KING ABDULAZIZ UNIVERSITY

FACULTY OF ENGINEERING AT RABIGH

ELECTRICAL ENGINEERING DEPARTMENT

# Engineering Numerical Methods

## Home Work

4

50/50

Excellent work

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# EEN 271: Engineering Numerical Methods Spring 2019

## HW # 4

Q1 For given data - Table (1)

| | | | | |
|---|---|---|---|---|
| 0.90 | 1.42 | 1.30 | 1.55 | 1.63 |
| 1.32 | 1.35 | 1.47 | 1.95 | 1.66 |
| 1.96 | 1.47 | 1.92 | 1.35 | 1.05 |
| 1.85 | 1.74 | 1.65 | 1.78 | 1.71 |
| 2.29 | 1.82 | 2.06 | 2.14 | 1.27 |

Determine **(a)** the mean, **(b)** the standard deviation, **(c)** the variance, **(d)** the coefficient of variation, and **(e)** the 95% confidence interval for the mean. **(f)** construct a histogram using a range from 0.6 to 2.4 with intervals of 0.2.

**Q2 For** given data -Table (2)

| x | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| y | 17 | 24 | 31 | 33 | 37 | 37 | 40 | 40 | 42 | 41 |

Aanswer the following:

a- Do data regression using **least Square Error Criterion** with the appropriate order to obtain the model with its coefficients. Show all details of your calculations.
b- **Using MATLAB**, solve the system obtained in **a**.
c- **Plot** of the original data and fitting model in the same graph.
d- **Compute** and **plot** the error value ***e(i):***

$$e(i) = y(i) - y_{model}(i)$$

**Q3** From the data in Table (2) above;

a- **Determine** the value of the ***y*** at ***x=22*** using the following interpolation methods:
- Second order direct polynomial method
- Second order Lagrange polynomial method
- Second order Newton's divided difference polynomial method

**b-** Using Matlab, **Plot** in one graph the **original** and **interpolated** data for all **three** above methods.

*Dr. Abdullah Alshehri*

Q1

a) The mean $\Rightarrow \quad \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = 1.6244$

b) The standard deviation $S = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = 0.33939$

c) The Variance $S^2 = (0.33939)^2 = 0.115185$

d) The coefficient of Variation $C.V = \frac{S}{\overline{x}} * 100 = \frac{0.33939}{1.6244} \times 100 = 20.89\%$

e) The 95% confidence interal for mean

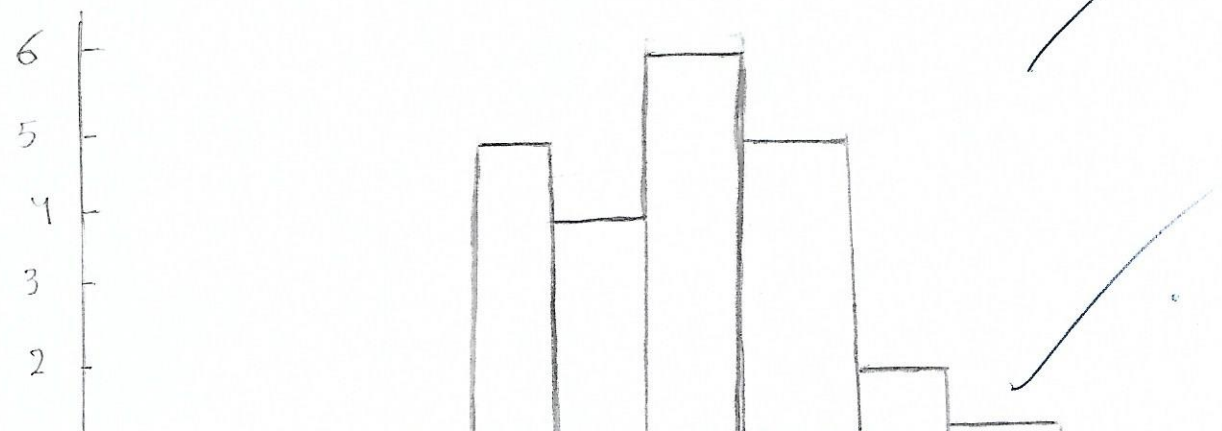
$= [\ \mu - 2S \quad , \quad \mu + 2S\ ]$

$= [(1.6244) - 2(0.33939) \ , \ (1.6244) + 2(0.33939)$

$= [\ 0.94562 \quad , \quad 2.30318\ ]$

F) Number of classes $= \frac{2.4 - 0.6}{0.2} = 9$

| 0.6-0.8 | 0.8-1 | 1-1.2 | 1.2-1.4 | 1.4-1.6 | 1.6-1.8 | 1.8-2 | 2-2.2 | 2.2-2.4 |
|---|---|---|---|---|---|---|---|---|
| — | 1 | 1 | 5 | 4 | 6 | 5 | 2 | 1 |



Q2 $\sum x_i = 275$ , $\sum x_i^2 = 9625$ , $\sum x_i^3 = 378125$ , $\sum x_i^4 =$ 15833125

$n = 10$ , $\sum y_i = 342$ , $\sum y_i x_i = 10425$ , $\sum x_i^2 y_i = 379975$

$$y = a_0 + a_1 x + a_2 x^2$$ ✓

$$\begin{bmatrix} n & \sum (x_i)^1 & \sum (x_i)^2 \\ \sum (x_i)^1 & \sum (x_i)^2 & \sum (x_i)^3 \\ \sum (x_i)^2 & \sum (x_i)^3 & \sum (x_i)^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

$$10 a_0 + 275 a_1 + 9625 a_2 = 342$$

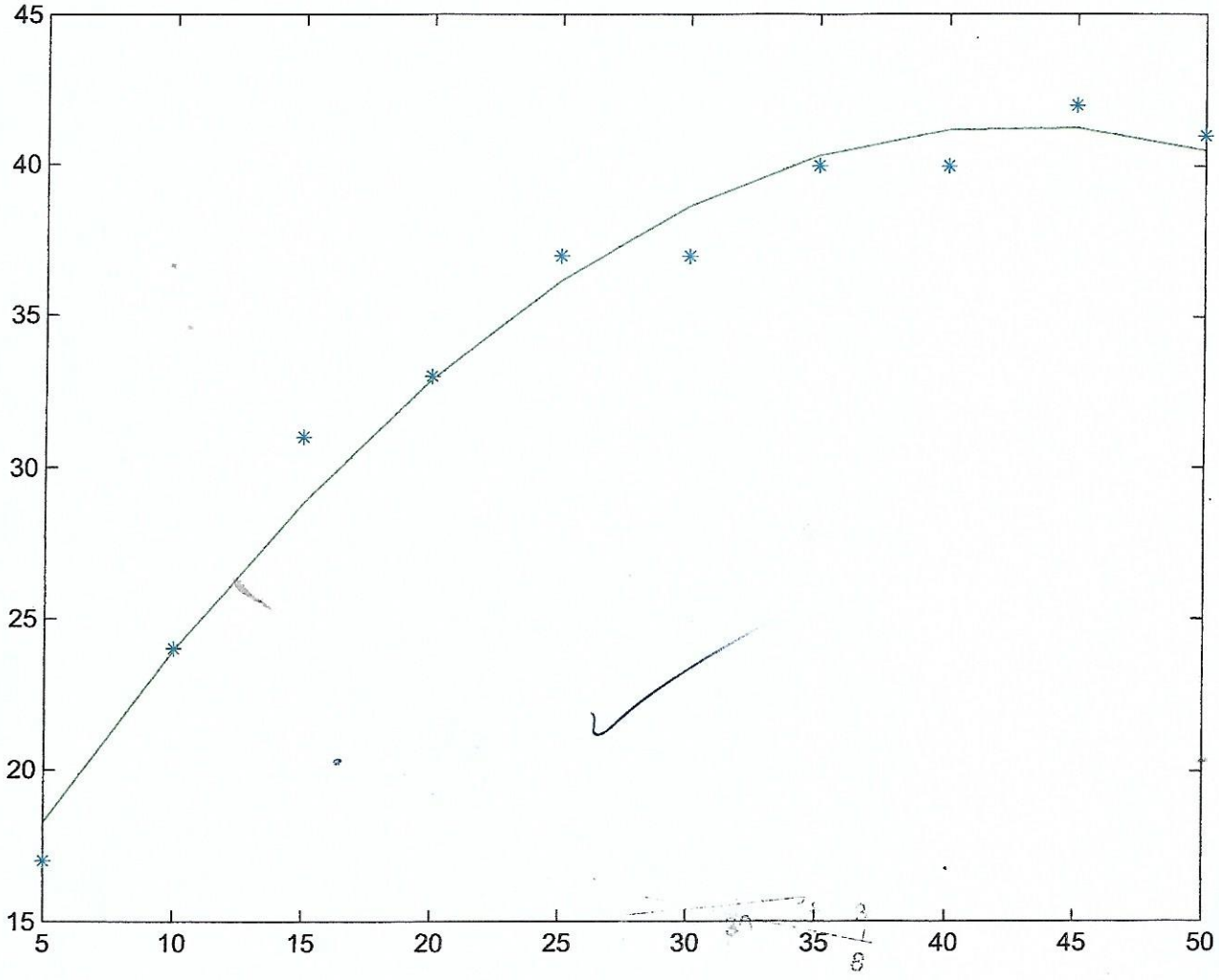
$$275 a_0 + 9625 a_1 + 378125 a_2 = 10425$$

$$9625 a_0 + 378125 a_1 + 15833125 a_2 = 379975$$

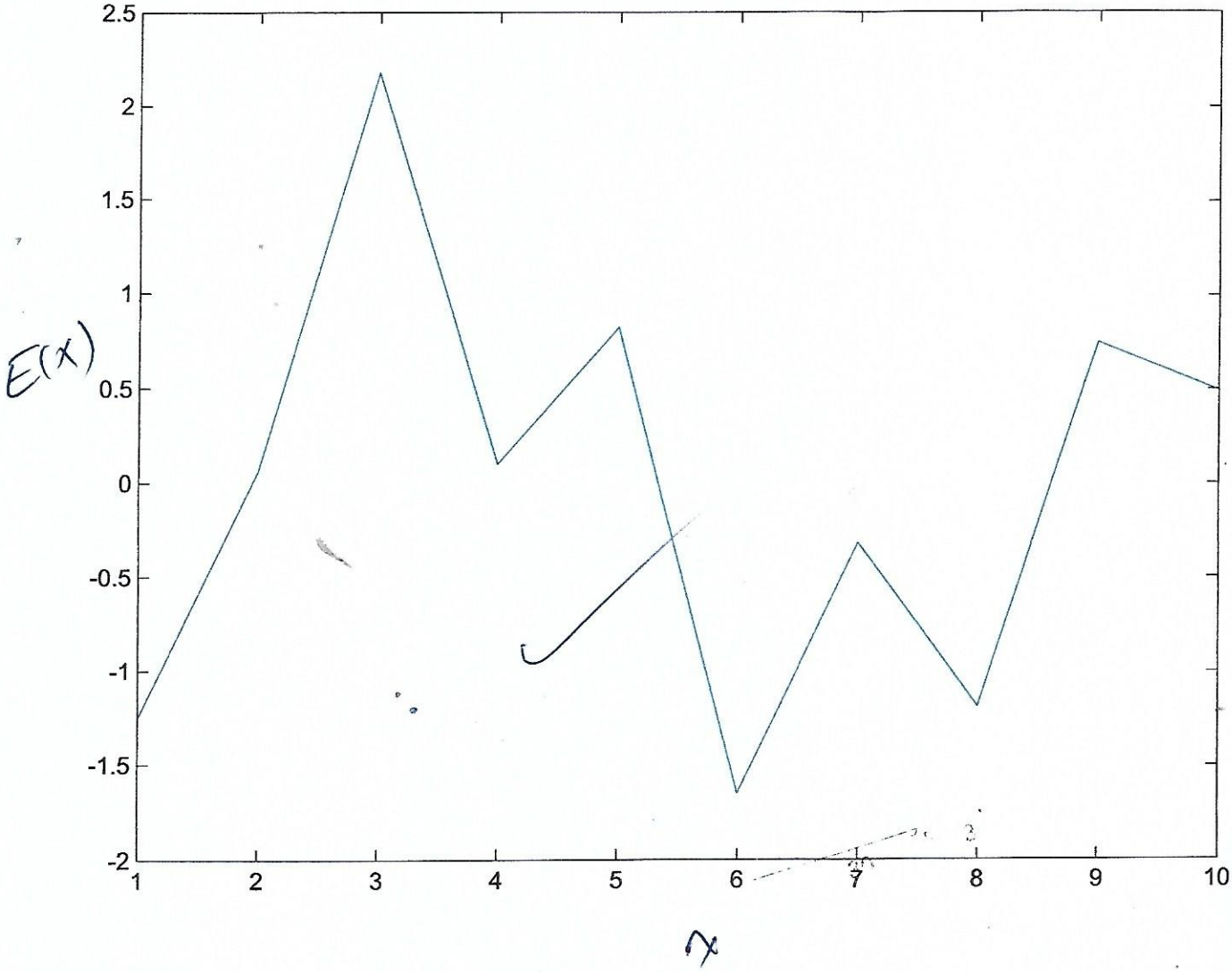
$\therefore a_0 = 11.76667$ ✓ $a_1 = 1.3778$ ✓ , $a_2 = -0.01606$ ✓

```
>> clear
>> x=[5 10 15 20 25 30 35 40 45 50];
>> x

x =

     5    10    15    20    25    30    35    40    45    50

>> y=[17 24 31 33 37 37 40 40 42 41];
>> y

y =

    17    24    31    33    37    37    40    40    42    41

>> A=[10 sum(x) sum(x.^2);sum(x) sum(x.^2) sum(x.^3);sum(x.
^2) sum(x.^3) sum(x.^4)];
>> A

A =

          10         275        9625
         275        9625      378125
        9625      378125    15833125

>> b=[sum(y) sum(x.*y) sum(y.*x.^2)];
>> b

b =

         342       10425      379975

>> a=inv(A)*b';
>> a

a =

   11.7667
    1.3779
   -0.0161

>>
```

```
>> clear
>> x=[5 10 15 20 25 30 35 40 45 50];
>> x

x =

     5    10    15    20    25    30    35    40    45    50

>> y=[17 24 31 33 37 37 40 40 42 41];
>> y

y =

    17    24    31    33    37    37    40    40    42    41

>> p=polyfit(x,y,2);
>> p

p =

   -0.0161    1.3779   11.7667

>> pe=fliplr(p);
>> pe

pe =

   11.7667    1.3779   -0.0161

>> y_e2=pe(1)+pe(2)*x+pe(3)*x.^2;
>> plot(x,y,'*',x,y_e2)
>>
```


45
40
35
30
25
20
15
5
10
15
20
25
30
35
40
45
50

```
>> clear
>> x=[5 10 15 20 25 30 35 40 45 50];
>> x

x =

     5    10    15    20    25    30    35    40    45    50

>> y=[17 24 31 33 37 37 40 40 42 41];
>> y

y =

    17    24    31    33    37    37    40    40    42    41

>> p=polyfit(x,y,2);
>> pe=fliplr(p);
>> y_e2=pe(1)+pe(2)*x+pe(3)*x.^2;
>> e=y-y_e2;
>> e

e =

  Columns 1 through 6

   -1.2545    0.0606    2.1788    0.1000    0.8242   -1.6485

  Columns 7 through 10

   -0.3182   -1.1848    0.7515    0.4909

>> plot(e)
>>
```


2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1
2
3
4
5
6
7
8
9
10
E(X)
N

Q3

(a)

(i) Polynomial method

$$Y(X) = a_0 + a_1 X + a_2 X_2$$

$X_2 = 25 \Rightarrow Y_2 = 37$

$X_1 = 20 \Rightarrow Y_1 = 33$

$X_0 = 15 \Rightarrow Y_0 = 31$

$$Y(15) = a_0 + a_1(15) + a_2(15)^2 = 31$$

$$Y(20) = a_0 + a_1(20) + a_2(20)^2 = 33$$

$$Y(25) = a_0 + a_1(25) + a_2(25)^2 = 37$$

$a_0 = 37$ , $a_1 = -1$ , $a_2 = 0.04$

$$Y = 37 - X + 0.04X^2 \qquad 15 \leq X \leq 25$$

$$Y(22) = 37 - (22) + 0.04(22)^2 = 34.36$$

(ii) Lagrange polynomial method

$$L_0(x) = \left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)$$

$$L_1(x) = \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)$$

$$L_2(x) = \left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)$$

$x_0 = 15 \Rightarrow f(x_0) = 31$

$x_1 = 20 \Rightarrow f(x_1) = 33$

$x_2 = 25 \Rightarrow f(x_2) = 37$

$$Y(x) = \left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)(f(x_0)) + \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)(f(x_1)) + \left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)(f(x_2))$$

$$Y(22) = \left(\frac{22-20}{15-20}\right)\left(\frac{22-25}{15-25}\right)(31) + \left(\frac{22-15}{20-15}\right)\left(\frac{22-25}{20-25}\right)(33) + \left(\frac{22-15}{25-15}\right)\left(\frac{22-20}{25-20}\right)(37)$$

$$= 34.36$$

(iii) Newton's divided difference polynomial method

$$F_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = F(x_0)$$

$$b_1 = \frac{F(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\frac{F(x_2) - F(x_1)}{x_2 - x_1} - \frac{f(x_1) - F(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$x_0 = 15 \Rightarrow F(x_0) = 31$ ✓

$x_1 = 20 \Rightarrow f(x_1) = 33$

$x_2 = 25 \Rightarrow f(x_2) = 37$

$$b_0 = f(x_0) = f(15) = 31$$

$$b_1 = \frac{F(20) - f(15)}{20 - 15} = \frac{33 - 31}{20 - 15} = 0.4$$

$$b_2 = \frac{\frac{F(25) - F(20)}{25 - 20} - \frac{F(20) - F(15)}{20 - 15}}{25 - 15} = \frac{\frac{37 - 33}{5} - \frac{33 - 31}{5}}{10} = 0.04$$

$$F(22) = 31 + 0.4(22 - 15) + 0.04(22 - 15)(22 - 20) = 34.36$$ ✓

Q3

(B) Interpolation with Matlab

```
yi = interp1(x,y,xi)
yi = interp1(y.xi)
yi = interp1(x,y,xi,method)
x = 0:10;
y = sin(x);
xi = 0:.25:10;
yi = interp1(x,y,xi);
plot(x.y,'o',xi,yi)
```

we don't have Sine wave function !