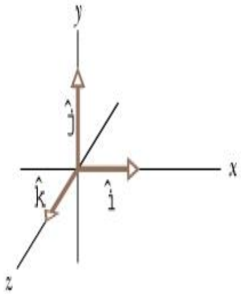
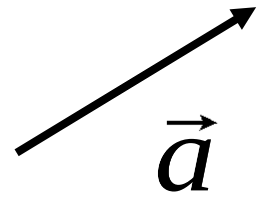
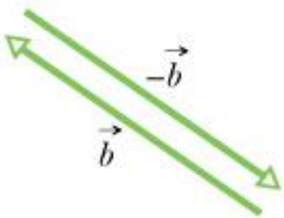
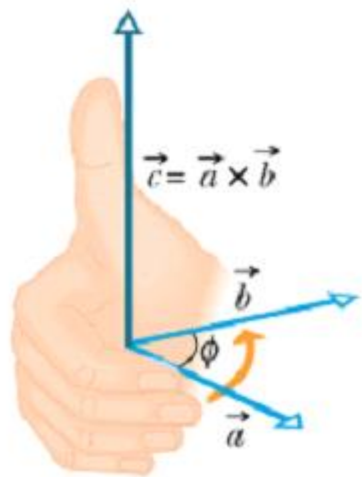


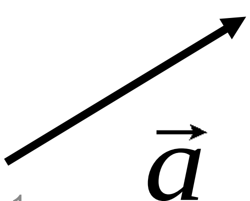
Chapter 3



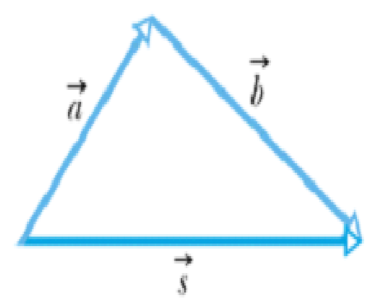




VECTORS



By Dr.Wajood Diery



3-1 VECTORS AND THEIR COMPONENTS

Vectors and Scalars

Physical Quantities

Vector Quantities

magnitude + direction

Follow certain rules of addition and multiplication

Scalar Quantities

+Ve Number

-Ve Number

Follow the rules of ordinary algebra

Vectors

Adding Vectors

Geometrically

- Adding vectors.
- Commutative Law.
- Associative Law.
- Vector Subtraction.

By Components

- resolving a vector.
- unit vectors.
- Adding vectors.

Multiplying Vectors

By scalar

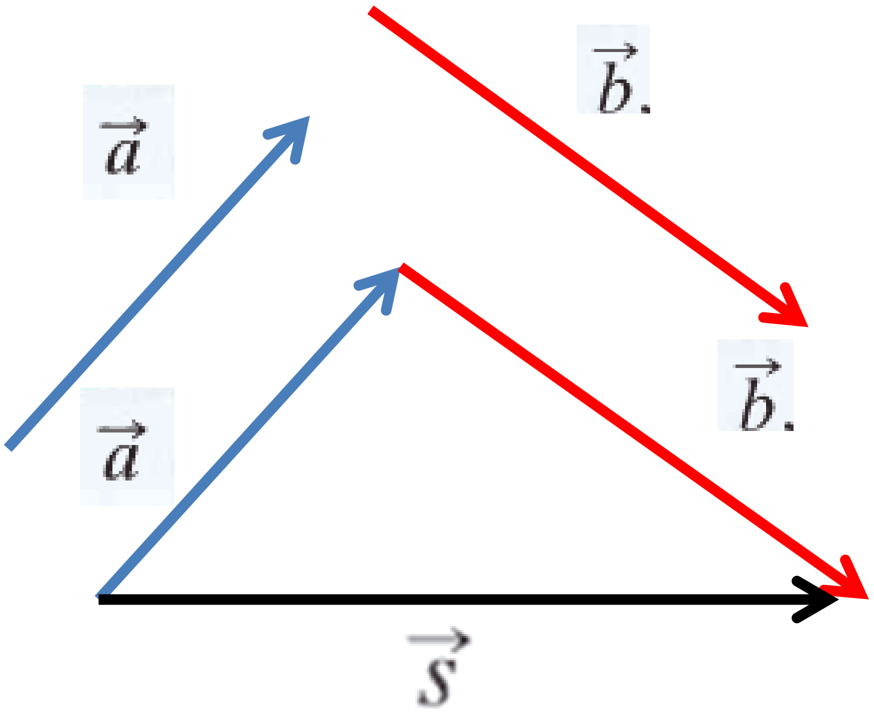
By a vector

Scalar Product

vector Product

Adding Vectors Geometrically

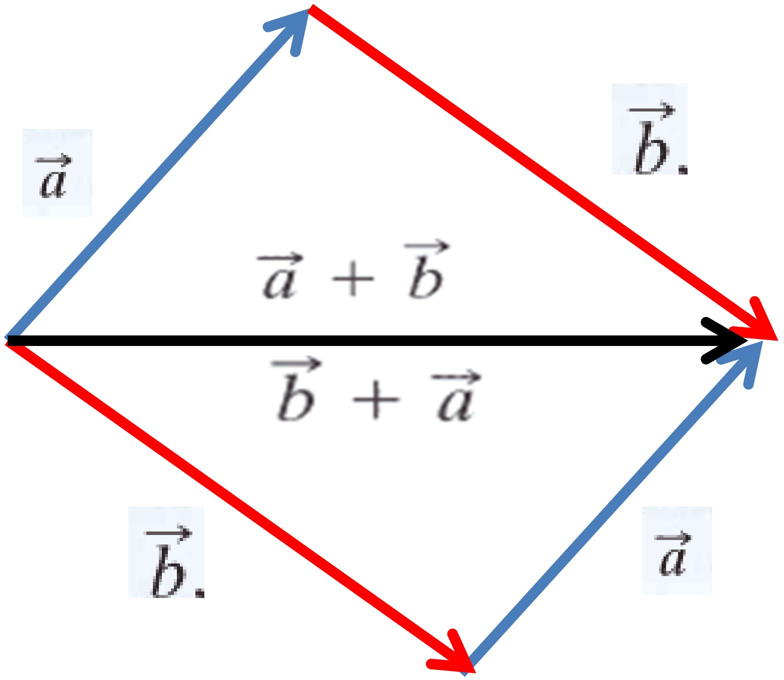
- Draw the first vector.
- From the end of the first vector draw the second vector.
- And so on.
- Draw a line from the start point to the end point and this will be the Sum or resultant.



- Vector equation

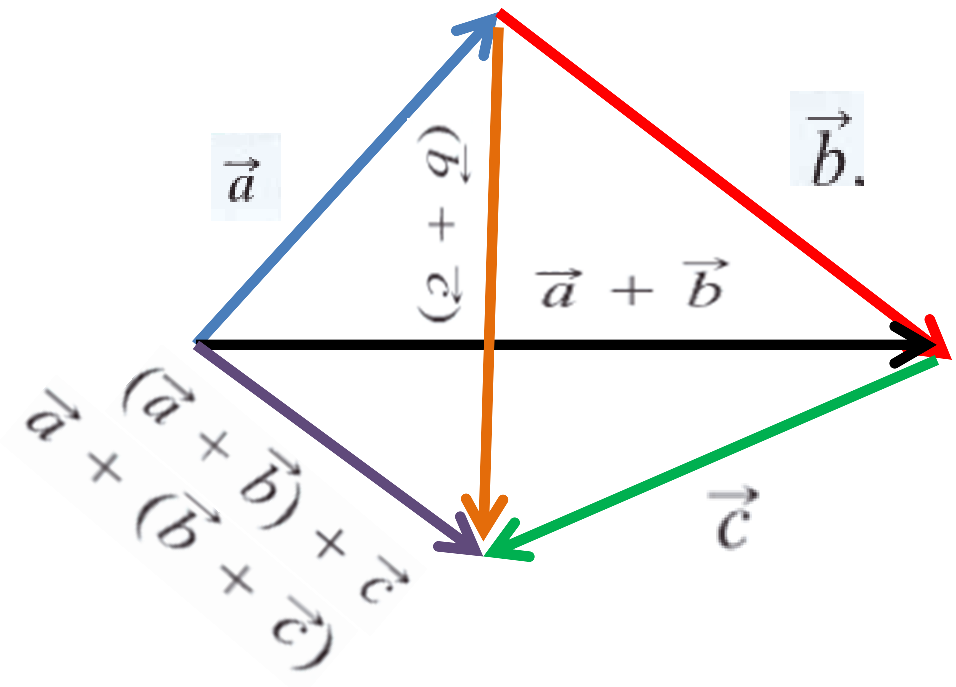
$$\vec{s} = \vec{a} + \vec{b},$$

• Commutative Law



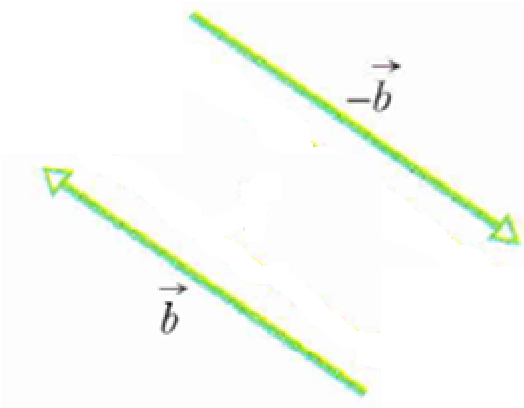
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

• Associative Law

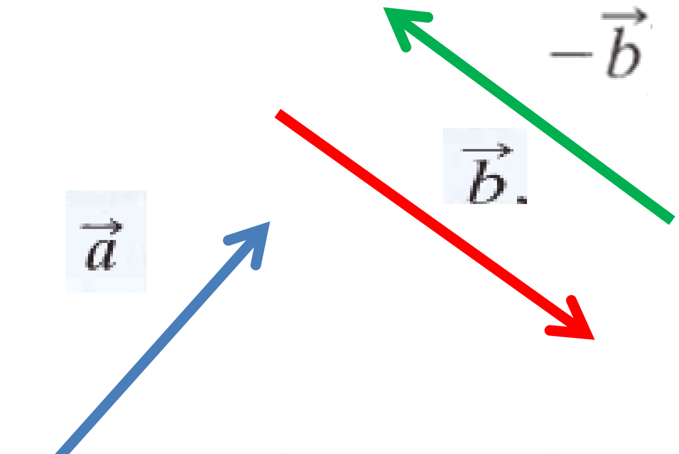


$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

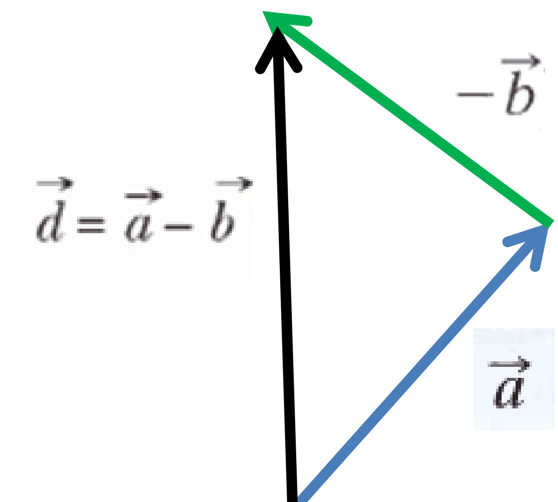
• Vector Subtraction



$$\vec{b} + (-\vec{b}) = 0.$$



$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

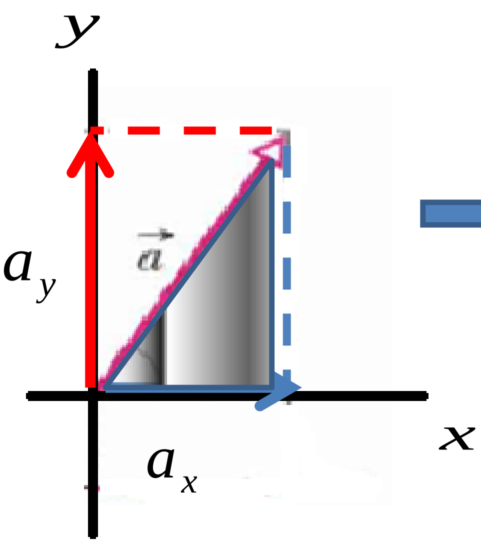


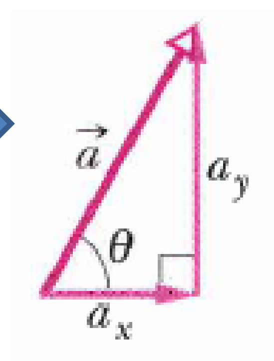
$$\boxed{\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})}$$

Components of Vectors

- Resolving the vector is the process of finding the components

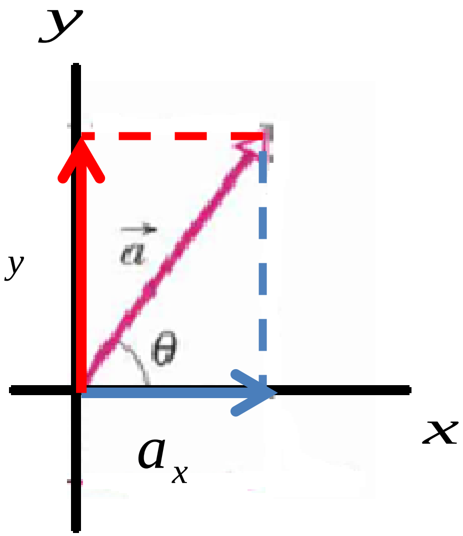
- Component is the projection of the vector on an axis

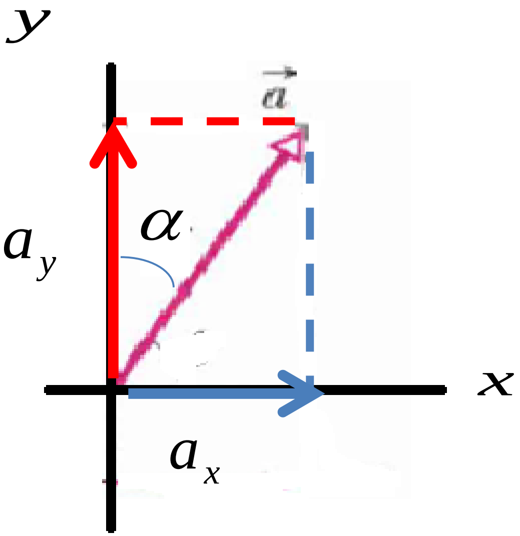




$$\cos\theta = \frac{a_x}{a} \quad , \quad \sin\theta = \frac{a_y}{a}$$

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$





$$\boxed{a_x = a\cos\theta \text{ and } a_y = a\sin\theta}$$

$$\boxed{a_x = a\sin\alpha \text{ and } a_y = a\cos\alpha}$$

- Writing a vector in magnitude- angle notation

$$\vec{a}: \; a_x \;\; and \;\; a_y$$

Magnitude

$$\boxed{a \;\;, \;\; |a| = \sqrt{a_x^2 + a_y^2}}$$

Direction (angle)

$$\boxed{\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)}$$

The angle is measured from positive X-axis.

$$\vec{a}$$

$$a \ and \ \theta$$

- Finding the components.

$$a_x = a\cos\theta$$

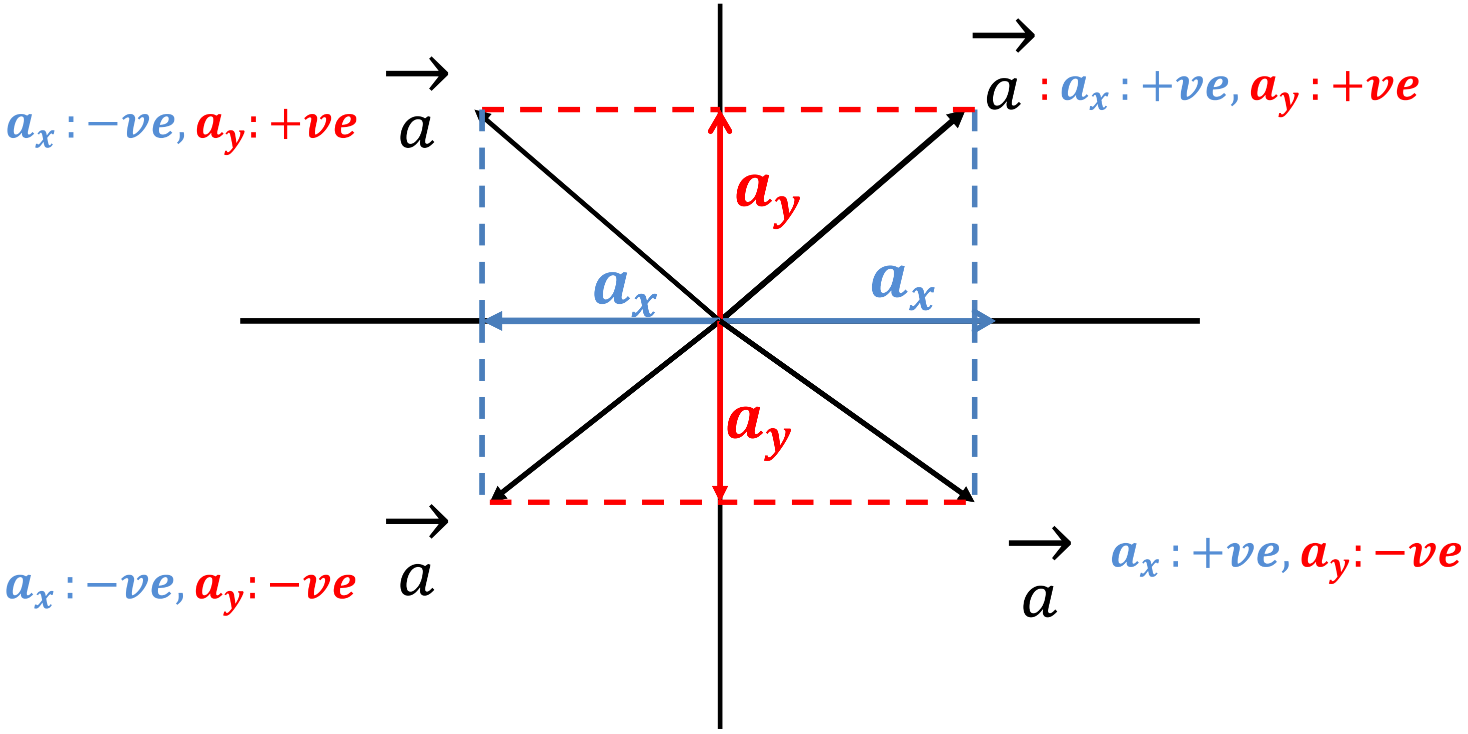
$$a_y = a\sin\theta$$

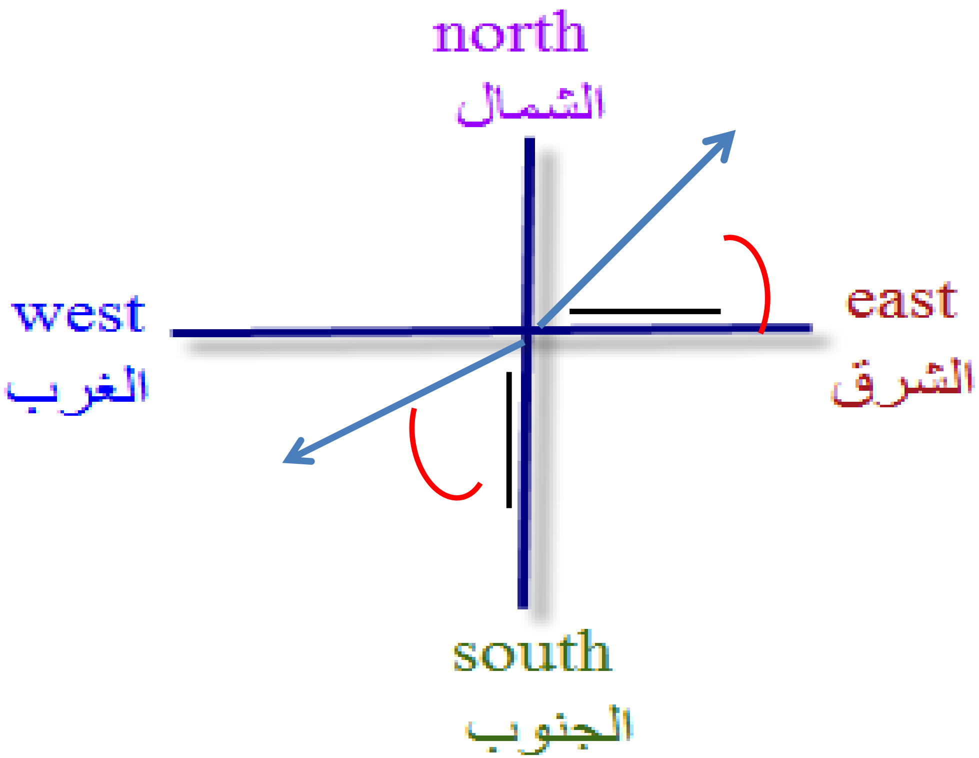
$$a_x \ and \ a_y$$

- Writing a vector in magnitude-angle notation

$$a = \sqrt{a_x^2 + a_y^2}$$

$$\theta = \tan^{-1}(\frac{a_y}{a_x})$$


$\vec{a}$: $a_x : +ve, a_y : +ve$
$a_x : -ve, a_y : +ve$ $\vec{a}$
$a_x : -ve, a_y : -ve$ $\vec{a}$
$\vec{a}$ $a_x : +ve, a_y : -ve$
a_y
a_x
a_x
a_y

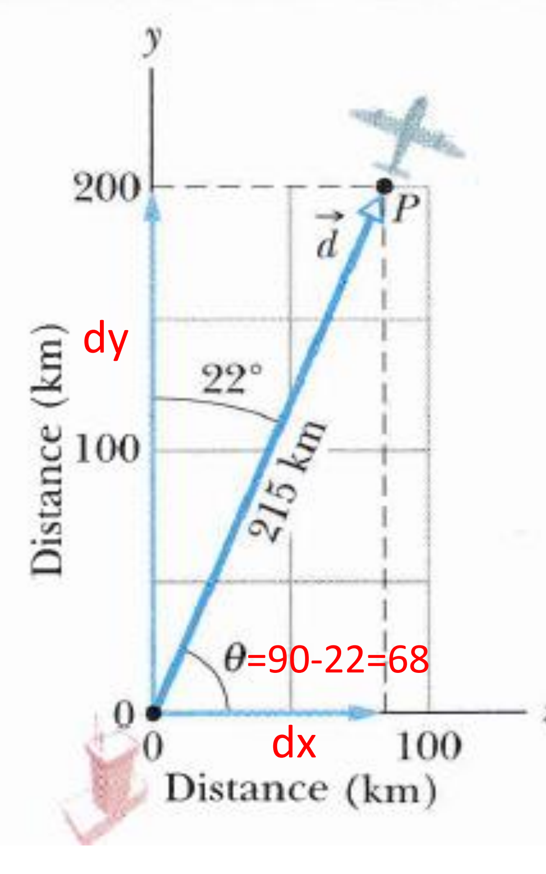


North of east

West of south

Sample Problem 3.02

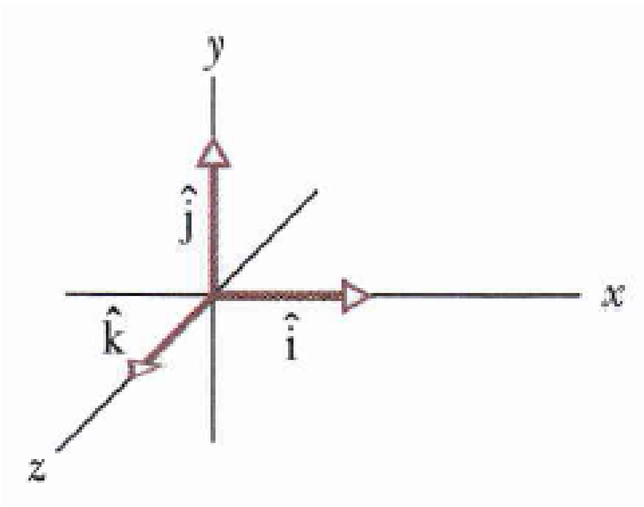
A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



3-2 UNIT VECTORS, ADDING VECTORS BY COMPONENT

Unit Vectors

- Unit vector is a vector of magnitude 1 and points in a particular direction



- **Writing a vector in Unit vector notation**

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{a} = \overbrace{\boxed{a_x\hat{i}} + \boxed{a_y\hat{j}} + \boxed{a_z\hat{k}}}^{\text{Vector Components}}$$

Scalar components

Adding vectors by Components

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{r} = \vec{a} \mp \vec{b}$$

$$\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}$$

$$r_x = a_x \mp b_x \qquad r_y = a_y \mp b_y \qquad r_z = a_z \mp b_z$$

Sample Problem 3.04

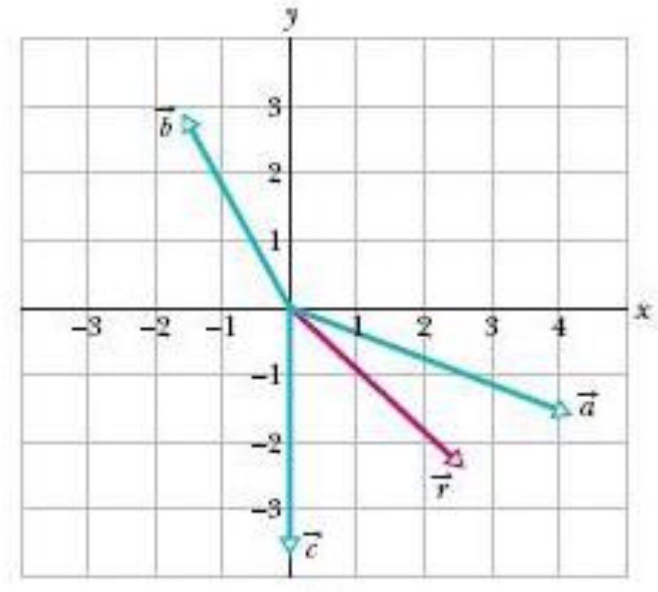
Figure 3-17a shows the following three vectors:

$$\vec{a} = (4.2\text{ m})\hat{\imath} - (1.5\text{ m})\hat{\jmath},$$

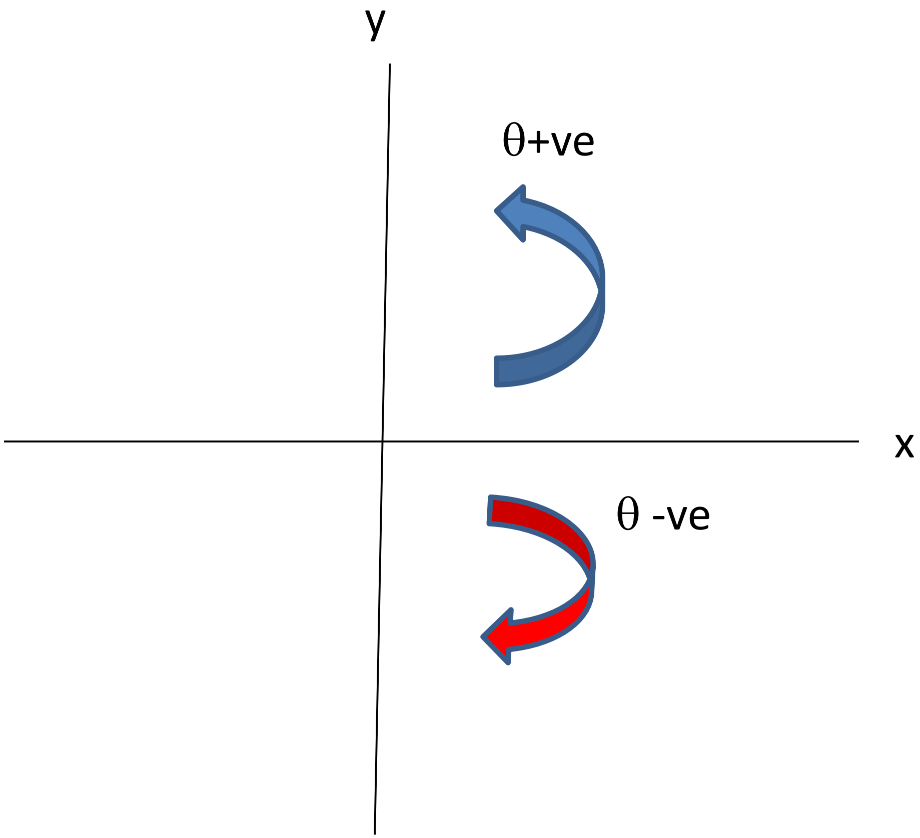
$$\vec{b} = (-1.6\text{ m})\hat{\imath} + (2.9\text{ m})\hat{\jmath},$$

and $$\vec{c} = (-3.7\text{ m})\hat{\jmath}.$$

What is their vector sum $\vec{r}$ which is also shown?



Rem : The angle θ must be measured from positive X-axis, if θ -ve if then move clockwise and if θ +ve move counterclockwise.



3-3 MULTIPLYING VECTORS

Multiplying vectors

Multiplying a vector by a scalar

+ve scalar

will produce a new vector in the same direction as the started vector

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

$$2\vec{a} = 4\hat{\imath} + 6\hat{\jmath}$$

-ve scalar

will produce a new vector in the opposite direction of the started vector

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

$$-2\vec{a} = -4\hat{\imath} - 6\hat{\jmath}$$

Multiplying a vector by a vector

Scalar product (or Dot product)

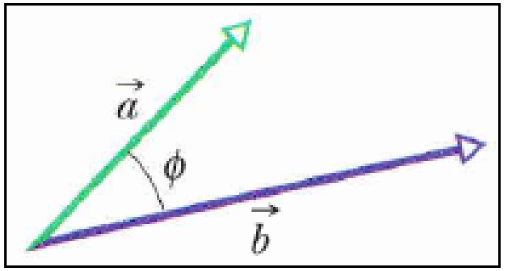
will produce a scalar

Vector product (or cross product)

will produce a new vector

Scalar (or Dot product)

If the two vectors are given in magnitude and the angle between them



$$\vec{a} \cdot \vec{b} = ab\cos\phi$$

If the two vectors are given in unit vector notation

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

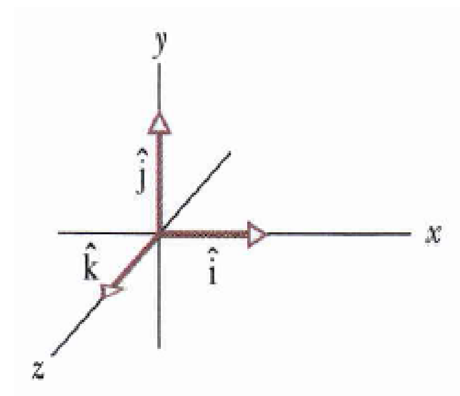
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\phi}$$

- The scalar product is commutative ⟹ $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
- If the two vectors are parallel ⟹ $\theta = 0 \Rightarrow \vec{a}\,.\,\vec{b} = a\,b$
- If the two vectors are perpendicular ⟹ $\theta = 90 \Rightarrow \vec{a}\,.\,\vec{b} = 0$
- If the two vectors are Antiparallel ⟹ $\theta = 180 \Rightarrow \vec{a}\,.\,\vec{b} = -\,a\,b$
- Multiplying Unit vectors

$$\hat{i}.\hat{i} = (1)(1)\cos 0 = 1 \quad \Longrightarrow \quad \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}.\hat{j} = (1)(1)\cos 90 = 0 \quad \Longrightarrow \quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$



Properties Of the scalar product

The scalar product is commutative

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

the angle between two vectors can be found

$$\vec{a} \cdot \vec{b} = ab\ \cos\theta$$

any two similar unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

any two different unit vectors

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0,$$

If $\theta = 0 \Rightarrow \vec{a} . \vec{b} = a\, b$ $\Rightarrow$ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} . \vec{b} = -\, a\, b$ $\Rightarrow$ vectors are anti parallel

$\theta = 90 \Rightarrow \vec{a} . \vec{b} = 0$ $\Rightarrow$ vectors are perpendicular

Sample Problem 3.05

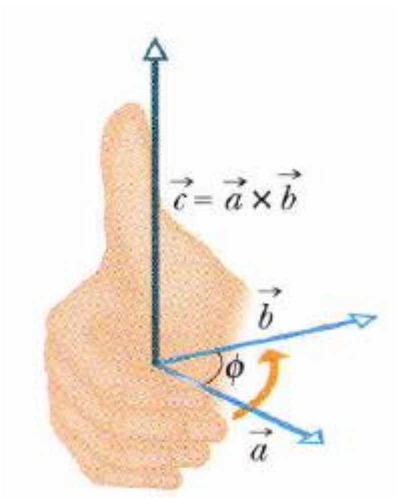
What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

Vector (or Cross product)

If the two vectors are given in magnitude and angle between them

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

The direction of the result vector



If the two vectors are given in unit vector notation

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_yb_z - b_ya_z)\hat{i} + (a_zb_x - b_za_x)\hat{j} + (a_xb_y - b_xa_y)\hat{k}$$

$$\boxed{|\vec{a}\times\vec{b}| = |c| = ab\ \sin\phi}$$

◊ The scalar product is Anti-commutative ➡ $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

◊ If the two vectors are parallel ➡ $\phi = 0 \Rightarrow \vec{a}\times\vec{b} = 0$

◊ If the two vectors are perpendicular ➡ $\phi = 90 \Rightarrow |\vec{a}\times\vec{b}| = a\,b$

◊ If the two vectors are Anti-parallel ➡ $\phi = 180 \Rightarrow \vec{a}\times\vec{b} = 0$

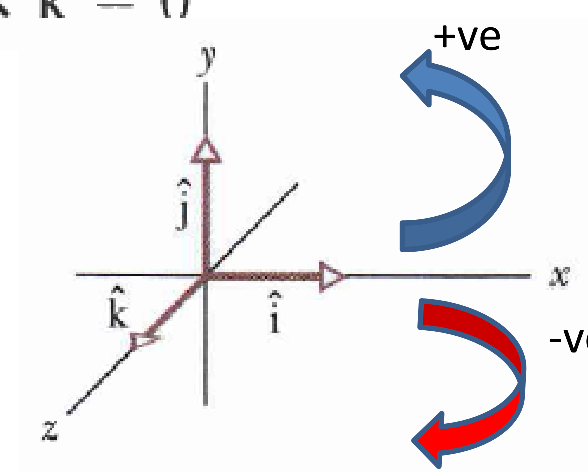
◊ Multiplying Unit vectors

$$\left|\hat{i}.\hat{i}\right| = (1)(1)\sin 0 = 0 \;\Rightarrow\; \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

$$\left|\hat{i}.\hat{j}\right| = (1)(1)\sin 90 = 1 \;\Rightarrow\; \hat{i}.\hat{j} = \hat{k}$$

$$\hat{i}\times\hat{j} = \hat{k}, \qquad \hat{j}\times\hat{k} = \hat{i}, \qquad \hat{k}\times\hat{i} = \hat{j}$$

$$\hat{j}.\hat{i} = -\hat{k} \qquad \hat{k}.\hat{j} = -\hat{i} \qquad \hat{i}.\hat{k} = -\hat{j}$$



Properties of the Vector product

any two different unit vectors

$$\hat{i} \times \hat{j} = \hat{k}, \qquad \hat{j} \times \hat{k} = \hat{i},$$

$$\hat{k} \times \hat{i} = \hat{j}$$

Anti- commutative

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

any two similar unit vectors

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

The small angle between the two vectors must be used because the odd property of the sin function

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

If $\phi = 0 \Rightarrow \vec{a} \times \vec{b} = 0$ ⇒ vectors are parallel

$\phi = 180 \Rightarrow \vec{a} \times \vec{b} = 0$ ⇒ vectors are anti parallel

$\phi = 90 \Rightarrow |\vec{a} \times \vec{b}| = a\,b$ ⇒ vectors are perpendicular

Sample Problem 3.07

If $\vec{a} = 3\hat{\text{i}} - 4\hat{\text{j}}$ and $\vec{b} = -2\hat{\text{i}} + 3\hat{\text{k}}$, what is $\vec{c} = \vec{a} \times \vec{b}$?


The End