SIGNALS AND SYSTEMS USING MATLAB
Chapter 0 — From the Ground Up!

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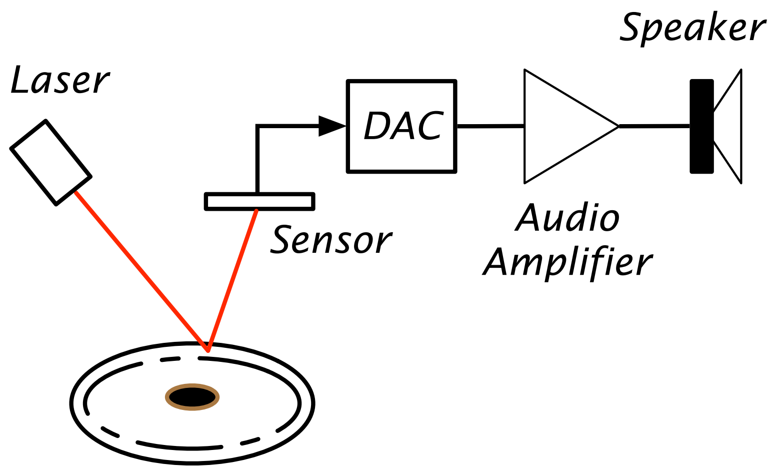
Digital signal processing

- 1948 – birth of digital technologies
  - Transistor (Bell Labs)
  - Stored–program computer (Manchester University, UK)
  - Publications
    - Shannon's digital communications
    - Hamming's error correcting codes
    - Wiener's *Cybernetics*

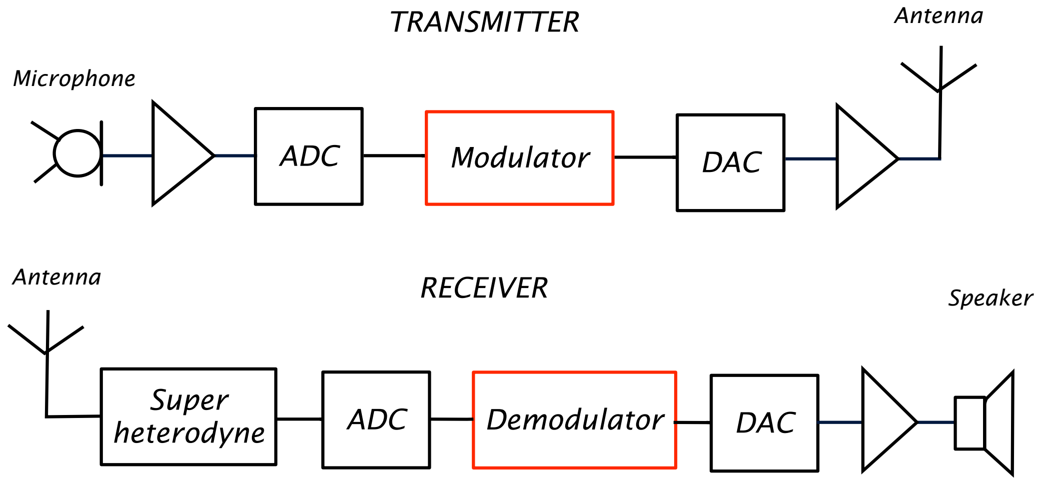
- Moore's Law, DSPs and FPGs
  - 1965 — Moore (Intel): number of transistors in a chip would double every 2 years
  - Digital Signal Processors (DSPs): optimized microprocessors for real–time processing
  - Field Programmable Gate Array (FPGA): device with programmable blocks and interconnects

Compact disc (CD) and compact disc player



When playing a CD, the CD player follows tracks in the disc, focus laser beam on them, as CD is spun. Light is reflected by pits and bumps on the surface of disc (corresponding to the coded digital signal from acoustic signal). Sensor detects reflected light and converts it into a digital signal and converted into an analog signal by DAC. Amplified and fed to speakers signal sounds like original recorded acoustic signal.

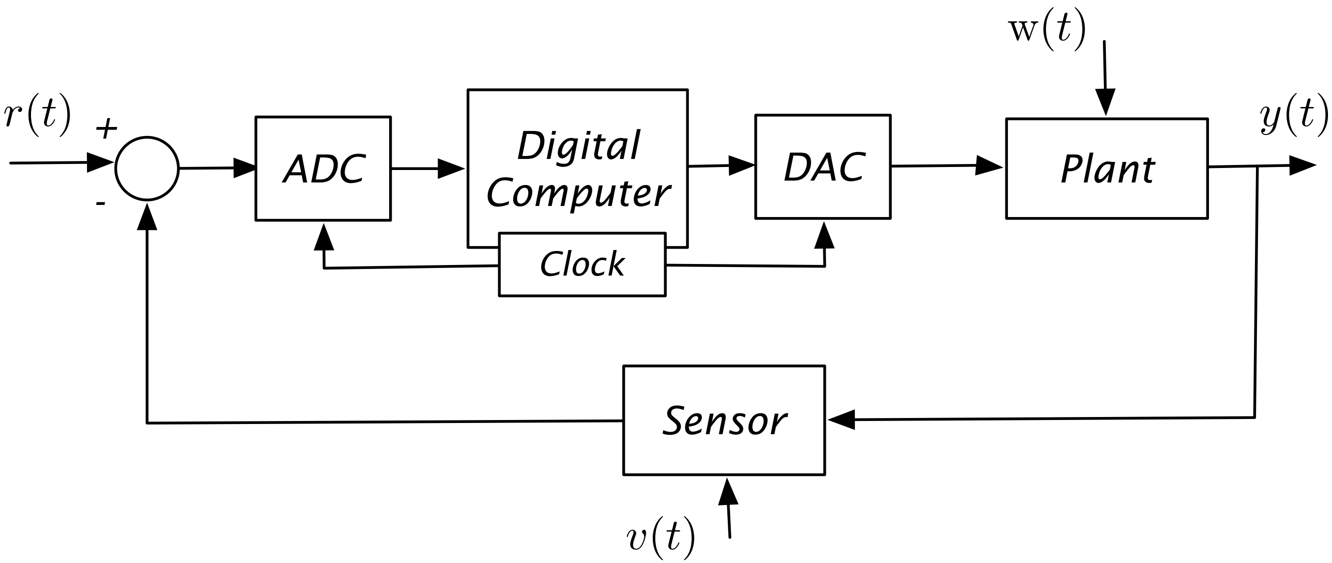
Software–defined radio (SDR)



Voice SDR mobile two-way radio

Transmitter: *voice signal inputted using microphone, amplified by an audio amplifier, converted into a digital signal by ADC, modulated using software, converted by DAC into analog signal which is amplified and radiated by antenna*

Receiver: *analog signal received by antenna is processed by a superheterodyne, converted by ADC, demodulated using software , converted by DAC, amplified and fed to speaker*

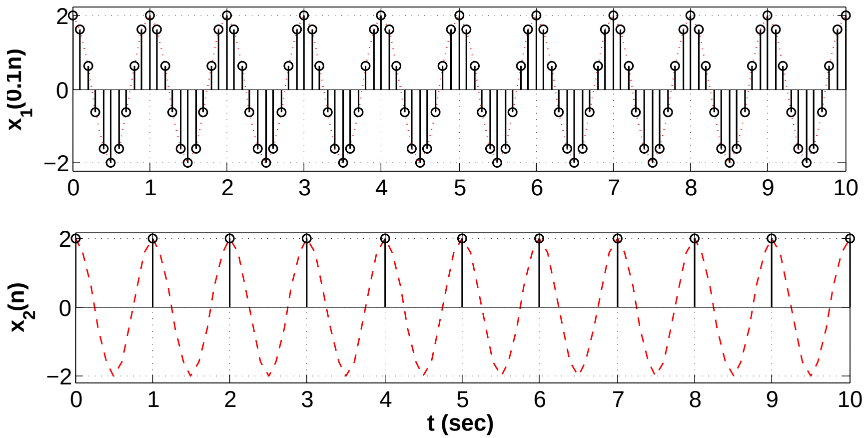


Computer control system for an analog plant (e.g., cruise control for a car)
Reference signal $r(t)$ (e.g., desired speed) and output $y(t)$ (e.g., car speed)
Signals $v(t)$ and $w(t)$: disturbances or noise in plant and sensor (e.g., electronic noise in the sensor and undesirable vibration in the car)

Sampling continuous–time signal $x(t)$ into discrete–time signal $x(nT_s)$ or discrete sequence $x[n]$:

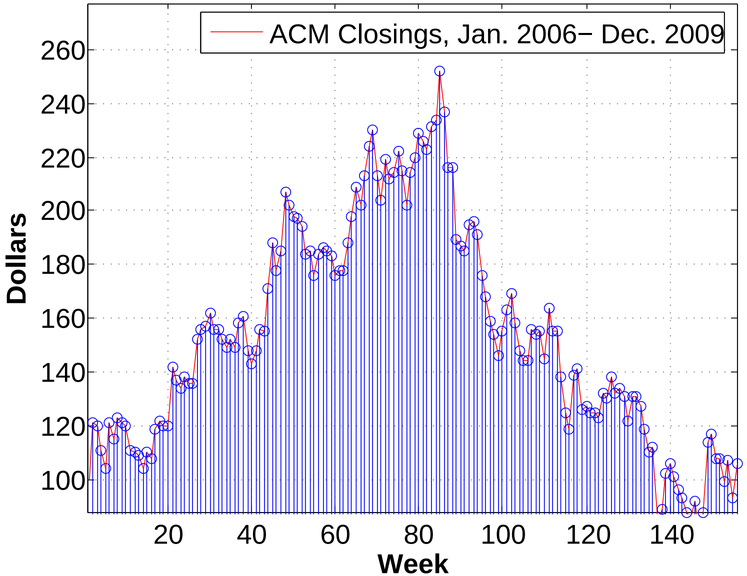
$$x[n] = x(nT_s) = x(t)_{|t=nT_s}$$

T_s: sampling period depends on frequency content of $x(t)$



Sampling $x(t) = 2\cos(2\pi t)$, $0 \le t \le 10$, *with* $T_{s1} = 0.1$ *(top) and* $T_{s2} = 1$ *(bottom) giving* $x_1(0.1n) = x_1[n]$ *and* $x_2(n) = x_2[n]$
Notice similarity between $x_1[n]$ *and* $x(t)$ *and loss of information when* $T_{s2} = 1$

Inherent discrete–time signal



Weekly closings of ACM stock for 160 weeks in 2006 to 2009. ACM is the trading name of the stock of the imaginary company ACME Inc. makers of everything you can imagine.

- Derivative and forward– difference

Derivative: rate of change of $x(t)$

$$D[x(t)] = \frac{dx(t)}{dt} = \lim_{h\to 0} \frac{x(t+h) - x(t)}{h}$$

Forward–difference: difference between $x((n+1)T_s)$ and $x(nT_s)$

$$\Delta[x(nT_s)] = x((n+1)T_s) - x(nT_s)$$

- Integral and summation

Integral and derivative

$$I(t) = \int_{t_0}^{t} x(\tau)d\tau, \quad x(t) = \frac{dI(t)}{dt}$$

Integral and summation

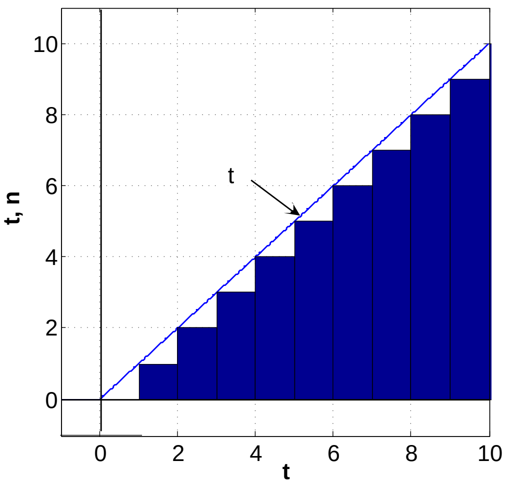
$$I(t) \approx \sum_n x(nT_s)p(n), \quad p(n) \text{ pulses of width } T_s$$

Area of $x(t) = t,\ 0 \leq t \leq 10$, and 0 otherwise

$$I(t) = \int_0^{10} t \ \ dt = \frac{t^2}{2} \Big|_{t=0}^{10} = 50$$

approximate $x(t)$ by aggregation of pulses $p[n]$ of width $T_s = 1$
and height $nT_s = n$

$$I(t) \approx \sum_{n=0}^{9} p[n] = \sum_{n=0}^{9} n = 0.5 \left[\sum_{n=0}^{9} n + \sum_{n=0}^{9} (9-n) \right] = \frac{10 \times 9}{2} = 45$$

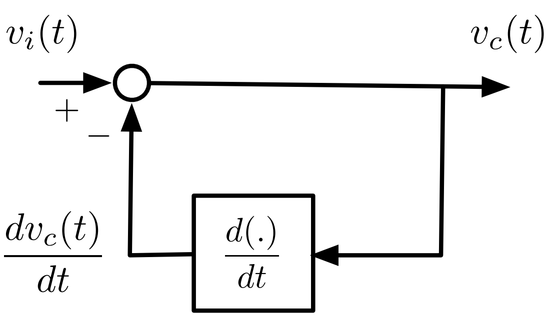


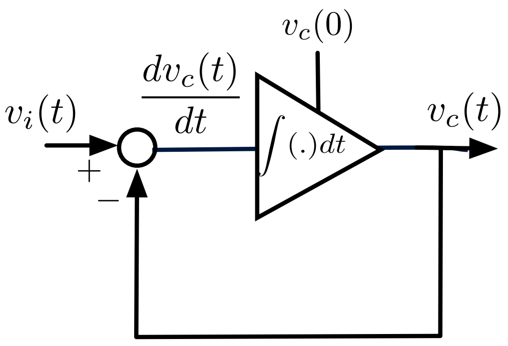
Solve d.e. from series RC circuit with a constant voltage source $v_i(t)$ as input and $R = 1$ Ω, $C = 1$ F (huge plates!)

$$v_i(t) = v_c(t) + \frac{dv_c(t)}{dt} \qquad t \geq 0$$

with initial voltage $v_c(0)$ across capacitor

- Use integrators

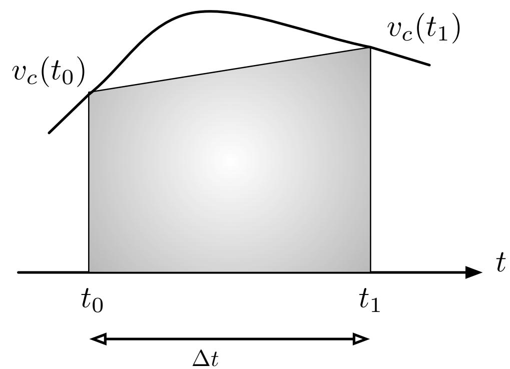




Block diagram for d.e. using differentiators (left) and integrators (right). Differentiators increase noise, integrators smooth out noise

- Approximate integral

$$v_c(t) = \int_0^t [v_i(\tau) - v_c(\tau)]d\tau + v_c(0) \qquad t \geq 0$$



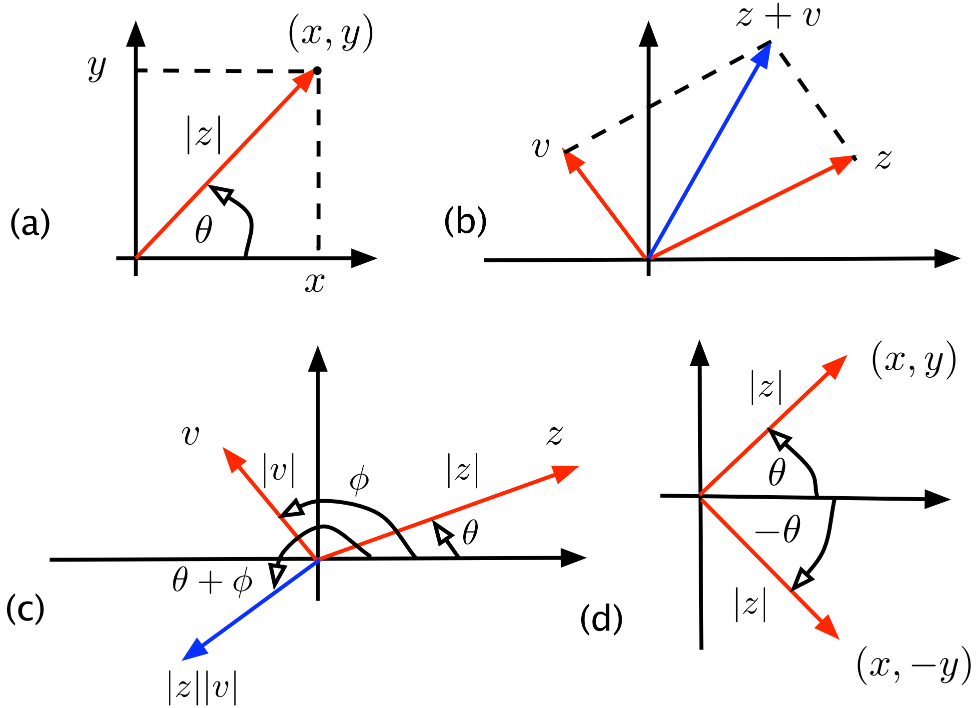
Trapezoidal approximation of area

- Difference equation

$$v_c(nT) = \frac{T}{2+T}[v_i(nT) + v_i((n-1)T)] + \frac{2-T}{2+T}v_c((n-1)T), \quad v_c(0) = 0, \quad n \geq 1$$

can be solved iteratively

- Damping and frequency of signals represented by complex variable
- Complex numbers and functions



(a) $z = x + jy$ as vector ; (b) addition of complex numbers; (c) multiplication of complex numbers; (d) complex conjugation of z.

Complex numbers

- Representations

$$\begin{aligned} z &= x + jy \qquad \text{rectangular} \\ &= |z|e^{j\angle z} \qquad \text{polar} \end{aligned}$$

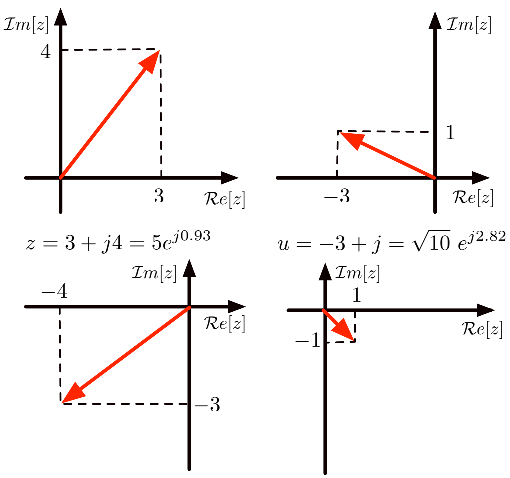
- Operations $\quad z = x + jy = |z|e^{j\angle(z)}, \quad v = u + jw = |v|e^{j\angle(v)}$

  Addition/subtraction: $z + v = (x + u) + j(y + w)$ rectangular

  Multiplication/division: $zv = |z||v|e^{j(\angle(z)+\angle(v))}$ polar

  Conjugation $z^* = x - jy = |z|e^{-j\angle z}$

- Rectangular to polar conversion



$z = 3 + j4 = 5e^{j0.93}$ $\quad u = -3 + j = \sqrt{10}\, e^{j2.82}$

$w = -4 - 3j = 5\, e^{j3.79}$ $\quad v = 1 - j = \sqrt{2}\, e^{-j\pi/4}$

$$\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta)}$$

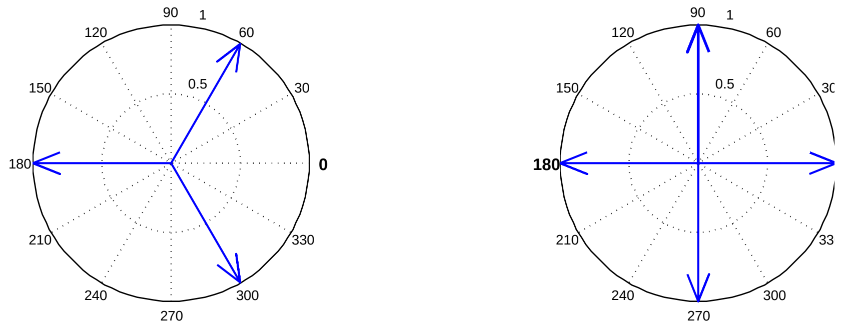
$$\cos(\theta) = \mathcal{R}e[e^{j\theta}] = \frac{e^{j\theta} + e^{-j\theta}}{2}$$
$$\sin(\theta) = \mathcal{I}m[e^{j\theta}] = \frac{e^{j\theta} - e^{-j\theta}}{2j}.$$

- Polar to rectangular conversion

$$\begin{aligned} z &= \sqrt{2}e^{j\pi/4} = \sqrt{2}\cos(\pi/4) + j\sqrt{2}\sin(\pi/4) = 1 + j, \quad \text{(first quadrant)} \\ u &= \sqrt{2}e^{-j\pi/4} = \sqrt{2}\cos(-\pi/4) + j\sqrt{2}\sin(-\pi/4) = \sqrt{2}\cos(\pi/4) - j\sqrt{2}\sin(\pi/4) \\ &= 1 - j, \quad \text{(fourth quadrant)} \\ w &= 5e^{j190^o} = 5e^{j180^o}e^{j10^o} = -5\cos(10^o) - j5\sin(10^o) \\ &= -4.92 - j0.87, \quad \text{(third quadrant)} \end{aligned}$$

- Roots and powers of j

$$z^3+1=0 \quad \Rightarrow \quad z_k^3=-1=e^{j(2k+1)\pi},\ \ k=0,1,2$$
$$z_k=e^{j(2k+1)\pi/3},\ \ k=0,1,2$$
$$z_0=e^{j\pi/3},\quad z_1=e^{j\pi}=-1,\quad z_2=e^{j(6-1)\pi/3}=e^{j2\pi}e^{-j\pi/3}=e^{-j\pi/3}$$



Left: roots of $z^3+1=0$. *Right: integer powers of* j, *periodic of period* 4, *with period of* $\{1,\ j,\ -1,\ -j\}$

- Trigonometric identities

$$\sin(-\theta) = \frac{e^{-j\theta} - e^{j\theta}}{2j} = -\sin(\theta)$$

$$\cos(\pi + \theta) = e^{j\pi}\frac{e^{j\theta} + e^{-j\theta}}{2} = -\cos(\theta)$$

$$\cos^2(\theta) = \left[\frac{e^{j\theta} + e^{-j\theta}}{2}\right]^2 = \frac{1}{4}[2 + e^{j2\theta} + e^{-j2\theta}] = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

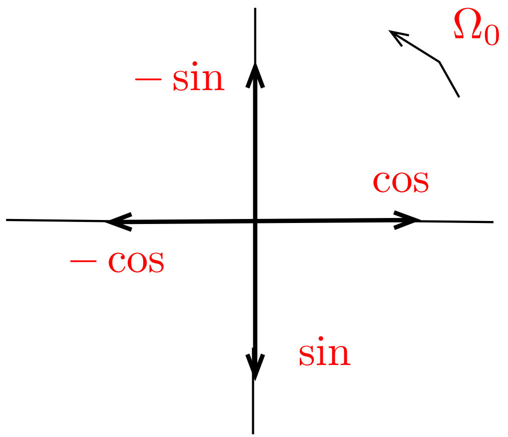
$$\sin(\theta)\cos(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}\,\frac{e^{j\theta} + e^{-j\theta}}{2} = \frac{e^{j2\theta} - e^{-j2\theta}}{4j} = \frac{1}{2}\sin(2\theta).$$

- Sinusoids and phasors

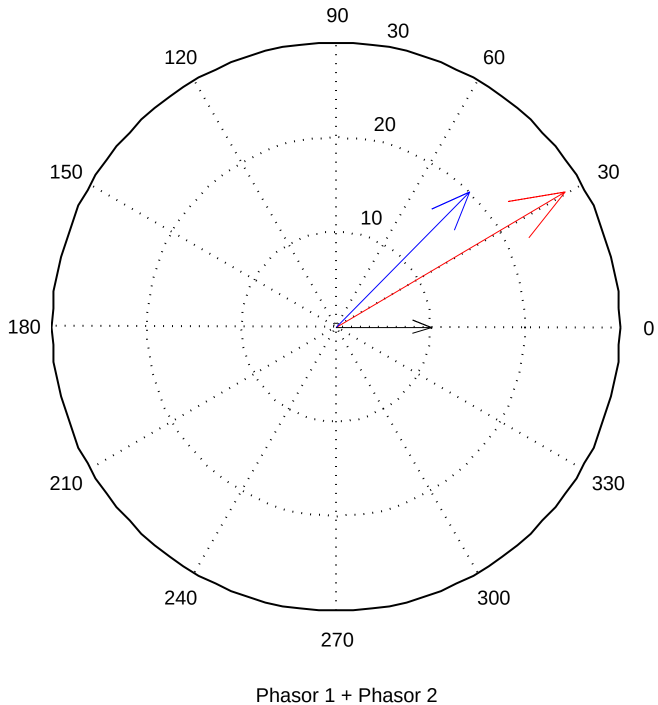
$$x(t) = A\cos(\Omega_0 t + \psi) \qquad -\infty < t < \infty$$

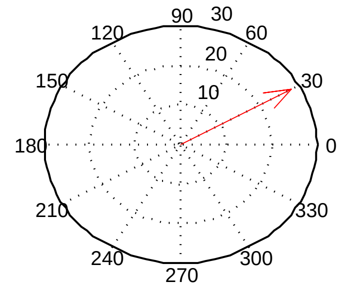
A amplitude, $\Omega_0 = 2\pi f_0$ frequency (rad/sec), ψ phase (rad)

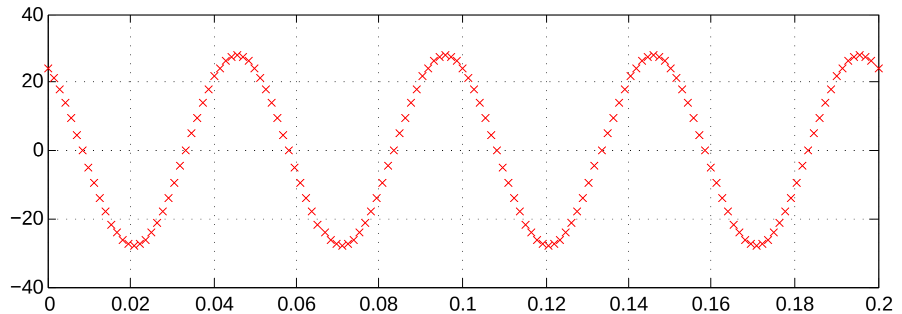
$$\text{Phasor}: X = Ae^{j\psi},\ x(t) = \mathcal{R}e[Xe^{j\Omega_0 t}]$$



Generation of sinusoids from phasors of a frequency Ω_0 shown at initial position





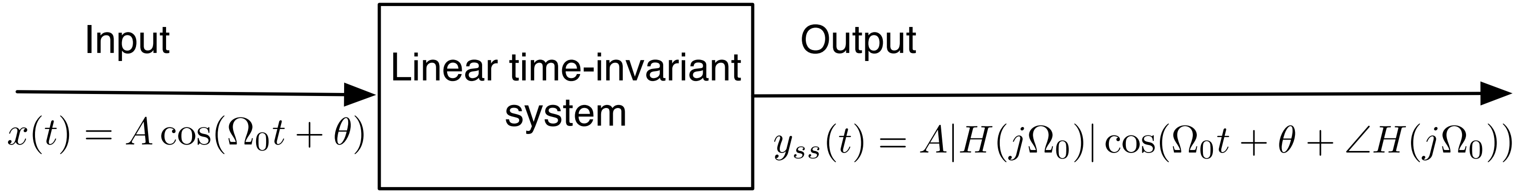


Sum of phasors $I_1 = 10e^{j0}$ *and* $I_2 = 20e^{j\pi/4}$ *with the result in the top left and the corresponding sinusoid (right bottom).*

- Eigenfunction property of LTI systems

$$\text{Input: } x(t) = \mathcal{R}e[Xe^{j\Omega_0 t}], \quad \text{input phasor } X = Ae^{j\theta}$$
$$\text{Output: } y(t) = \mathcal{R}e[Ye^{j\Omega_0 t}], \quad \text{output phasor } Y = XH(j\Omega_0)$$

- Steady–state response



Frequency response of system

$$H(j\Omega_0) = |H(j\Omega_0)|e^{j\angle H(j\Omega_0)}$$