SAMPLE PROBLEM 2-1

One drives a beat-up pickup truck along a straight road for *8.4* km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, he walks another 2.0 km farther along the road to a gasoline station.

(a) What is the overall displacement from the beginning of his drive to his arrival at the station?

Calculation:

$$\Delta x = x_2 - x_1 = 10.4 \text{ km} - 0 = 10.4 \text{ km}.$$

Thus, the overall displacement is *10.4* km in the positive direction of the *X* axis.

(b) What is the time interval Δt from the beginning of his drive to his arrival at the station?

Calculation: We first write $v_{avg,dr} = \frac{\Delta x_{dr}}{\Delta t_{dr}}$

Rearranging and substituting data then give us

$$\Delta t_{dr} = \frac{\Delta x_{dr}}{v_{avg,dr}} = \frac{8.4\ \text{km}}{70\ \text{km/h}} = 0.12\ \text{h}$$

So,

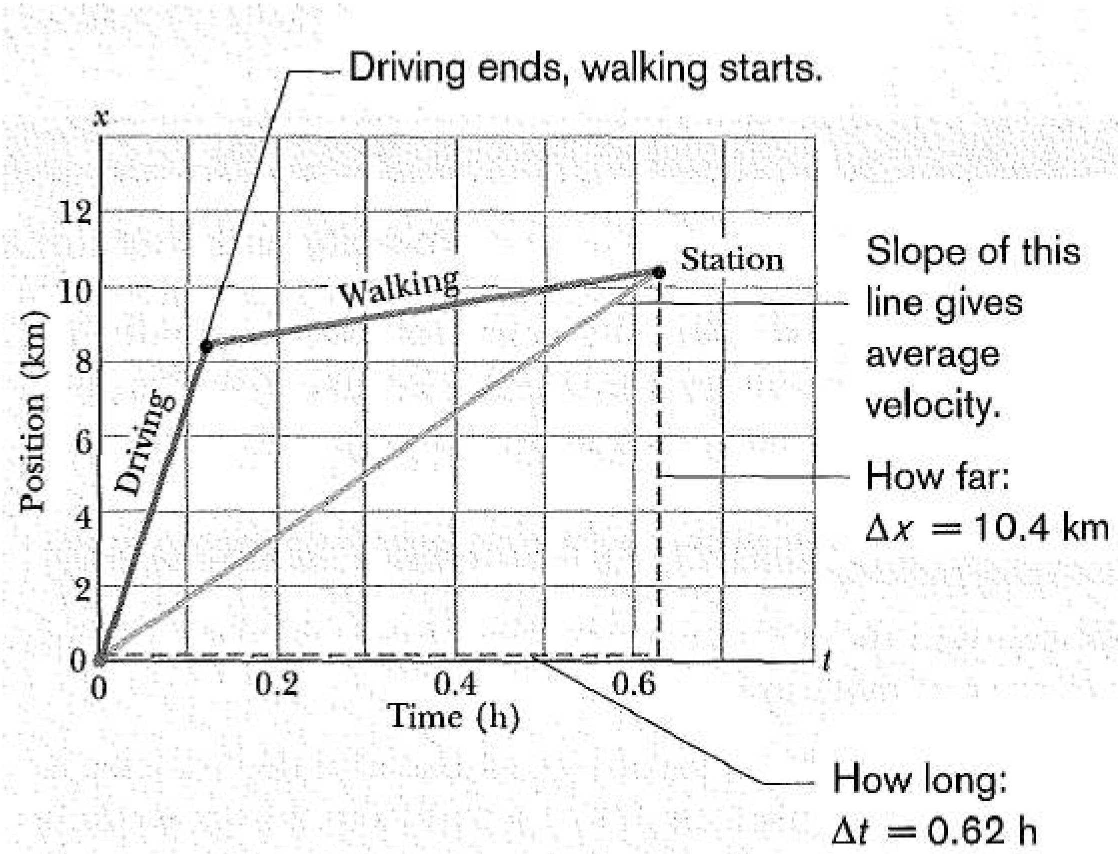
$$\Delta t = \Delta t_{dr} + \Delta t_{wlk}$$
$$= 0.12\ \text{h} + 0.50\ \text{h} = 0.62\ \text{h}.$$

(d) What is the average speed v_{avg} from the beginning of his drive to his arrival at the station? Find it both numerically and graphically.

Calculation: Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4\ \text{km}}{0.62\ \text{h}}$$
$$= 16.8\ \text{km/h} \approx 17\ \text{km/h}.$$

SAMPLE PROBLEM 2-1



The average velocity is the slope of the straight line connecting the origin to the final position

Sample Problem 2-3

The position of a particle moving on an x axis is given by

$$x = 7.8 + 9.2t - 2.1t^3, \tag{2-5}$$

with x in meters and t in seconds. What is its velocity at $t = 3.5$ s? Is the velocity constant, or is it continuously changing?

$$v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3),$$

which becomes

$$v = 0 + 9.2 - (3)(2.1)t^2 = 9.2 - 6.3t^2. \tag{2-6}$$

At $t = 3.5$ s,

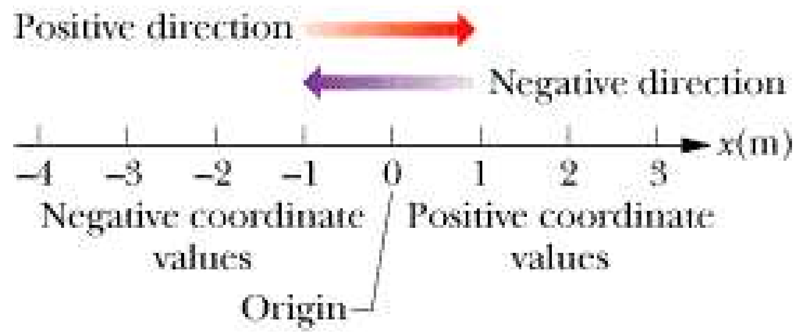
$$v = 9.2 - (6.3)(3.5)^2 = -68 \text{ m/s.} \quad \text{(Answer)}$$

Sample Problem 2-4 Build your skill

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.



(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with a in meters per second squared.

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

CHECKPOINT 4

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the above equations apply?

$$(1)\ v = \frac{dx}{dt} = 3 \qquad a = \frac{d^2x}{dt^2} = 0 \ldots.\ constant$$

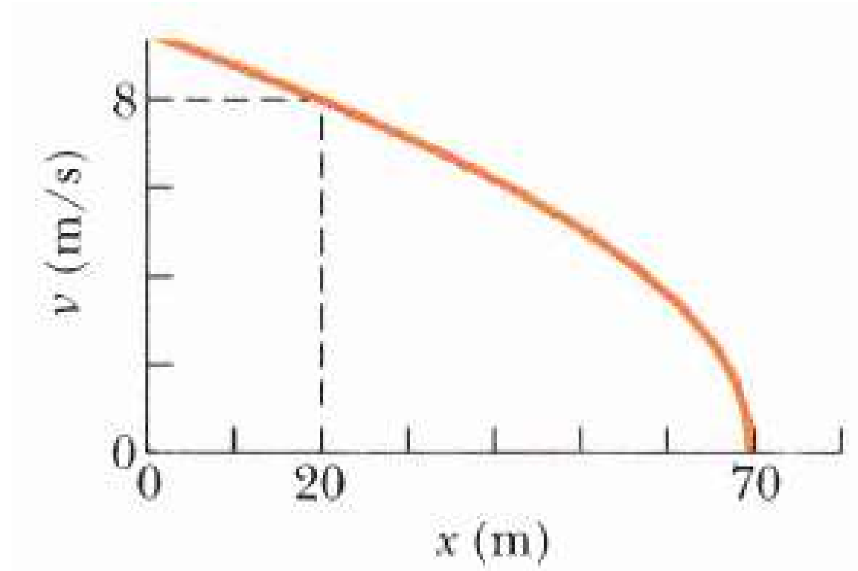
$$(2)\ v = \frac{dx}{dt} = -15t^2 + 8t \qquad a = \frac{d^2x}{dt^2} = -30t + 8 \ldots.\ not\ constant$$

$$(3)\ a = \frac{d^2x}{dt^2} = not\ constant$$

$$(4)\ v = \frac{dx}{dt} = 10t \qquad a = \frac{d^2x}{dt^2} = 10 \ldots.\ constant$$

Sample Problem

The figure gives a particle's velocity v versus its position as it moves along an x axis with constant acceleration. What is its velocity at position x = O?



From the graph, We have: *v = 0 and x = 70 m*. then using

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(0\ \mathrm{m/s})^2 = (8\ \mathrm{m/s})^2 + 2a(70\ \mathrm{m} - 20\ \mathrm{m})$$

which gives us $a = -0.64\ m/s^2$

Also we have: *v = 8 m/s and x = 20 m,*

$$(8\ \mathrm{m/s})^2 = v_0^2 + 2a(20\ \mathrm{m} - 0)$$

Then substituting for *a* and solving for v0 results in $v_0 = 9.5\ \mathrm{m/s}.$

CHECKPOINT 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

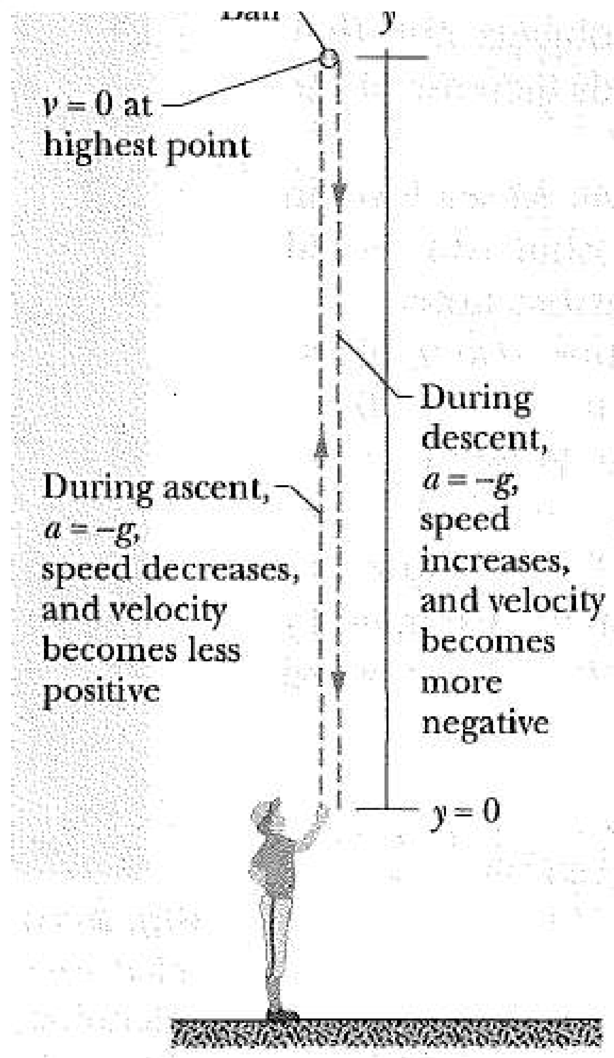
Sample Problem

A pitcher tosses a baseball up along a *y axis, with* an initial speed of 12 *m/s.*
(a) How long does the ball take to reach its maximum height?

Calculation: **Knowing** *v, a*, and the initial velocity Vo = 12 m/s, and seeking t, we solve the equation

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{0 - 12\ \mathrm{m/s}}{-9.8\ \mathrm{m/s^2}} = 1.2\ \mathrm{s}$$



Sample Problem

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point as $y_0 = 0$. Set *$y – y_0 = y$ and $v = 0$ (at the maximum height), and solve the equation*

$$v^2 = v_0^2 + 2ay \qquad y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 7.3\text{ m}$$

(C) How long does the ball take to reach a point 5.0 m above its release point?

Calculation: We know y_0, $a=-g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we set $y_0=0$ and use the equation

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \longrightarrow y = v_0 t - \tfrac{1}{2}gt^2$$

$$5.0\text{ m} = (12\text{ m/s})t - (\tfrac{1}{2})(9.8\text{ m/s}^2)t^2$$

$$\longrightarrow 4.9t^2 - 12t + 5.0 = 0 \longrightarrow t = 0.53\text{ s} \quad \text{and} \quad t = 1.9\text{ s}$$

Sample Problem

A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

SOLUTION

(a)
$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$40 \text{ m} = \frac{1}{2}\left(v_0 + 2.8\frac{\text{m}}{\text{s}}\right)(8.5\ s) \qquad v_0 = 6.61 \text{ m/s}$$

(b)
$$a = \frac{v - v_0}{t} = \frac{2.8 - 6.61 \text{ m/s}}{8.5 \text{ s}} = -0.448 \text{ m/s}^2$$

Samples of Exam Questions

Displacement

Q.14 The position of a ball thrown vertically upward is given by the equation y = 10.0+12.0t- 5.00t^2 (SI units), the height at t=0 is:

(A) 15 m (B) 1 m (C) 5 m (D) Zero (E) 10 m

$$\boxed{y = 10 + 12t - 5t^2 \quad \Rightarrow \quad y(t=0) = 10\ \text{m}}$$

Average & instantaneous Velocity

Q.8 A bicycle travels 12 km in 90 min. Its average speed is:
(A) 48 km/h (B) 18 km/h (C) 8 km/h (D) 0.3 km/h (E) 36 km/h

$$\boxed{\begin{aligned} t &= 90\ \text{min} = \frac{90}{60} = 1.5\ \text{h} \\ v_{avg} &= \frac{\Delta x}{\Delta t} = \frac{12}{1.5} = 8\ \text{km/h} \end{aligned}}$$

Q.7 A bicycle travels 15 km in 30 min. Its average speed is:
(A) 48 km/h (B) 18 km/h (C) 8 km/h (D) 0.3 km/h (E) 30 km/h

$$\boxed{\begin{aligned} t &= 30\ \text{min} = \frac{30}{60} = 0.5\ \text{h} \\ v_{avg} &= \frac{\Delta x}{\Delta t} = \frac{15}{0.5} = 30\ \text{km/h} \end{aligned}}$$

Average & instantaneous Velocity

Q.6 A car moves along a straight line with velocity in m/s given by $\upsilon = t^2 + 3$. The velocity at t=0 is:

(A) zero (B) 4 m/s (C) 3 m/s (D) 2 m/s (E) 6 m/s

$$\boxed{v = t^2 + 3 \quad \Rightarrow \quad v(t=0) = 3 \ \text{m/s}}$$

Q.13 A car moves along the x-axis with constant deceleration, the speed of the car is:

(A) Decreasing (B) Increasing (C) 9.8 m/s^2 (D) Zero (E) none of these

Q.12 An object falling toward the earth's surface will have velocity that its magnitude is: (Ignore air resistance)

(A) Decreasing (B) Zero (C) 9.8 m/s^2 (D) Increasing (E) none of these

Average & instantaneous Velocity

Q.8 The position of an object is given by $x = 4t + 2t^2$. Its average velocity over the time interval from t = 0 to t = 4 s is:

(A) 8 m/s (B) 10 m/s (C) 12 m/s (D) 14 m/s (E) 16 m/s

$$\mathrm{x(t=4) = 4x4 + 2x4^2 = 48m} \qquad \mathrm{x(t=0) = 0}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{\mathrm{x(t=4) - x(t=0)}}{4-0} = \frac{48-0}{4-0} = 12\ \mathrm{m/s}$$

Average & instantaneous Acceleration

Q.4 The instantaneous acceleration $\vec{a}$ is given as:

(A) $\frac{dx}{dt}$ (B) $\frac{d}{dt}\left(\frac{d^2x}{dt^2}\right)$ (C) $\frac{d^2}{dt^2}\left(\frac{dx}{dt}\right)$ (D) $\frac{d^2}{dt^2}\left(\frac{dv}{dt}\right)$ $\boxed{\text{(E) } \frac{d}{dt}\left(\frac{dx}{dt}\right)}$

Q.5 A particle is moving along the negative x-axis with constant velocity. The magnitude of its acceleration is:

(A) -9.8 m/s^2 (B) zero (C) constant (D) 9.8 m/s^2 (E) 980 cm/s^2

Since the particle moves with constant velocity, its acceleration is zero

Average & instantaneous Acceleration

Q.6 A car moves along a straight line with velocity in m/s given by $\upsilon = t^2 + 3$. The velocity at t=0 is:

(A) zero (B) 4 m/s (C) 3 m/s (D) 2 m/s (E) 6 m/s

Q.7 Referring to question 6, the acceleration of the car at t = 4s is:

(A) 6 m/s^2 (B) 8 m/s^2 (C) 10 m/s^2 (D) 12m/s^2 (E) 4m/s^2

$$\boxed{a = \frac{dv}{dt} = 2t \quad \Rightarrow \quad a(t=4) = 2\text{x}4 = 8\ \text{m/s}^2}$$

Average & instantaneous Acceleration

Q.9 A particle is moving along a straight line. At t=3s its velocity is 20 m/s and at t=8s its velocity is zero. The average acceleration is:

(A) -6 m/s^2 (B) -2 m/s^2 (C) -3 m/s^2 (D) -4m/s^2 (E) -5 m/s^2

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t=8) - v(t=3)}{8-3} = \frac{0-20}{8-3} = \frac{-20}{5} = -4 \text{ m/s}^2$$

Constant Acceleration

Q.10 A car travels in a straight line with an initial velocity of 4 m/s and an acceleration of 2 m/s^2. The distance traveled in 4s is:

((A) 36 m (B) 40 m (C) 24 m (D) 28 m (E) 32 m

$v_0 = 4\ \mathrm{m/s} \qquad a = 2\ \mathrm{m/s^2} \qquad t = 4\ \mathrm{s} \qquad x\text{-}x_0\ (?) \qquad \Rightarrow \qquad v\ \text{(missed)}$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = 4\mathrm{x}4 + \frac{1}{2}\mathrm{x}2\mathrm{x}4^2 = 32\ \mathrm{m}$$

Constant Acceleration

Q.11 A car, initially at rest, travels 32 m in 4 s along a straight line with constant acceleration. The acceleration of the car is:

(A) 4 m/s^2 (B) 5 m/s^2 (C) 6 m/s^2 (D) 2 m/s^2 (E) 3 m/s^2

$$v_0 = 0\ \text{m/s} \qquad a = ? \qquad t = 4\ \text{s} \qquad x\text{-}x_0 = 32\ \text{m} \quad \Rightarrow \quad v\ \text{(missed)}$$

$$\boxed{x - x_0 = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad \Rightarrow \quad a = \frac{2(x\text{-}x_0)}{t^2} = \frac{2\text{x}32}{4\text{x}4} = 4\ \text{m/s}^2}$$

Constant Acceleration

Q.12 What is the initial speed of a car moving a distance of 60 m in 6 s if the final speed was 15 m/s?

(A) 15 m/s (B) 10 m/s (C) 5 m/s (D) zero (E) 20 m/s

$v_0 = ?$ $\quad v = 15\ \text{m/s}$ $\quad t = 6\ \text{s}$ $\quad x\text{-}x_0 = 60\ \text{m}$ $\quad \Rightarrow \quad a$ (missed)

$$x - x_0 = \frac{1}{2}(v + v_0)t \quad \Rightarrow \quad v + v_0 = \frac{2(x - x_0)}{t}$$

$$\Rightarrow \quad v_0 = \frac{2(x - x_0)}{t} - v = \frac{2\text{x}60}{6} - 15 = 5\ \text{m/s}$$

Constant Acceleration

Q.30 A car moving with constant acceleration covers the distance between two points 60 m apart in 4 seconds. If its speed as it passes the second point is 20 m/s, its speed at the first point is:

(A) 20 m/s (B) 10 m/s (C) 5 m/s (D) 45 m/s (E) 30 m/s

$v_0 = ?$ $\quad v = 20\text{ m/s}$ $\quad t = 4\text{ s}$ $\quad x\text{-}x_0 = 60\text{ m}$ $\quad \Rightarrow \quad a$ (missed)

$$x - x_0 = \frac{1}{2}(v + v_0)t \quad \Rightarrow \quad v + v_0 = \frac{2(x - x_0)}{t}$$

$$\Rightarrow \quad v_0 = \frac{2(x - x_0)}{t} - v = \frac{2\text{x}60}{4} - 20 = 10\text{ m/s}$$

Constant Acceleration

Q.7 A car uniformly changes its speed from 20 m/s to 5 m/s in 5 s. The distance moved in the third second is:
(A) 56 m (B) 46.5 m (C) 34 m (D) 12.5 m (E) 9.5 m

(1) $v_0 = 20\text{ m/s}$ $\quad v = 5\text{ m/s}$ $\quad t = 5\text{ s}$ $\quad x\text{-}x_0(t = 5\text{ s}) = \text{missed}$ $\quad \&\ a = ?$

$$v = v_0 + at \quad \Rightarrow \quad v - v_0 = at \quad \Rightarrow \quad a = \frac{v - v_0}{t} = \frac{5-20}{5} = \text{-3 m/s}^2$$

(2) $v_0 = 20\text{ m/s}$ $\quad t = 3\text{ s}$ $\quad x\text{-}x_0(t = 3\text{ s}) = ?$ $\quad a = \text{-}3\text{ m/s}^2$ $\quad v\ \text{(missed)}$

$$x\text{-}x_0 = v_0 t + \frac{1}{2}at^2 = 20\text{x}3 + \frac{1}{2}\text{x(-3)x}3^2 = 60 - 13.5 = 46.5\text{ m}$$

Q.6 A car uniformly changes its speed from 20 m/s to 5 m/s in 5 s. The distance moved in the fourth second is:
(A) 56 m (B) 9.5 m (C) 62.5 m (D) 3 m (E) 46.5 m

(1) $v_0 = 20\text{ m/s}$ $\quad v = 5\text{ m/s}$ $\quad t = 5\text{ s}$ $\quad x\text{-}x_0(t = 5\text{ s}) = \text{missed}$ $\quad \&\ a = ?$

$$a = \frac{v - v_0}{t} = \frac{5-20}{5} = \text{-3 m/s}^2$$

(2) $v_0 = 20\text{ m/s}$ $\quad t = 4\text{ s}$ $\quad x\text{-}x_0(t = 4\text{ s}) = ?$ $\quad a = \text{-}3\text{ m/s}^2$ $\quad v\ \text{(missed)}$

$$x\text{-}x_0 = v_0 t + \frac{1}{2}at^2 = 20\text{x}4 + \frac{1}{2}\text{x(-3)x}4^2 = 80 - 24 = 56\text{ m}$$

Free fall acceleration

Q.6 An object thrown vertically upwards will have velocity that its magnitude is: (Ignore air resistance)
(A) Zero (B) Increasing (C) Constant $\boxed{\text{(D) Decreasing}}$ (E) none of these

Q.10 At the earth's surface, a ball thrown straight up from a bridge would have an acceleration of magnitude:
(A) less than 9.8 m/s^2 $\boxed{\text{(B) 9.8 } m/s^2}$ (C) more than 9.8 m/s^2 (D) Zero (E) none of these

Q.13 A baseball is thrown vertically up into the air. The acceleration of the ball at its highest point is:
(A) -19.6 m/s^2 (B) 19.6 m/s^2 (C) + 9.8 m/s^2 (D) - 9.8 m/s^2 (E) zero

The acceleration is a vector, then it is equal to -9.8 m/s^2

Free fall acceleration

Q.14 An object is thrown straight up from ground level with a speed of 30 m/s. Its height after 1.0 s is:

(A) 15.1 m (B) 5.1 m (C) 45.1 m (D) 35.1 m (E) 25.1 m

$$v_0 = 30\,\text{m/s} \qquad g = 9.8\,\text{m/s}^2 \qquad t = 1\,\text{s} \qquad y\text{-}y_0 = ? \quad \Rightarrow \quad v\ (\text{missed})$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 = 30\text{x}1 - \frac{1}{2}\text{x}9.8\text{x}1^2 = 30 - 4.9 = 25.1\,\text{m}$$

Free fall acceleration

Q.16 A stone dropped off a 75 m high building reaches the ground in:

(A) 3.91 s (B) 2.86 s (C) 1.35 s (D) 5.53 s (E) 4.95 s

$$v_0 = 0\ \text{m/s (free drop)} \qquad g = 9.8\ \text{m/s}^2 \qquad t = ? \qquad y\text{-}y_0 = -75\ \text{m} \quad \Rightarrow \quad v\ \text{(missed)}$$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2 = 0 - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2 \quad \Rightarrow \quad t^2 = -\frac{2(y\text{-}y_0)}{g}$$

$$t = \sqrt{-\frac{2(y\text{-}y_0)}{g}} = \sqrt{-\frac{2\text{x}(-75)}{9.8}} = 3.91\ \text{s}$$

Q.17 Referring to question 16,the speed of the stone just before reaching the ground is: Morouj Q

(A) 54.2 m/s (B) 48.5 m/s (C) 38.3 m/s (D) 28 m/s (E) zero

$$v_0 = 0\ \text{m/s} \qquad g = 9.8\ \text{m/s}^2 \qquad t = 3.91\ \text{s} \qquad y\text{-}y_0 = -75\ \text{m} \qquad v = ?$$

$$v = v_0 + at = 0 - gt = -9.8\text{x}3.91 = -38.8\ \text{m/s} \Rightarrow \text{speed} = 38.8\ \text{m/s}$$

Free fall acceleration

Q.9 A ball is thrown vertically upward at a speed of 21 m/s. It will reach its maximum height in:
(A) 1.8 s (B) 2.1 s (C) 0.60 s (D) 0.33 s (E) 1.2 s

$$v_0 = 21\,\mathrm{m/s} \qquad v = 0\,\mathrm{m/s} \qquad g = 9.8\,\mathrm{m/s^2} \qquad t = ? \qquad y\text{-}y_0\,(\text{missed})$$

$$v = v_0 - gt \quad \Rightarrow \quad gt = v_0 - v \quad \Rightarrow \quad \mathrm{t} = \frac{v_0 - v}{g} = \frac{21-0}{9.8} = 2.1\,\mathrm{s}$$

Q.12 A ball is thrown vertically upward from ground level to reach a maximum height of 98 m. The initial speed is:
(A) 43.8 m/s (B) 100 m/s (C) 25 m/s (D) 31.3 m/s (E) 49 m/s

$$v = 0\,\mathrm{m/s} \qquad g = 9.8\,\mathrm{m/s^2} \qquad v_0 = ? \qquad y\text{-}y_0 = 98\,\mathrm{m} \qquad t\,(\text{missed})$$

$$v^2 = v_0^2 - 2g(y - y_0) \quad \Rightarrow \quad v_0^2 = v^2 + 2g(y - y_0) = 0 + 2x9.8x98 = 1920.8$$

$$v = 43.8s\,\mathrm{m/s}$$

Free fall acceleration

Q.29 A boy shot a football vertically up with an initial speed v_o. When the ball was 2 m above the ground, the speed was 0.4 of the initial speed. The initial speed is :

(A) 6.8 m/s (B) 3.4 m/s (C) 11.8 m/s (D) 4.8 m/s (E) 19.6 m/s

$$v = 0.4v_0 \qquad g = 9.8\,\text{m/s}^2 \qquad v_0 = ? \qquad y\text{-}y_0 = 2\,\text{m} \qquad t\ \text{(missed)}$$

$$v^2 = v_0^2 - 2g(y - y_0) \quad \Rightarrow \quad v_0^2 = v^2 + 2g(y - y_0) = 0.4\text{x}0.4v_0^2 + 2\text{x}9.8\text{x}2 = 0.16v_0^2 + 39.2$$

$$v_0^2 = 0.16v_0^2 + 39.2 \;\Rightarrow\; v_0^2 - 0.16v_0^2 = 39.2 \quad \Rightarrow \quad 0.84v_0^2 = 39.2$$

$$v_0^2 = \frac{39.2}{0.84} \qquad \Rightarrow \qquad v = 6.8\,\text{m/s}$$