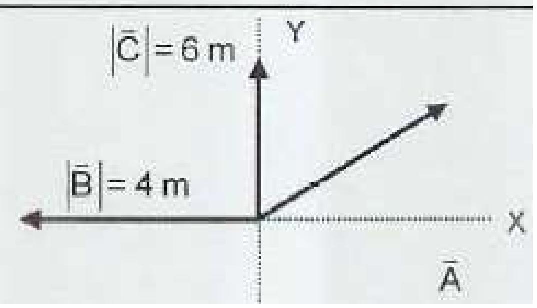
Q.22 Given $\bar{A} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\bar{B} = 2\hat{i} - 6\hat{j} + 7\hat{k}$, $\bar{C} = 2\hat{i} - \hat{j} + 4\hat{k}$. Then the vector $\bar{D} = 2\bar{A} + \bar{B} - \bar{C}$ is:
(A) $-\hat{i} - 2\hat{j} + 3\hat{k}$ (B) $3\hat{i} + 2\hat{j} - 5\hat{k}$ (C) $3.5\hat{i}$ (D) $4\hat{i} - 3\hat{j} + 9\hat{k}$ (E) $\hat{i} + 2\hat{j} - 5k$

$$A_x = 2 \quad A_y = 1 \quad A_z = 3$$
$$B_x = 2 \quad B_y = -6 \quad B_z = 7$$
$$C_x = 2 \quad C_y = -1 \quad C_z = 4$$
$$\begin{aligned}\vec{D} = 2\vec{A} + \vec{B} - \vec{C} &= (2A_x + B_x - C_x)\hat{i} + (2A_y + B_y - C_y)\hat{j} + (2A_z + B_z - C_z)\hat{k} \\ &= (2x2 + 2 - 2)\hat{i} + (2x1 - 6 - (-1))\hat{j} + (2x3 + 7 - 4)\hat{k} \\ &= (4 + 2 - 2)\hat{i} + (2 - 6 + 1)\hat{j} + (6 + 7 - 4)\hat{k} \\ &= 4\hat{i} - 3\hat{j} + 9\hat{k}\end{aligned}$$

Q.19 Refer to question (18) the angle between the vector $\bar{A}$ and the positive z-axis is:
(A) 36.7° (B) Zero (C) 180° (D) 315° (E) 90°

Vector Addition

Q.19 In figure, if $\vec{A}+\vec{B}-\vec{C}=4\hat{i}$ then the vector $\vec{A}$ in unit vector notation is:



(A) $4\hat{i}+2\hat{j}$ (B) $9\hat{i}+4\hat{j}$ (C) $8\hat{i}+6\hat{j}$ (D) $5\hat{i}-4\hat{j}$ (E) $4\hat{i}$

$$B_x = -4\text{ m} \qquad B_y = 0$$

$$C_x = 0 \qquad C_y = 6\text{ m}$$

$$\vec{A}+\vec{B}-\vec{C}=4\hat{i} \qquad \Rightarrow \vec{A}=4\hat{i}-\left(\vec{B}-\vec{C}\right)$$

$$\vec{B}-\vec{C}=\left(B_x-C_x\right)\hat{i}+\left(B_y-C_y\right)\hat{j}=\left(-4-0\right)\hat{i}+\left(0-6\right)\hat{j}=-4\hat{i}-6\hat{j}$$

$$\vec{A}=4\hat{i}-\left(\vec{B}-\vec{C}\right)=4\hat{i}-\left(-4\hat{i}-6\hat{j}\right)=4\hat{i}+4\hat{i}+6\hat{j}=8\hat{i}+6\hat{j}$$

Scalar Product

Q.25 If the magnitude of two vectors are 10 units and 20 units and the angle between them is 60° then their scalar product is:

(a) 100 (B) 125 (C) zero (D) 25 (E) 75

$$A = 10\ \text{units} \qquad B = 20\ \text{units} \qquad \varphi = 60^0$$

$$\vec{A} \bullet \vec{B} = AB\cos\varphi = 10x20x\cos60^0 = 200x\frac{1}{2} = 100$$

Q.26 Two vectors are given as $\bar{A} = 5\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j}$, their scalar product $\vec{A} \cdot \vec{B}$ is:

(A) 4 (B) 5 (C) 6 (D) 7 (E) 3

$$A.B = A_xB_x + A_yB_y + A_zB_z = 0\times(-1) + 5\times1 + 4\times0 = 5\ \text{units}$$

Scalar Product

Q.26 Two vectors are given as $\bar{A} = 5\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j}$, their scalar product $\vec{A} \cdot \vec{B}$ is:

(A) 4 (B) 5 (C) 6 (D) 7 (E) 3

$$A_x = 0 \qquad A_y = 5 \qquad A_z = 4$$

$$B_x = -1 \qquad B_y = 1 \qquad B_z = 0$$

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z = 0x(-1) + 5x1 + 4x0 = 0 + 5 + 0 = 5$$

Scalar Product

Q.24 Given $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, Then $(\bar{a} \cdot \bar{b})$ is:

(A) $3\hat{i} + 4\hat{j} - 5\hat{k}$ (B) 40 (C) 8 (D) $\hat{i} + \hat{j} - 5\hat{k}$ (E) $\hat{i} + 2\hat{j}$

$$A_x = 1 \qquad A_y = 2 \qquad A_z = 3$$

$$B_x = 2 \qquad B_y = -3 \qquad B_z = 4$$

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z = 1x2 + 2x(-3) + 3x4 = 2 - 6 + 12 = 8$$

Scalar Product

(A) 74.5°

Q.24 Given $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then $(5\vec{a} \bullet \vec{b})$ is:

(A) $3\hat{i} + 4\hat{j} - 5\hat{k}$ (B) 40 (C) 8 (D) $\hat{i} + 2\hat{j} - 5\hat{k}$ (E) 60

... then the angle between vector $\vec{c}$ and $\vec{d}$ is:

$$A_x = 1 \qquad A_y = 2 \qquad A_z = 3$$

$$B_x = 2 \qquad B_y = -3 \qquad B_z = 4$$

$$5\vec{A} \bullet \vec{B} = 5A_xB_x + 5A_yB_y + 5A_zB_z = 5x1x2 + 5x2x(-3) + 5x3x4 = 10 - 30 + 60 = 40$$

Scalar Product

Q.25 Given $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{d} = 2\hat{i} - \hat{j} + 4\hat{k}$,then the angle between vector $\vec{c}$ and $\vec{d}$ is:

(A) 45.6° (B) 15° (C) 120° (D) 90° (E)Zero

$$\vec{c} \bullet \vec{d} = c\,d\cos\varphi \quad \Rightarrow \quad \cos\varphi = \frac{\vec{c} \bullet \vec{d}}{cd} \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{\vec{c} \bullet \vec{d}}{cd}\right)$$

$$c_x = 1 \qquad c_y = 2 \qquad c_z = 3$$

$$d_x = 2 \qquad d_y = -1 \qquad d_z = 4$$

$$c = \sqrt{c_x^2 + c_y^2 + c_z^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{4+1+16} = \sqrt{21}$$

$$\vec{c} \bullet \vec{d} = c_x d_x + c_y d_y + c_z d_z = 1x2 + 2x(-1) + 3x4 = 2 - 2 + 12 = 12$$

$$\varphi = \cos^{-1}\left(\frac{12}{\sqrt{14}\sqrt{21}}\right) = \cos^{-1}\left(\frac{12}{\sqrt{294}}\right) = \cos^{-1}\left(\frac{12}{17.15}\right) = 45.6^0$$

Vector Product

Q.30 If the angle between $\vec{A}$ *and* $\vec{B}$ is 30°, and A = 5 units, B = 10 units, then the magnitude of the vector product $\vec{A} \times \vec{B}$ is:

(A) 25 (B) 20 (C) 15 (D) 30 (E) 35

$$A = 5 \text{ units} \qquad B = 10 \text{ units} \qquad \varphi = 30^0$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\varphi = 5x10x\sin 30^0 = 50x\frac{1}{2} = 25 \quad \text{unit}^2$$

Vector Product

Q.21 If $\bar{A}$ and $\bar{B}$ are vectors with magnitudes 5 and 4, respectively, and the magnitude of their cross product is 17.32, then the angle between $\bar{A}$ and $\bar{B}$ is:

(A) 90° (B) 60° (C) 45° (D) 180° (E) 30°

$$\mathrm{A} = 5 \text{ units} \qquad \mathrm{B} = 4 \text{ units} \qquad \left|\vec{\mathrm{A}} \times \vec{\mathrm{B}}\right| = 17.32$$

$$\left|\vec{\mathrm{A}} \times \vec{\mathrm{B}}\right| = \mathrm{AB}\sin\varphi \quad \Rightarrow \quad \sin\varphi = \frac{\left|\vec{\mathrm{A}} \times \vec{\mathrm{B}}\right|}{\mathrm{AB}} = \frac{17.32}{5\mathrm{x}4} = \frac{17.32}{20} \quad \Rightarrow \quad \varphi = \sin^{-1}\frac{17.32}{20} = 60^0$$

Q.21 If $\bar{A}$ and $\bar{B}$ are vectors with magnitudes 5 and 4, respectively, and the magnitude of their cross product is 10, then the angle between $\bar{A}$ and $\bar{B}$ is:

(A) 90° (B) 60° (C) 45° (D) 180° (E) 30°

Vector Product

Q.29 Two vectors $\vec{A} = 8\hat{i} + 6\hat{j}$ and $\vec{B} = -6\hat{i}$, their vector product $\vec{A} \times \vec{B}$ is:

(A) $48\hat{k}$ (B) $30\hat{k}$ (C) $36\hat{k}$ (D) $42\hat{k}$ (E) $48\hat{k}$

$$\boxed{A \times B = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 6 & 0 \\ -6 & 0 & 0 \end{vmatrix} = 0\hat{i} + 0\hat{j} + (8 \times 0 - 6 \times (-6))\hat{k} = 36\hat{k}}$$

Vector Product

Q.27 Given that $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\bar{b} = 2\hat{i} - \hat{j} + 4\hat{k}$,then $\bar{a} \times \bar{b}$ is:

(A) $11\hat{i} + 2\hat{j} - 5\hat{k}$ (B) $-\hat{i} - 2\hat{j} + 3\hat{k}$ (C) $3.5\hat{i}$ (D) 4 (E) $\hat{i} + 2\hat{j} - 5k$

$$a_x = 1 \qquad a_y = 2 \qquad a_z = 3$$

$$b_x = 2 \qquad b_y = -1 \qquad b_z = 4$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{vmatrix} = \hat{i}\left(2\text{x}4 - 3\text{x}(-1)\right) - \hat{j}\left(1\text{x}4 - 3\text{x}2\right) + \hat{k}\left(1\text{x}(-1) - 2\text{x}2\right)$$

$$= \hat{i}\left(8+3\right) - \hat{j}\left(4-6\right) + \hat{k}\left(-1-4\right) = 11\hat{i} + 2\hat{j} - 5\hat{k}$$