

## Motion in TWo And

## Three Dimensions



## 4-1 position and displacement

## Position and displacement

## Position

$$
\vec{r}=x \hat{\mathrm{i}}+y \hat{\mathrm{j}}+z \hat{\mathrm{k}}
$$

## Displacement

$$
\Delta \vec{r}=\vec{r}_{2}-\vec{r}_{1}
$$

$$
\vec{r}_{1}=x_{1} \hat{\mathrm{i}}+y_{1} \hat{\mathrm{j}}+z_{1} \hat{\mathrm{k}} \text { and } \vec{r}_{2}=x_{2} \hat{\mathrm{i}}+y_{2} \hat{\mathrm{j}}+z_{2} \hat{\mathrm{k}}
$$



$$
\Delta \vec{r}=\left(x_{2} \hat{\mathbf{i}}+y_{2} \hat{\mathbf{j}}+z_{2} \hat{\mathbf{k}}\right)-\left(x_{1} \hat{\mathbf{i}}+y_{1} \hat{\mathbf{j}}+z_{1} \hat{\mathrm{k}}\right)
$$

$$
\Delta \vec{r}=\left(x_{2}-x_{1}\right) \hat{\mathrm{i}}+\left(y_{2}-y_{1}\right) \hat{\mathrm{j}}+\left(z_{2}-z_{1}\right) \hat{\mathrm{k}}
$$

$$
\Delta \vec{r}=\Delta x \hat{\mathrm{i}}+\Delta y \hat{\mathrm{j}}+\Delta z \hat{\mathrm{k}}
$$

## Problem 3 page 73

An elementary particle is subjected to a displacement of

$$
\Delta \vec{r}=2.0 \hat{\imath}-4.0 \hat{\jmath}+8.0 \hat{k}
$$

ending with the position vector $\vec{r}=4.0 \hat{\jmath}-5.0 \hat{k}$ What was the particle's initial position vector?

$$
\Delta \vec{r}=\vec{r}_{2}-\vec{r}_{1}
$$

$$
\overrightarrow{r_{1}}=\overrightarrow{r_{2}}-\Delta \vec{r}
$$

$$
=4.0 \hat{\jmath}-5.0 \hat{k}-(2.0 \hat{\imath}-4.0 \hat{\jmath}+8.0 \hat{k})
$$

$$
\overrightarrow{r_{1}}=-2 \hat{\imath}+8 \hat{\jmath}-13 \hat{k}
$$

## Sample Problem 4.01

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time $t$ (seconds) are given by

$$
\begin{align*}
& x=-0.31 t^{2}+7.2 t+28  \tag{4-5}\\
& y=0.22 t^{2}-9.1 t+30 . \tag{4-6}
\end{align*}
$$

(a) At $t=15 \mathrm{~s}$, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?


$$
\vec{r}=x \hat{\dot{\mathrm{i}}}+y \hat{\mathrm{j}}
$$

$$
\text { At } t=15 \mathrm{~s}, \quad x=(-0.31)(15)^{2}+(7.2)(15)+28=66 \mathrm{~m}
$$

$$
y=(0.22)(15)^{2}-(9.1)(15)+30=-57 \mathrm{~m}
$$

$$
\vec{r}=(66 \mathrm{~m}) \hat{\mathrm{i}}-(57 \mathrm{~m}) \hat{\mathrm{j}}
$$

$$
\begin{aligned}
r=\sqrt{x^{2}+y^{2}} & =\sqrt{(66 \mathrm{~m})^{2}+(-57 \mathrm{~m})^{2}}=87 \mathrm{~m} \\
4 & \theta
\end{aligned}=\tan ^{-1} \frac{y}{x}=\tan ^{-1}\left(\frac{-57 \mathrm{~m}}{66 \mathrm{~m}}\right)=-41^{\circ} .
$$

4.2 Average velocity and instantaneous velocity

## Average velocity

$$
\vec{v}_{\mathrm{avg}}=\frac{\Delta \vec{r}}{\Delta t} .
$$

$\vec{v}_{\text {avg }}=\frac{\Delta x \hat{\mathrm{i}}+\Delta y \hat{\mathrm{j}}+\Delta z \hat{\mathrm{k}}}{\Delta t}=\frac{\Delta x}{\Delta t} \hat{\mathrm{i}}+\frac{\Delta y}{\Delta t} \hat{\mathrm{j}}+\frac{\Delta z}{\Delta t} \hat{\mathrm{k}}$.
Instantaneous velocity

$$
\begin{aligned}
& \vec{v}=\frac{d \vec{r}}{d t} . \text { but } \vec{r}=x \hat{\mathrm{i}}+y \hat{\mathrm{j}}+z \hat{\mathrm{k}}, \\
& \vec{v}=\frac{d}{d t}(x \hat{\mathrm{i}}+y \hat{\mathrm{j}}+z \hat{\mathrm{k}})=\frac{d x}{d t} \hat{\mathrm{i}}+\frac{d y}{d t} \hat{\mathrm{j}}+\frac{d z}{d t} \hat{\mathrm{k}} \\
& v_{x}=\frac{d x}{d t} \quad v_{y}=\frac{d y}{d t}, \quad v_{z}=\frac{d z}{d t} \Rightarrow \vec{v}=v_{x} \hat{\mathrm{i}}+v_{y} \hat{\mathrm{j}}+v_{z} \hat{\mathrm{k}}
\end{aligned}
$$

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.


## Sample Problem 4.02

For the rabbit in Sample Problem 4.01 find the velocity $\vec{v}$ at time $t=15 \mathrm{~s}$.

$$
\vec{v}=v_{x} \hat{i}+v_{y} \hat{\mathrm{j}}
$$

$$
\begin{aligned}
v_{x} & =\frac{d x}{d t} \quad x=-0.31 t^{2}+7.2 t+28 \\
& =(-0.31)(2) t+7.2 \\
& =-0.62 t+7.2
\end{aligned}
$$

$$
\begin{aligned}
v_{y}= & \frac{d y}{d t} \quad y=0.22 t^{2}-9.1 t+30 \\
& =(0.22)(2) t-9.1 \\
& =0.44 t-9.1
\end{aligned}
$$

At $t=15 \mathrm{~s}, \quad v_{x}=-2.1 \mathrm{~m} / \mathrm{s} \quad$ At $t=15 \mathrm{~s}, \quad v_{y}=-2.5 \mathrm{~m} / \mathrm{s}$.

$$
\vec{v}=-2.1 \hat{\imath}-2.5 \hat{\jmath}
$$

$$
v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(-2.1 \mathrm{~m} / \mathrm{s})^{2}+(-2.5 \mathrm{~m} / \mathrm{s})^{2}}=3.3 \mathrm{~m} / \mathrm{s}
$$

$$
\theta=\tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1}\left(\frac{-2.5 \mathrm{~m} / \mathrm{s}}{-2.1 \mathrm{~m} / \mathrm{s}}\right)=50^{\circ}
$$

## 4-3 Average Acceleration and Instantaneous Acceleration

## Average Acceleration

$$
\vec{a}_{\mathrm{avg}}=\frac{\vec{v}_{2}-\vec{v}_{1}}{\Delta t}=\frac{\Delta \vec{v}}{\Delta t}
$$

Instantaneous Acceleration

$$
\begin{gathered}
\vec{a}=\frac{d \vec{v}}{d t} \quad \text { but } \vec{v}=v_{x} \hat{\mathrm{i}}+v_{y} \hat{\mathrm{j}}+v_{z} \hat{\mathrm{k}} \\
\vec{a}=\frac{d}{d t}\left(v_{x} \hat{\mathrm{i}}+v_{y} \hat{\mathrm{j}}+v_{z} \hat{\mathrm{k}}\right)=\frac{d v_{x}}{d t} \hat{\mathrm{i}}+\frac{d v_{y}}{d t} \hat{\mathrm{j}}+\frac{d v_{z}}{d t} \hat{\mathrm{k}} \\
a_{x}=\frac{d v_{x}}{d t} \quad a_{y}=\frac{d v_{y}}{d t} \quad a_{z}=\frac{d v_{z}}{d t} \Rightarrow \vec{a}=a_{x} \hat{\mathrm{i}}+a_{y} \hat{\mathrm{j}}+a_{z} \hat{\mathrm{k}}
\end{gathered}
$$

For the rabbit in Sample Problems 4.01 and 4.02 find the acceleration $\vec{a}$ at time $t=15 \mathrm{~s}$.

$$
\vec{a}=a_{x} \hat{\mathrm{i}}+a_{y} \hat{\mathrm{j}}
$$

$$
\begin{aligned}
a_{x} & =\frac{d v_{x}}{d t} \quad v_{x}=-0.62 t+7.2 & a_{y} & =\frac{d v_{y}}{d t} \quad v_{y}=0.44 t-9.1 \\
& =-0.62 \mathrm{~m} / \mathrm{s}^{2} . & & =0.44 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
\begin{gathered}
\vec{a}=\left(-0.62 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathrm{i}}+\left(0.44 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathrm{j}} \\
a=\sqrt{a_{x}^{2}+a_{y}^{2}}=\sqrt{\left(-0.62 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(0.44 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=0.76 \mathrm{~m} / \mathrm{s}^{2} . \\
\theta=\tan ^{-1} \frac{a_{y}}{a_{x}}=\tan ^{-1}\left(\frac{0.44 \mathrm{~m} / \mathrm{s}^{2}}{-0.62 \mathrm{~m} / \mathrm{s}^{2}}\right)=-35^{\circ} .
\end{gathered}
$$

## Problem 15 page 74

From the origin, a particle starts at $\mathrm{t}=0 \mathrm{~s}$ with a velocity $\vec{v}=7.0 \hat{\imath} \mathrm{~m} / \mathrm{s}$ and moves in the xy plane with a constant acceleration of $\vec{a}=-9.0 \hat{\imath}+3.0 \hat{\jmath}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$. At the time particle reaches the maximum x coordinate what is it's (a) velocity and (b) position vector?

$$
\text { (a) } \vec{v}=? ? ?
$$

$$
\vec{v}=2.34 \hat{\jmath}
$$

$$
\begin{aligned}
& \overrightarrow{v_{o}}=7.0 \hat{\imath} \\
& \overrightarrow{v_{o}}=v_{o x} \hat{\imath}+v_{o y} \hat{\jmath} \\
& \vec{a}=-9.0 \hat{\imath}+3.0 \hat{\jmath} \quad v_{x}=0 \\
& \begin{array}{r}
\vec{a}=a_{x} \hat{\imath}+a_{y} \hat{\jmath} \\
v_{y}=? ?
\end{array} \\
& v_{y}=v_{0 y}+a_{y} t<t=? ? ? \\
& v_{x}=v_{0 x}+a_{x} t \\
& 0=7-9 t \\
& t=0.78 \\
& v_{y}=v_{0 y}+a_{y} t \\
& v_{y}=v_{0 y}+a_{y} t \\
& y-y_{0}=v_{0,} t+\frac{1}{2} a_{y} t^{2} \\
& v^{2}=v_{0}^{2}+2 a_{y}\left(y-y_{0}\right) \\
& y-y_{0}=\frac{1}{2}\left(\nu_{0 y}+v_{y}\right) t \\
& y-y_{0}=v_{y} t-\frac{1}{2} a_{y} t^{2} \\
& =0+(3)(0.78)=2.34 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) position vector?

$$
\begin{aligned}
& \begin{array}{lcc}
\overrightarrow{v_{o}}=7.0 \hat{\imath} & \vec{a}=-9.0 \hat{\imath}+3.0 \hat{\jmath} & \vec{v}=2.34 \hat{\jmath} \\
\overrightarrow{v_{o}}=v_{o x} \hat{\imath}+v_{o y} \hat{\jmath} & \vec{a}=a_{x} \hat{\imath}+a_{y} \hat{\jmath}
\end{array} \quad t=0.78 s \\
& \overrightarrow{v_{o}}=v_{o x} \hat{\imath}+v_{o y} \hat{\jmath} \\
& \vec{r}=x \hat{\dot{i}}+y \hat{\mathrm{j}} \\
& x-x_{0}=v_{0 x} t+\frac{1}{2} a_{x} t^{2} \\
& x-0=(7)(0.78)+\frac{1}{2}(-9)(0.78)^{2} \\
& x=2.7 m \\
& y-y_{0}=v_{0 y} t+\frac{1}{2} a_{y} t^{2} \quad v_{y}=v_{0 y}+a_{y} t \\
& y-0=0+\frac{1}{2}(3)(0.78)^{2} \\
& y=0.91 m \\
& \vec{r}=2.7 \hat{\mathrm{i}}+0.91 \hat{\mathrm{j}} \\
& y-y_{0}=v_{0 y} t+\frac{1}{2} a_{y} t^{2} \\
& v^{2}=v_{0}^{2}+2 a_{y}\left(y-y_{0}\right) \\
& y-y_{0}=\frac{1}{2}\left(v_{0 y}+v_{y}\right) t \\
& y-y_{0}=v_{y} t-\frac{1}{2} a_{y} t^{2}
\end{aligned}
$$

### 4.4 Projectile Motion

is the motion of a particle that is launched with an initial Velocity $\vec{v}_{0}$ and its acceleration is always the free fall Acceleration -g.



Types of Projectiles





In projectile motion, a particle is launched into the air with a speed vo and angle $\theta$ o (measured From $x$-axis)



In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

Projectile motion



$$
H=\frac{\left(v_{0} \sin \theta_{0}\right)^{2}}{2 g}
$$

## Problem 22 page 74 :

A small ball rolls horizontally off the edge of a tabletop that is 1.50 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b)What is its speed at the instant it leaves the table?

$$
\theta_{o}=0 \quad y=1.5 m \quad x=1.52 m
$$

a)

$$
t=? ?
$$

$$
y=v_{0} \sin \theta_{0} t-\frac{1}{2} g t^{2}
$$

$$
-1.5=v_{o} \sin (0)-\frac{1}{2}(9.8) t^{2}
$$

$$
t=0.55 \mathrm{~s}
$$

b) $\quad v_{0}=? ?$


$$
\begin{aligned}
& x=v_{0} \cos \theta_{0} t \\
& 1.52=v_{o} \cos (0) 0.55 \quad v_{0}=2.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Problem 32 page 75:

You throw a ball toward a wall at speed $25.0 \mathrm{~m} / \mathrm{s}$ and at angle $\theta_{0}=40.0^{\circ}$ above the horizontal . The wall is distance $\mathrm{d}=22.0 \mathrm{~m}$ from the release point of the ball.
(a)How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall?

$$
v_{o}=25 \mathrm{~m} / \mathrm{s} \quad \theta_{0}=40.0 \quad \mathrm{x}=22.0 \mathrm{~m}
$$

(a)

$$
\begin{aligned}
& \boldsymbol{y}=v_{\mathbf{0}} \sin \theta_{\mathbf{0}} \boldsymbol{t}-\frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{g} \boldsymbol{t}^{2} \\
&=(25) \sin (40)(1.15)-0.5(9.8)(1.15)^{2} \\
& y=12 m
\end{aligned}
$$

$$
\mathrm{t}=? ?
$$

(b) $v_{x}=v_{o x}=v_{o} \cos \theta_{o}$

$$
=25 \cos (40)=19 \mathrm{~m} / \mathrm{s}
$$

$$
22=(25) \cos (40) t
$$

(c) $v_{y=v_{0} \sin \theta_{0}-g t}$

$$
t=1.15 \mathrm{~s}
$$

$$
\begin{aligned}
& =(25) \sin (40)-(9.8)(1.15) \\
& =4.73 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
y=? ?
$$

$$
x=v_{0} \cos \theta_{0} t
$$

d)When it hits, has it passed the highest point on its trajectory?

$$
v_{o}=25 \mathrm{~m} / \mathrm{s} \quad \theta_{0}=40.0
$$

Maximum height (H)

$$
\begin{aligned}
\boldsymbol{H} & =\frac{\left(v_{0} \sin \boldsymbol{\theta}_{\mathbf{0}}\right)^{2}}{2 \boldsymbol{g}} \\
& =\frac{(25 \sin 40)^{2}}{2(9.8)}=13 \mathrm{~m}
\end{aligned}
$$


$\mathrm{Y}<\mathrm{H}$ then no it has not passed the highest point when it hits the wall $v_{y}=4.73 \mathrm{~m} / \mathrm{s} \quad$ So the ball hits the wall before the max height

## The Horizontal Range

horizontal range $R$, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$
R=\frac{v_{0}^{2}}{g} \sin 2 \theta_{0}
$$

Maximum range

$$
\theta_{0}=45^{0} \rightarrow R_{\max }=\frac{v_{0}^{2}}{g}
$$




## The Equation of the Projectile Path (TRAJECTORY)



This is the equation of a parabola, so the projectile path is parabolic

## EXTERNAL EXAMPLE

Figure $4-16$ shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_{0}=82 \mathrm{~m} / \mathrm{s}$.
(a) At what angle $\theta_{0}$ from the horizontal must a ball be fired to hit the ship?

$$
R=560 \mathrm{~m} \cdot \quad v_{0}=82 \mathrm{~m} / \mathrm{s} . \quad \theta_{o}=? ?
$$

$$
R=\frac{v_{0}^{2}}{g} \sin 2 \theta_{0}
$$

$$
560=\frac{(82)^{2}}{9.8} \sin 2 \theta_{o}
$$

$$
\theta_{0}=\frac{1}{2} \sin ^{-1} 0.816=27^{\circ} \text { and } 63^{\circ}
$$



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(b) What is the maximum range of the cannonballs?

$$
v_{0}=82 \mathrm{~m} / \mathrm{s} .
$$

## Maximum range

$$
\boldsymbol{\theta}_{0}=45^{0} \rightarrow \boldsymbol{R}_{\max }=\frac{v_{0}^{2}}{g}=\frac{(82)^{2}}{9.8}=686 \mathrm{~m}
$$



## 4-5 Uniform Circular Motion

A particle is in uniform circular motion if it travels around a circle or circular arc at constant speed.

1-Velocity:
-magnitude constant v.
-direction :tangent to the circle in the direction of motion.

2-Acceleration:


Why is the particle accelerating even though the speed does not vary?

- magnitude $\quad a=\frac{v^{2}}{r}$
- direction: toward the center.
- It is called Centripetal(meaning seeking center) acceleration


3- Period: is the time for a particle go around the circle once.

$$
\text { Time }=\frac{\text { distance }}{\text { velocity }}
$$

For one round $\Rightarrow$ distance $=$ circumference of the circle

$$
T=\frac{2 \pi r}{v}
$$

$$
\vec{v}=v_{x} \hat{i}+v_{y} \hat{j}
$$



$$
\vec{a}=a_{x} \hat{i}+a_{y} \hat{j}
$$



## Sample Problem 4.06

What is the magnitude of the acceleration, in $g$ units. of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_{i}=(400 \hat{\mathrm{i}}+500 \hat{\mathrm{j}}) \mathrm{m} / \mathrm{s}$ and 24.0 s later leaves the turn with a velocity of $\vec{v}_{f}=(-400 \hat{\mathrm{i}}-500 \hat{\mathrm{j}}) \mathrm{m} / \mathrm{s}$ ?

$$
\begin{array}{cc}
a=\frac{v^{2}}{r} & v=\sqrt{(400)^{2}+(400)^{2}}=640.31 \mathrm{~m} / \mathrm{s} \\
T=\frac{2 \pi r}{v} & r=\frac{T v}{2 \pi} \quad T=? ? ? \\
r=\frac{(48)(640.31)}{2 \pi}=4891.6 \mathrm{~m} \\
a=\frac{v^{2}}{r} \\
& =\frac{(640.31)^{2}}{4891.6} \\
& =83.81 \mathrm{~m} / \mathrm{s}^{2}
\end{array}
$$

$$
\begin{aligned}
a & =83.81 \times \frac{g}{g} \\
& =\frac{83.81 \times g}{9.8} \\
& =8.6 \mathrm{~g}
\end{aligned}
$$



