KINGDOM OF SAUDI ARABIA
Ministry of Higher Education
KING ABDULAZIZ UNIVERSITY
Faculty of Sciences
Department of Statistics

Entrance Examination
for PhD Degree
20/06/1439
Time: 3 Hours

You have four sections (A, B, C, D). Select 2 questions from each section and answer them. Best Wishes.

Section A:

1) Suppose a random sample of size n is available from the distribution

$$f(x;p) = p(1-p)^{x-1}\ ;\ x = 1,2,3,\ldots\ldots,\infty.$$

Find the maximum likelihood estimator of p.

2) Find the moment estimator of θ if a random sample of size n is available from the distribution

$$f(x;\theta) = \theta x^{\theta-1}\ ;\ 0 < x < 1.$$

3) Suppose we have a sample of n observation from $N(0;\theta)$. Show that $\sum_{i=1}^{n} X_i^2$ is a sufficient statistics for θ.

Section B:

1) Suppose a random sample of size 10 is available from the distribution

$$P(X = x) = p^x(1-p)^{1-x}\ ;\ x = 0,1.$$

Using a sample observation, a hypothesis $H_0: p = 0.2$ is to be tested against $H_1: p = 0.6$. The hypothesis will be rejected if $\sum_{i=1}^{10} X_i > 4$.
Compute the probability of Type–I error for the test and its power.

2) A sample of size 4 is drawn from Poisson distribution with parameter λ. Find the most powerful test of size α for testing $H_0: \lambda = 2$ against $H_1: \lambda = 3$.

3) A random sample of size n is available from the distribution

$$f(x;\theta) = \frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right);\ x > 0\ ;\ \theta > 0.$$

Use the Generalized Likelihood Ratio method to develop a test statistic for testing the hypothesis $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.

Section C:

1) Let X be a random variable having pdf,

$$f(x) = \frac{2}{35}(x+3);\ -2 < x < 3.$$

Find the distribution of $z = x^2$.

2) Find the distribution for which the Characteristic function (cf) is $\phi(t) = (q + pe^{it})^n$.

3) Let X be a Poisson random variable with probability function,

$$p_k = P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!};\ k = 0,1,2,\ldots$$

Find the mean and variance using the probability generating function (pgf) of Poisson distribution.

Section D:

1) For the regression model $Y_i = \beta_0 + \epsilon_i$. Derive the least squares estimator of β_0 for this model and Prove that the least squares estimator of β_0 obtained is unbiased.

2) Consider the simple linear model, $Y = \alpha + \beta X + e$ with $e \sim N(0, \sigma_e^2)$ in usual notation. Prove that $Var(\hat{\alpha}) = \frac{\sum_{i=1}^{n} X_i^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sigma_e^2$.

3) Let $y = X\beta + e$, where X is an $n\times(k+1)$ matrix of full rank, β is a $(k+1)\times 1$ vector of unknown parameters, and e is an $n\times 1$ random vector with mean 0 and variance $\sigma^2 I$. Show that the least squares estimator b is the best linear unbiased estimator (BLUE) for β.