Chapter 16

Waves I

In this chapter we will start the discussion on wave phenomena. We will study the following topics:

Types of waves
Amplitude, phase, frequency, period, propagation speed of a wave
Mechanical waves propagating along a stretched string
Wave equation
Principle of superposition of waves
Wave interference
Standing waves, resonance

(16–1)

16-2: Types of Waves

A **wave** is defined as motion of a disturbance that is self-sustained and propagates in space with a constant speed.

Waves transfer energy without transferring matter.

Waves can be classified in the following three categories:

Mechanical waves
- Need medium to propagate

Matter waves
particles have a wave associated with them

Electromagnetic waves
- Can propagate in vacuum

Mechanical waves

All mechanical waves require
(1) some source of disturbance,
(2) a medium that can be disturbed, and
(3) some physical mechanism through which elements of the medium can influence each other.

16-3: Transverse wave and Longitudinal Waves

Mechanical waves

Waves can be divided into the following two categories depending on the orientation of the disturbance with respect to the wave propagation velocity $\vec{v}$.

Motion of the wave

Motion of particles

Transverse waves

Examples: water waves, waves on a string and electromagnetic waves (Radio waves, light waves).

Longitudinal Waves

Examples: sound waves and waves on a slinky spring.(which consists of regions of rarefaction and compression).

Both a transverse wave and a longitudinal wave are said to be **traveling waves** because they both travel from one point to another.

All types of traveling waves transport energy.

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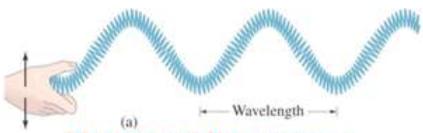
16-3: Transverse wave and Longitudinal Waves

Motion of wave and particles

Transverse waves

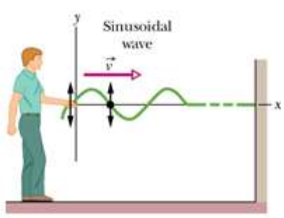
Motion of particles



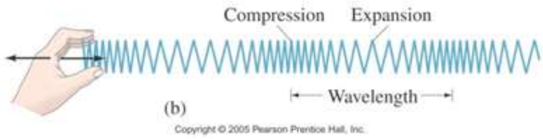
Motion of the wave

direction of propagation

Wave on a string



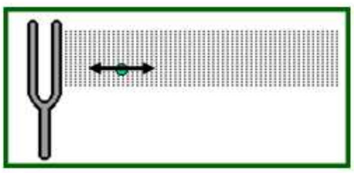
Longitudinal Waves

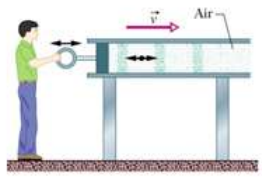


Motion of particles

Motion of the wave

Sound waves





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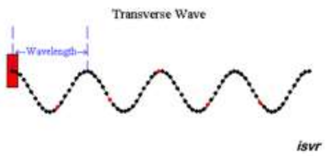
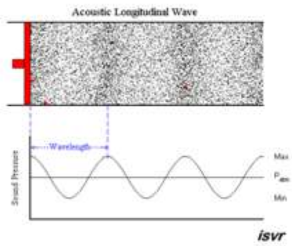
16-3: Transverse wave and Longitudinal Waves





Wave characteristics (properties):

- **Amplitude, y_m**
- **Frequency f and period T**
- **wave function**
- **Wavelength, λ**
- **Wave velocity v**
- **Particle velocity u**

The amplitude of a wave is the maximum displacement of the medium from the equilibrium position.

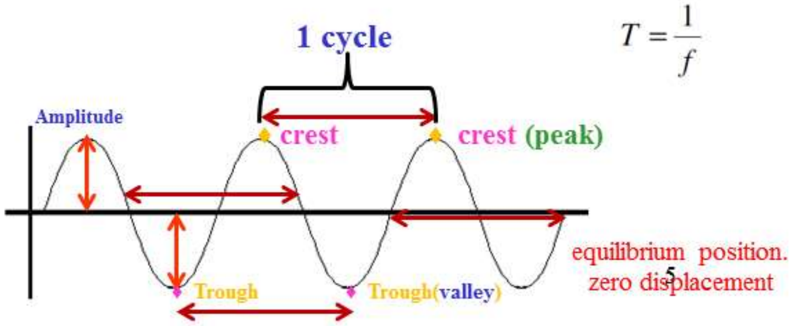
Frequency (f) : Number of vibration cycles per second.

Period (T) : Time taken to complete one cycle.

$$T = \frac{1}{f}$$

Crests and troughs

A *crest* is a point on the wave where the displacement of the medium is at a maximum. A point on the wave is a *trough* if the displacement of the medium at that point is at a minimum.



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16-4: Wavelength and Frequency

Traveling waves

The mathematical function of wave moves toward ***the right*** will be:

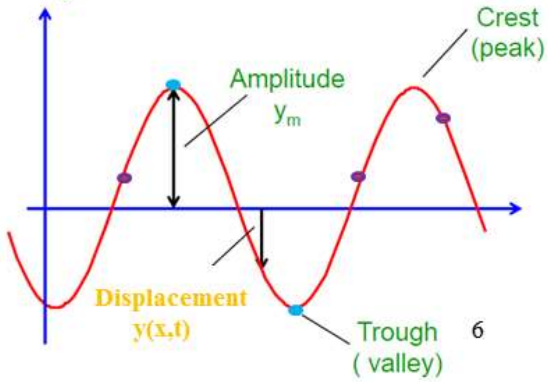
Oscillating term (±1)

$$\boxed{y\,(x,t)} = \boxed{y_m} \boxed{\sin(kx - \omega t)}$$

Phase (Argument)

Displacement — **Amplitude** — **angular wave number** — **Position** — **Angular frequency** — **Time**

The y(x, t) represents the y coordinate of any element located at position x at any time t.



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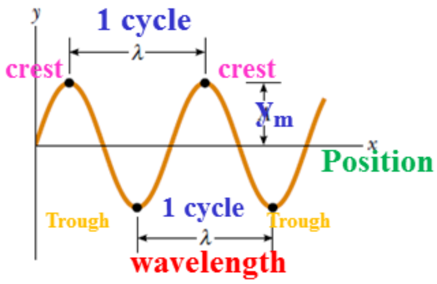
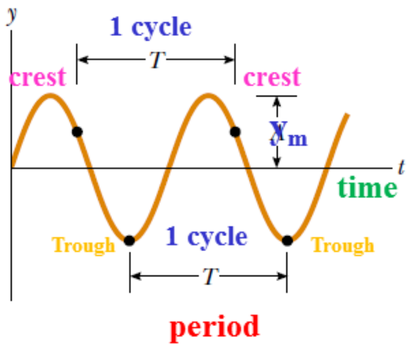
16-4: Wavelength and Frequency

Two ways to show waves on paper

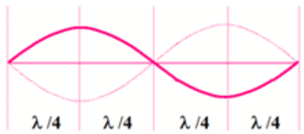
At certain time
(waveform)



At certain position
(Snapshot)



Wavelength (λ) is the distance from any point on the wave to an exactly similar point (two consecutive- successive-next crest).
SI unit is meter (m)



Period (T) : Time taken to complete one oscillation.

SI unit is second (s)

16-4: Wavelength and Frequency

Wavelength (λ) and Angular wave number (k)

at $t = 0$.

k (usually called simply the wave number)

$$k = \frac{2\pi}{\lambda}$$

- **Angular wave number**
- **SI unit rad/m**

Period (T), Angular Frequency (ω) and Frequency (f)

$x = 0$

$$\omega = \frac{2\pi}{T}$$

- **Angular Frequency**
- **SI unit rad/s**

$$f = \frac{1}{T}$$

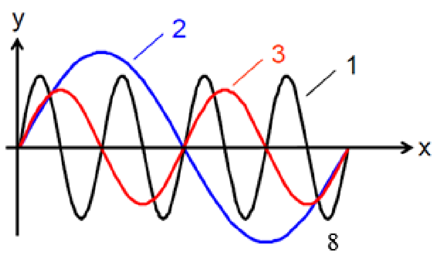
- **Frequency**
- **SI unit Hz (Hertz)**

$$\omega = 2\pi f$$

Checkpoint 1

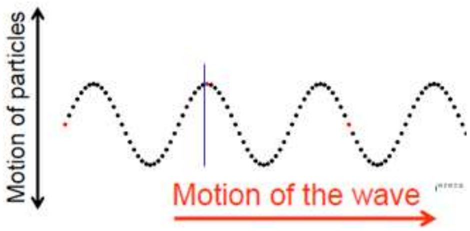
Match the snapshots of the three waves to the correct phase

(a) 2x -4t, (b) 4x-8t, (c) 8x-16t



16-5: The Speed of a Traveling Wave

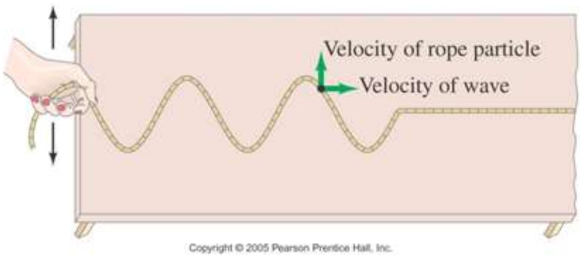
Velocity of particle
($v_y = u$)



Velocity of wave (v)

- ***Wave speed:*** $v \equiv$ velocity at which wave crests (or any part) move.

- ***particle velocity*** ($v_y = u$) $\equiv$ velocity at which particle position moves
  - ***Velocity of particle*** ($v_y = u$) $\neq$ ***Velocity of wave*** (v)



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16-5: The Speed of a Traveling Wave

Direction of a travelling wave

If the wave travels to the **right** (**positive x-axis**), the transverse positions of elements of the string are described by

$$y = y_m \sin(kx - \omega t)$$

The minus (-) sign means the wave is traveling to the right.

If the wave travels to the **left** (**negative x-axis**), the transverse positions of elements of the string are described by

$$y = y_m \sin(kx + \omega t)$$

The plus (+) sign means the wave is traveling to the left.

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16-5: The Speed of a Traveling Wave

Direction of a travelling wave-summary

$y\,(x,t) \;=\; y_m\; sin(+kx - \;\omega t\,)$ moves to the right.

$y\,(x,t) \;=\; y_m\;\; sin(-\,kx + \;\omega t\,)$ moves to the right.

A wave moves to the right if the signs of the kx and ωt terms are opposite.

$y\,(x,t) \;=\; y_m\;\; sin(+kx + \;\omega t\,)$ moves to the left.

$y\,(x,t) \;=\; y_m\;\; sin(-\,kx - \;\omega t\,)$ moves to the left.

A wave moves to the left if the signs of the kx and ωt terms are the same.

16-5: The Speed of a Traveling Wave

If the wave propagates along:

positive x-axis

$$y = y_m \sin\,(kx - \omega t)$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$$y\,(x,t) \;=\; y_m\;\; sin(\,k(x - \frac{\omega}{k}t)\,)$$

$$y\,(x,t) \;=\; y_m\;\; sin(\,k(x - vt)\,)$$

negative x-axis

$$y = y_m \sin\,(kx + \omega t)$$

$$v = -\,\frac{\omega}{k} = -\,\frac{\lambda}{T} = -\,\lambda f$$

$$y\,(x,t) \;=\; y_m\;\; sin(\,k(x + \frac{\omega}{k}t)\,)$$

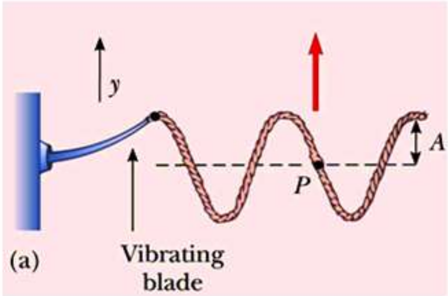
$$y\,(x,t) \;=\; y_m\;\; sin(\,k(x + vt)\,)$$

16-5: The Speed of a Traveling Wave

The velocity of the particles

- The ***transverse velocity u*** of an element of the string (a point P on the string) is:

$$v_y = u = \frac{\partial y}{\partial t} \qquad \text{at } x = \text{constant}$$



$$y(x,t) = y_m \sin(kx - \omega t)$$

$$v_y = u = -\omega\, y_m \cos(kx - \omega t)$$

$$u_{max} = \omega\, y_m$$

- This is different than the speed of the wave (v) as it propagates along the string:

v is constant u varies sinusoidally (**SHM**)

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16-5: The Speed of a Traveling Wave

The acceleration of the particles

- The ***transverse acceleration*** of the element of the string is

$$u = -\omega\, y_m \cos(kx - \omega t)$$

$$a = \frac{\partial u}{\partial t} = -\omega^2\, y_m \sin(kx - \omega t)$$

$$a_{max} = \omega^2\, y_m$$

$$a = -\omega^2\, y(x,t)$$

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16-5: The Speed of a Traveling Wave

Checkpoint 2

Here are the equations of three waves

(1) $y(x,t) = 2\sin(4x - 2t)$, (2) $y(x,t) = \sin(3x - 4t)$, (3) $y(x,t) = 2\sin(3x - 3t)$

Rank the waves according to

(a) wave speed, and (b) maximum transverse speed, greatest first

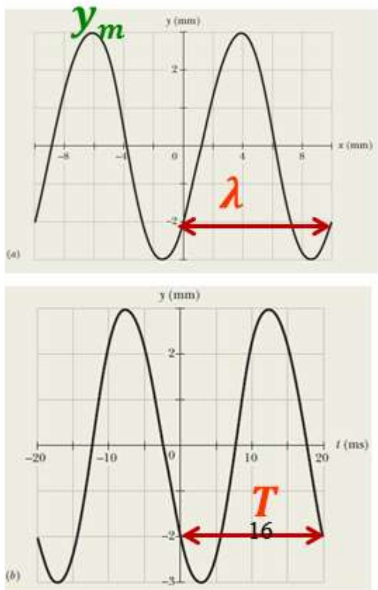
16-5: The Speed of a Traveling Wave

Sample Problem 16-1

A transverse wave traveling along an $+x$ axis has the form given by

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

Figure (a) gives the displacements of string elements as a function of x, all at time $t = 0$. Figure (b) gives the displacements of the element at $x = 0$ as a function of t. Find the values of the quantities shown in $y(x,t)$.



16-5: The Speed of a Traveling Wave

Sample Problem 16-2

A wave traveling along a string is described by

$$y(x,t) = 0.00327\sin(72.1x - 2.72t),$$

where all constants are in SI units.

(a) What is the amplitude of this wave?

(b) What are the wavelength, period, and frequency of this wave?

(c) What is the velocity of this wave?

(d) What is the displacement y at x = 22.5 cm and at t =18.9 s.

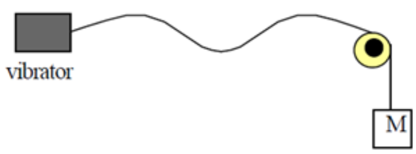
(e) What is the transverse velocity u of the string element at x =22.5 cm at time t=18.9 s?

(f) What is the transverse acceleration a_y of our string element at x = 22.5 cm and at t =18.9 s.

16-6: Wave Speed on Stretched String

The speed of a transverse wave traveling on a string is related to the wave's wavelength and frequency by

$$\boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f}$$



But it is set by the properties of the medium.

The medium transfers energy.

Kinetic energy

Medium's mass
Linear Density μ

$$\mu = \frac{\text{mass of the sting}}{\text{length of the sting}} = \frac{m}{L}$$

Potential energy

Medium's elasticity
(Stretch)
Tension τ

Speed of a wave on a stretched string $\quad v = \sqrt{\dfrac{\tau}{\mu}}$

where τ *is the tension in the string in* (N), μ *is the linear density* (kg/m) and v is the *speed* of the wave (m/s).

16-6: Wave Speed on Stretched String

Checkpoint 3

You send a wave along a string by oscillating one end.
If you increase the frequency of oscillation,
the speed of the wave (a) increases, (b) decreases, (c) remains the same,

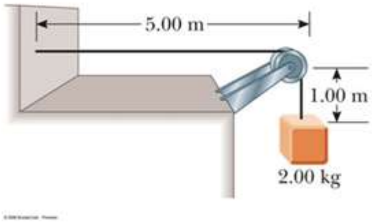
and the wavelength (a) increases, (b) decreases, (c) remains the same.

If you increase the tension in the string,
the speed of the wave (a) increases, (b) decreases, (c) remains the same

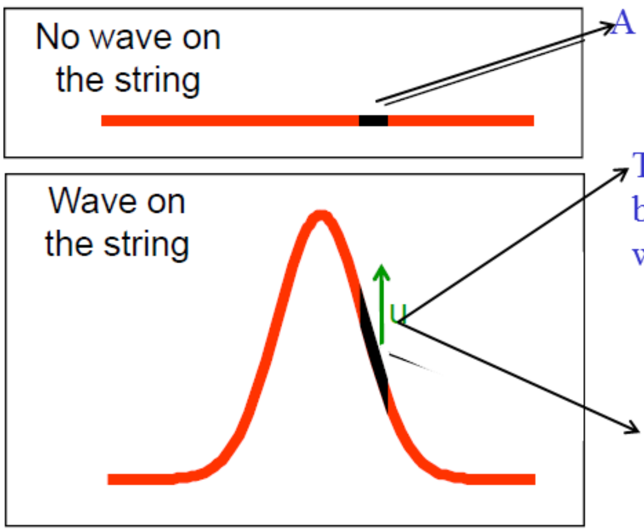
and the wavelength (a) increases, (b) decreases, (c) remains the same.

Problem:

A uniform cord has a mass of 0.300 kg and a total length of 6.00 m. Tension is maintained in the cord by suspending an object of mass 2.00 kg from one end



16-7: Energy and Power of a Traveling String Wave



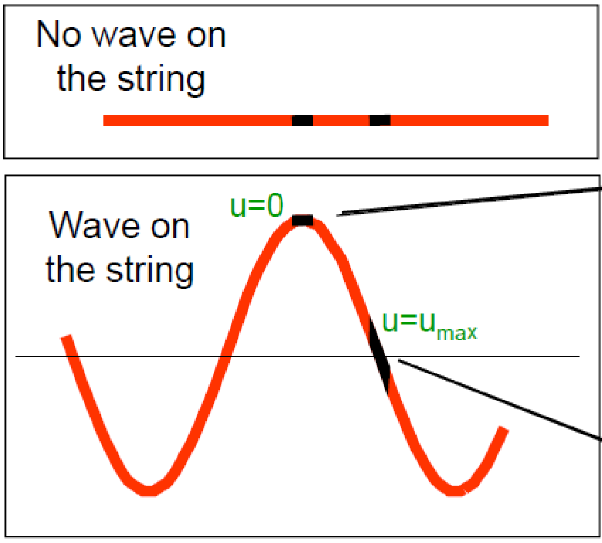
A string element

The string element has kinetic energy because it is moving transversely with speed u

The elastic potential energy of the string element increases because it is stretched more by the wave.

16-7: Energy and Power of a Traveling String Wave

Maximum and minimum energies



At maximum displacement $y = y_m$
zero transverse velocity ➔ zero kinetic energy
no extra stretch ➔ zero elastic potential energy

At zero displacement $y = 0$
Max. transverse velocity ➔ Max. kinetic energy
Max. stretch ➔ Max. elastic potential energy

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16-7: Energy and Power of a Traveling String Wave

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = K_\lambda + U_\lambda = \tfrac{1}{2}\mu\,\omega^2\,\lambda\,y_m^2$$

Average rate at which energy transmitted = Average power transmitted

$$\text{Average power transmitted} = \frac{\text{Energy transmitted in one preiod}}{\text{One period}}$$

$$P_{avg} = \frac{\Delta E}{\Delta t} = \frac{E_\lambda}{T}$$

$$P_{avg} = \tfrac{1}{2}\mu\,\omega^2\,v\,y_m^2$$

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16-7: Energy and Power of a Traveling String Wave

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

This expression shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to
(a) the wave speed,
(b) the square of the angular frequency, and
(c) the square of the amplitude.

½ of the average power transmitted is kinetic power and the other ½ is elastic power

$$(P_{avg})_{kinetic} = (P_{avg})_{potential} = \frac{1}{4}\mu v \omega^2 y_m^2$$

the total kinetic energy K_λ *in one wavelength* is $K_\lambda = \frac{1}{4}\mu\, \omega^2\, y_m^2\, \lambda$

the total potential energy U_λ *in one wavelength* is $U_\lambda = \frac{1}{4}\mu\, \omega^2\, y_m^2\, \lambda$

16-7: Energy and Power of a Traveling String Wave

S.P. 16-5: P. (424)

A string has $\mu = 525g/m$ and $\tau = 45N$. A sinusoidal wave on a string is send with $f = 120Hz$ and $y_m = 8.5mm$. At which average rate does the wave transport energy?(Average power transmitted)

16-9: The Principle of Superposition for Waves

The principle of superposition for waves:

If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

The displacement of the resultant wave= displacement of wave 1 + displacement of wave 2

$$\boldsymbol{y'(x,t) = y_1(x,t) + y_2(x,t)}$$

Some *Results* of Superposition:

- Interference: **Two waves, same wavelength (λ) and frequency (f), similar direction, different phase:**

$$y_1(x,t) = y_m \sin(kx - \omega t) \quad \text{and} \quad y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

- Standing Wave: **Two waves, same wavelength and frequency, opposite direction:**

- $$y_1(x,t) = y_m \sin(kx - \omega t) \quad \text{and} \quad y_2(x,t) = y_m \sin(kx + \omega t)$$

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16-10: Interference of Waves

Two waves moving in the same direction

In phase and out of phase

•Interference two waves, same wavelength (λ) and frequency (f), similar direction but **different phase**.

$$\boldsymbol{y_1(x,t) = y_m \sin(kx - \omega t)}$$

$$\boldsymbol{y_2(x,t) = y_m \sin(kx - \omega t + \phi)}$$

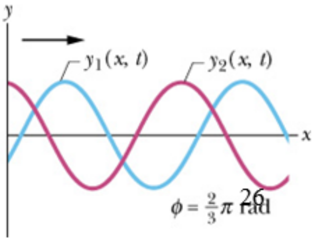
The phase difference between the two waves = phase of wave 2 - phase of wave 1

$= (kx - \omega t + \phi) - (kx - \omega t) \qquad = \phi$

Two waves are said to be out of phase by ϕ (or ϕ out of phase)

Two waves have phase difference ϕ,

wave1 is *phase–shifted* from wave 2 by ϕ.



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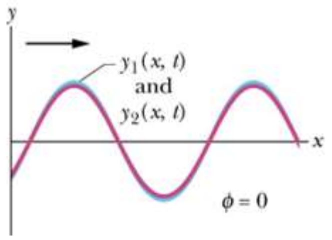
16-10: Interference of Waves

Two waves moving in the same direction

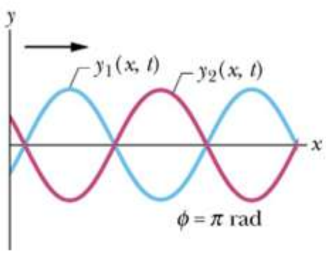
In phase and out of phase

$$y_1\,(x,t) = \ y_m \sin(kx\ -\ \omega t)$$

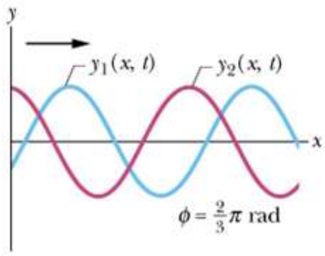
$$y_2\,(x,t) = \ y_m \sin(kx\ -\ \omega t\ + \phi)$$



If the phase difference $\phi = 0$ or even integer multiple of π, we say that wave1 and wave 2 are in phase.



If the phase difference $\phi = \pi$ or odd integer multiple of π, we say that wave1 and wave 2 are out of phase.



Two waves have phase difference ϕ,
Two waves have $\frac{2}{3}\pi$ out of phase

16-10: Interference of Waves

Two waves moving in the same direction

Let us now apply the principle of superposition to two sinusoidal waves traveling in the same direction in a linear medium. If the two waves are traveling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$y_1\,(x,t) = \ y_m\, sin(kx\ -\ \omega t) \quad \text{and}$$
$$y_2\,(x,t) = \ y_m\, sin(kx\ -\ \omega t\ + \phi)$$

resultant wave function y' *is:*

$$y' \ = \ y_1 \ + \ y_2$$

$$y'\,(x,t) = 2\,y_m\, cos\left(\frac{\phi}{2}\right)\, sin\,(kx - \omega t\ + \frac{\phi}{2})$$

16-10: Interference of Waves

Two waves moving in the same direction

$$\boxed{y'(x,t)} = \boxed{2\,y_m \cos\left(\frac{\phi}{2}\right)}\ \boxed{\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

Displacement of resultant wave

Magnitude gives amplitude of resultant wave (y'_m)

Oscillating term

The resultant wave has the same wavelength and frequency as that of $y_1(x,t)$ and $y_2(x,t)$

This result has several important features:

•The resultant wave function y' *also is sinusoidal* and has the same frequency, same wavelength and same direction as the individual waves.

•Its phase constant is ($\frac{\phi}{2}$)

•The amplitude of the resultant wave (y'_m) is the magnitude of $2\,y_m \cos\left(\frac{\phi}{2}\right)$

where ϕ is the phase difference between wave 1 and wave 2

$$y'_m = \left|2\,y_m \cos\left(\frac{\phi}{2}\right)\right|$$

16-10: Interference of Waves

Two waves moving in the same direction

$$y'(x,t) = 2\,y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - wt + \frac{\phi}{2}\right)$$

$$y'_m = \left|2\,y_m \cos\left(\frac{\phi}{2}\right)\right|$$

y'_m = maximum

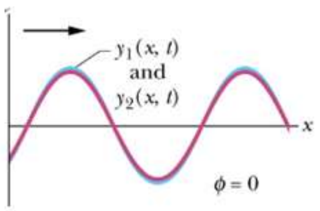
The phase difference is

$\phi = 0, 2\pi, 4\pi, 6\pi$

(wave 1 and wave 2 are in phase)

$$y'_m = 2\,y_m$$

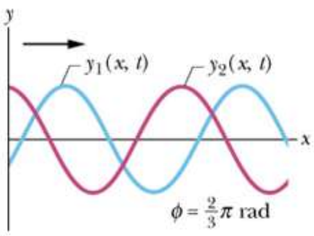
Fully Constructive Interference



The phase difference $\phi \neq 0, \pi$

Intermediate Interference

Partial Interference



y'_m = minimum

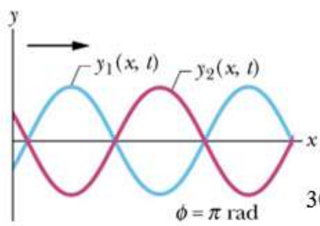
The phase difference is

$\phi = \pi, 3\pi, 5\pi$

(wave 1 and wave 2 are out of phase)

$$y'(x,t) = 0$$

Fully Destructive Interference



16-10: Interference of Waves

Two waves moving in the same direction

$$y'_m = \left| 2\, y_m \cos\left(\frac{\phi}{2}\right) \right|$$

Fully Constructive Interference

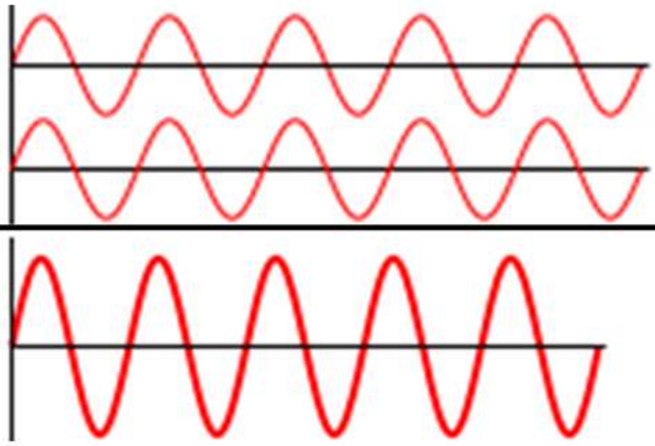
Wave 1

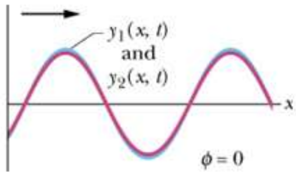
+

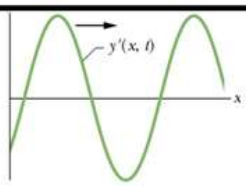
Wave 2

=

Resultant Wave







$$y'(x,t) = 2\, y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - wt + \frac{\phi}{2}\right)$$

$$y'_m = 2\, y_m$$

$\phi = 0$ ➔ Maximum amplitude

$$y'(x,t) = 2\, y_m \sin\left(kx - wt + \frac{\phi}{2}\right)$$

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16-10: Interference of Waves

Two waves moving in the same direction

$$y'_m = \left| 2\, y_m \cos\left(\frac{\phi}{2}\right) \right|$$

Fully Destructive Interference

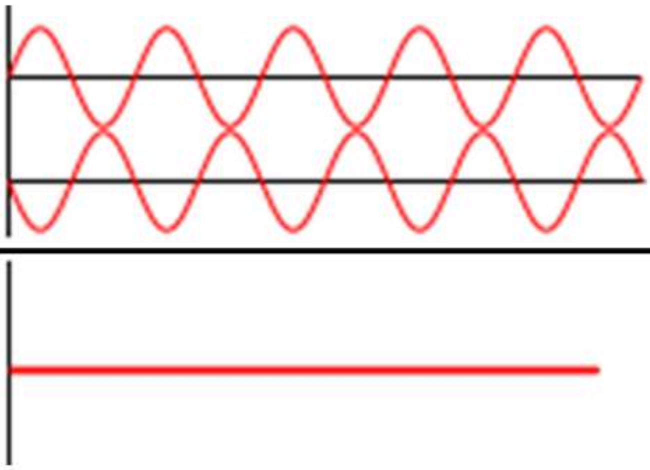
Wave 1

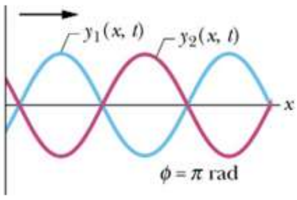
+

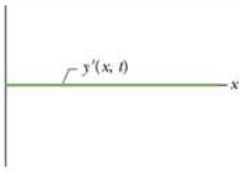
Wave 2

=

Resultant Wave







$$y'(x,t) = 2\, y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - wt + \frac{\phi}{2}\right)$$

$\phi = \pi$ ➔ $y'_m = 0$

$$y'(x,t) = 0$$

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16-10: Interference of Waves

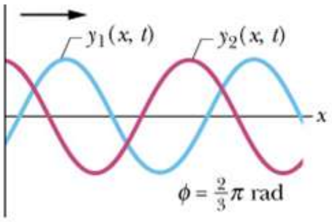
Two waves moving in the same direction

Intermediate Interference or partially Interference

Wave 1

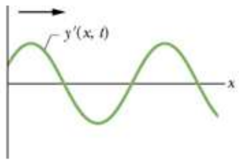
+

Wave 2



=

Resultant Wave



$$\phi = \frac{2\pi}{3} \rightarrow \cos\left(\frac{\pi}{3}\right) = \tfrac{1}{2}$$

$$y`_m = 0 < y`_m < y`_m = 2\, y_m$$

$$y'(x,t) = y_m \sin\left(kx - wt + \frac{\pi}{3}\right)$$

$$y'(x,t) = 2\, y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - wt + \frac{\phi}{2}\right)$$

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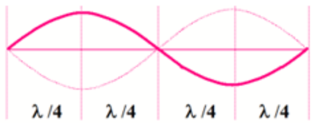
16-10: Interference of Waves

Phase difference expressed in terms of wavelengths

In general, we can say two waves are ϕ out of phase



Note: It is sometimes useful to express the phase difference in terms of wavelength λ. In this case, remember that

2 π radians = 1 λ

Example

1- We can say the two waves are 2π rad out of phase,
Or the two waves are one wavelength out of phase.

2π out of phase,
1λ out of phase.

2- We can say the two waves are π rad out of phase,
Or the two waves are half a wavelength out of phase.

π out of phase,
½ λ out of phase.

3- We can say the two waves are 3.8 π rad out of phase,
Or the two waves are 0.6 wavelength out of phase.

3.8 π out of phase,
0.6 λ out of phase.

$$\text{Distance} = 3.8 \text{ rad} \left(\frac{\lambda}{2\pi \text{ rad}}\right) = 0.60\,\lambda$$

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16-10: Interference of Waves

Checkpoint 5

The phase difference between two identical waves moving on a string is
(a) $\phi = 0.20$ wavelength (b) $\phi = 0.45$ wavelength
(c) $\phi = 0.60$ wavelength (d) $\phi = 0.80$ wavelength
Rank according to the amplitude of the resultant wave, greatest first.

16-10: Interference of Waves

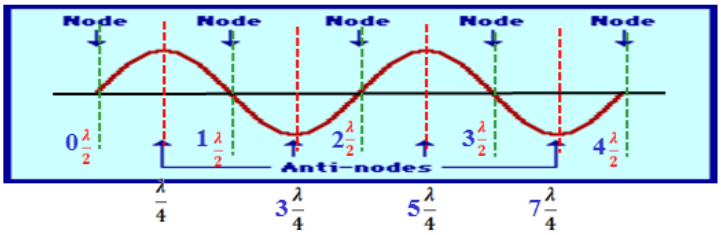
Sample Problem 16-6

Two identical sinusoidal waves, moving in the same direction along a streached string, interfere with each other.
Amplitude of each wave $y_m = 9.8$ mm and Phase difference between them $\phi = 100^o$.
(a) What is the amplitude of the resultant wave? What type of interference occurs?
(b) What phase difference, in radian and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

16-12: Standing Waves Two waves moving in opposite directions

Two waves $y_1(x,t)$ and $y_2(x,t)$ of the same amplitude and frequency moving in opposite directions on the same string interfere to produce the standing wave

$$y_1(x,t) = y_m \sin(kx - \omega t)$$
$$y_2(x,t) = y_m \sin(kx + \omega t)$$
$$y' = y_1 + y_2$$



$$y'(x,t) = \underbrace{\boxed{2\, y_m \sin(kx)}}_{y'_m} \cos(\omega t)$$

$y'_m \rightarrow$ Amplitude is a function of position

position of the ***zero amplitude (nodes)***

$$x = n\,\frac{\lambda}{2} \qquad \textit{for } n = 0, 1, 2, 3, \ldots$$

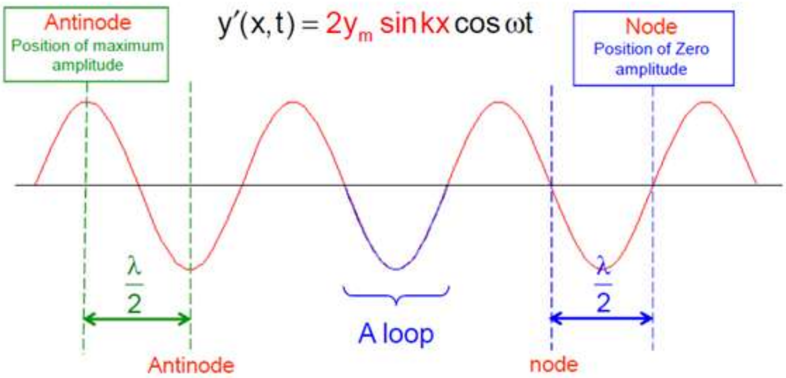
Position of nodes

position of the ***maximum amplitude (anti-nodes)***

$$x = n\,\frac{\lambda}{4} \qquad \textit{for } n = 1, 3, 5, \ldots$$

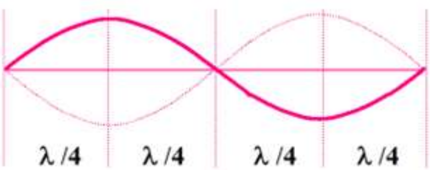
Position of anti-nodes

16-12: Standing Waves Two waves moving in opposite directions



Note

The distance between adjacent antinodes is equal to $\lambda/2$.
The distance between adjacent nodes is equal to $\lambda/2$.
The distance between a node and an adjacent antinode is $\lambda/4$.



16-10: Interference of Waves

Two waves moving in the same direction

same frequency, wavelength, and amplitude
but differ in phase

$$y_1(x,t) = y_m \sin(kx - \omega t)$$
$$y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

$$y'(x,t) = 2\,y_m \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

y'_m = **maximum**

$\phi = 0, 2\pi, 4\pi, 6\pi$

(2 waves are in phase)

$y'_m = 2\,y_m$

Constructive Interference

y'_m = **minimum**

$\phi = \pi, 3\pi, 5\pi$

(2 waves are out of phase)

$y'(x,t) = 0$

Destructive Interference

$\phi \neq 0, \pi$

Intermediate Interference

Partial Interference

16-12: Standing Waves

Two waves moving in opposite directions

same amplitude and frequency
But moving in opposite directions

$$y_1(x,t) = y_m \sin(kx - \omega t)$$
$$y_2(x,t) = y_m \sin(kx + \omega t)$$

$$y'(x,t) = 2\,y_m \sin(kx)\cos(\omega t)$$

zero amplitude (nodes)

$$x = n\frac{\lambda}{2}$$

for n = 0, 1, 2, 3,…

Position of nodes

maximum amplitude (anti-nodes)

$$x = n\frac{\lambda}{4}$$

for n = 1, 3,5,…

Position of anti-nodes

16-12: Standing Waves Two waves moving in opposite directions

Checkpoint 5

Two identical waves interfere to produce

(1) $y'(x,t) = 4\sin(5x - 4t)$

(2) $y'(x,t) = 4\sin(5x)\cos(4t)$

(3) $y'(x,t) = 4\sin(5x + 4t)$

In which situation are the two combining waves traveling

(a) toward positive x,

(b) toward negative x, and

(c) in opposite directions?

16-12: Standing Waves   Two waves moving in opposite directions

Example

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are:

$$y_1 = (4.0\ cm)\ sin(3.0\ x - 2.0\ t)\ ,$$
$$y_2 = (4.0\ cm)\ sin(3.0\ x + 2.0\ t)$$

where x *and* y *are measured in centimeters.*

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at x =2.3 *cm.*

(B) Find the positions of the nodes and antinodes if one end of the string is at x = *0.*

(C) What is the maximum value of the position in the simple harmonic motion of an element located at an antinode?

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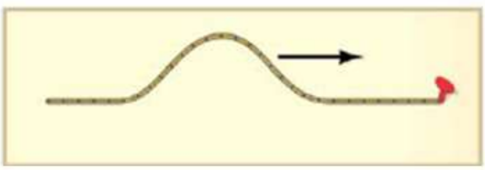
16-12: Standing Waves   Two waves moving in opposite directions

Standing Waves Due To Reflections from Hard and Soft Boundaries

Standing waves can form under a variety of conditions, but they are easily demonstrated in a medium which is finite or bounded.

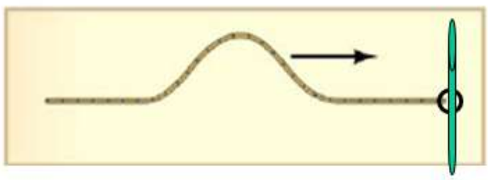
One way to get standing wave (two waves traveling in opposite directions) is to have a single wave train reflect from a boundary.

Hard Boundaries
Fixed end



Hard Reflection

Soft Boundaries
Free end



Soft Reflection

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16-12: Standing Waves Two waves moving in opposite directions

Reflections at a Boundary

Hard Reflection

- The reflected and incident waves are out of phase ($\phi = \pi$).
- A pulse is inverted when it is reflected from a fixed end.
- The fixed end of the string is a node.

Soft Reflection

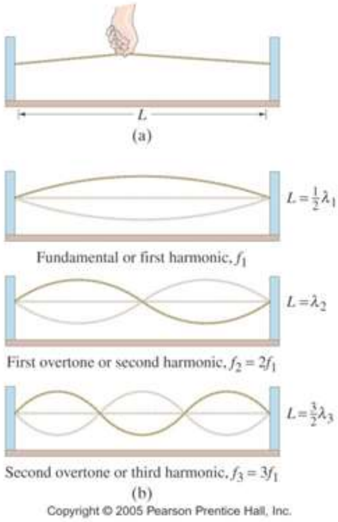
- A pulse is not inverted when it is reflected from a free end
- The free end of the string is a antinode.

16-13: Standing Waves and Resonance

Standing waves occur when both ends of a string are fixed. Consider a string of length *L fixed at both ends, as shown in Figure. Standing* waves are set up in the string by a continuous superposition of waves incident on and reflected from the ends. Note that there is a boundary condition for the waves on the string. The ends of the string, because they are fixed, must necessarily have zero displacement (nodes).

In general, standing waves can be produced by any two identical waves traveling in opposite directions that have the right wavelength. In a bounded medium, standing waves occur when a wave with the correct wavelength meets its reflection. The interference of these two waves produces a resultant wave that does not appear to move.

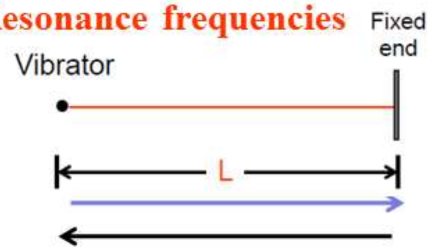
16-13: Standing Waves and Resonance



*The frequencies of the standing waves on a particular string are called resonant frequencies and the corresponding standing wave pattern is an oscillation mode.

*They are also referred to as the fundamental and harmonics.

Resonance frequencies



The distance between two consecutive right-going waves is 2L

16-13: Standing Waves and Resonance

Resonance frequencies

The wavelengths and frequencies of standing waves are:

$$\lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \cdots$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1, \qquad n = 1, 2, 3, \cdots$$

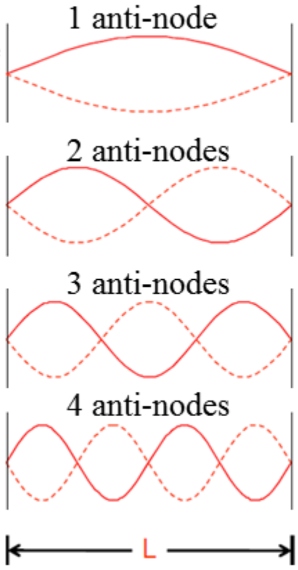
n : harmonic number

f_1 lowest resonant frequency, n=1

- **The simplest of the harmonics is called the fundamental or first harmonic. Subsequent standing waves are called the second harmonic, third harmonic, etc.**

16-13: Standing Waves and Resonance

Harmonics



	Oscillation mode	Harmonic Number n=1,2,3	Resonant wavelength $\lambda = \frac{2L}{n}$ $L = n\frac{\lambda_n}{2}$	Resonant frequency $f_n = n\frac{v}{2L} = n f_1$
Fundamental mode 1st harmonic	1 anti-node	n=1	$L = 1\frac{\lambda_1}{2} = \frac{\lambda_1}{2}$	$f_1 = 1\frac{v}{2L} = 1 f_1$
2nd harmonic	2 anti-nodes	n=2	$L = 2\frac{\lambda_2}{2} = \lambda_2$	$f_2 = 2\frac{v}{2L} = 2 f_1$
3rd harmonic	3 anti-nodes	n=3	$L = 3\frac{\lambda_3}{2} = 3\frac{\lambda_3}{2}$	$f_3 = 3\frac{v}{2L} = 3 f_1$
4th harmonic	4 anti-nodes	n=4	$L = 4\frac{\lambda_4}{2} = 2\lambda_4$	$f_4 = 4\frac{v}{2L} = 4 f_1$

16-13: Standing Waves and Resonance

Checkpoint 7

What is the missing resonant frequency (less than 400Hz) from the following series?

150 Hz,
225 Hz,
300 Hz,
375 Hz

What is the frequency of the seven harmonic?