Chapter 12

Elasticity

In this chapter we will examine how a "rigid" body can be deformed by an external force. In this section we will introduce the following concepts:

Stress and strain
Young's modulus (in connection with tension and compression)
Shear modulus (in connection with shearing)
Bulk modulus (in connection with hydraulic stress)

(12-7) 1

12-7 *ELASTIC PROPERTIES OF MATERIALS*

- **In our first course, phys110, we firstly examined structures in which we assumed the object were particle. Then, we assumed the object were rigid.**
- **Rigid means that the object did not deform due to the applied force.**
- **In real, we have deformation "change in shape and size ".**

2

12-7 *ELASTIC PROPERTIES OF MATERIALS*

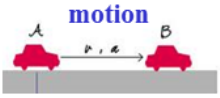
States of Matter

Solid | **Liquid** | **Gas**

F applied

The body is moving

motion



x, x_0, v_0, v, t, a

↓

Phys110

The body is change its dimension size/shape/volume

Elasticity

Deformation of soild

↓

Phys203

Fluid

Properties of fluid

↓

Phys203

3

12-7 *ELASTIC PROPERTIES OF MATERIALS* Changing shape

Elasticity is a material property, that describes its stiffness and is therefore one of the most important properties of solid materials.

Elasticity is a fundamental property of materials.

Deformation of solid

A **deformation** is the change in size or shape of an object.

المرونة : هي قابلية المادة للتشوه عند تأثير قوة خارجية عليها مع قدرة المادة على استعادة وضعها الأصلي بعد زوال السبب الذي أدى إلى تشوهها .

4

12-7 ***ELASTIC PROPERTIES OF MATERIALS***

Deformation

- When a object crash a car, the car may not move but it will noticeably change shape.
- A change in the shape due to the application of a force is a deformation.
- Even very small forces are known to cause some deformation.

Force causes Deformation

12-7 ***ELASTIC PROPERTIES OF MATERIALS***

Deformation

When force acts on a body, it can change its shape, the amount of deformation will depend on a number of things:

- The size of the specimen.
- The size of the force.
- The material of which the specimen is made.

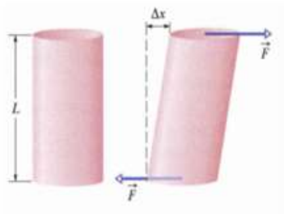
Types of Deformation

There are three ways to change the shape of a solid.

Change in length

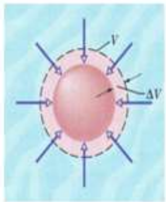
Longitudinal change
Tensile change

Change in shape without change in volume



Shear change

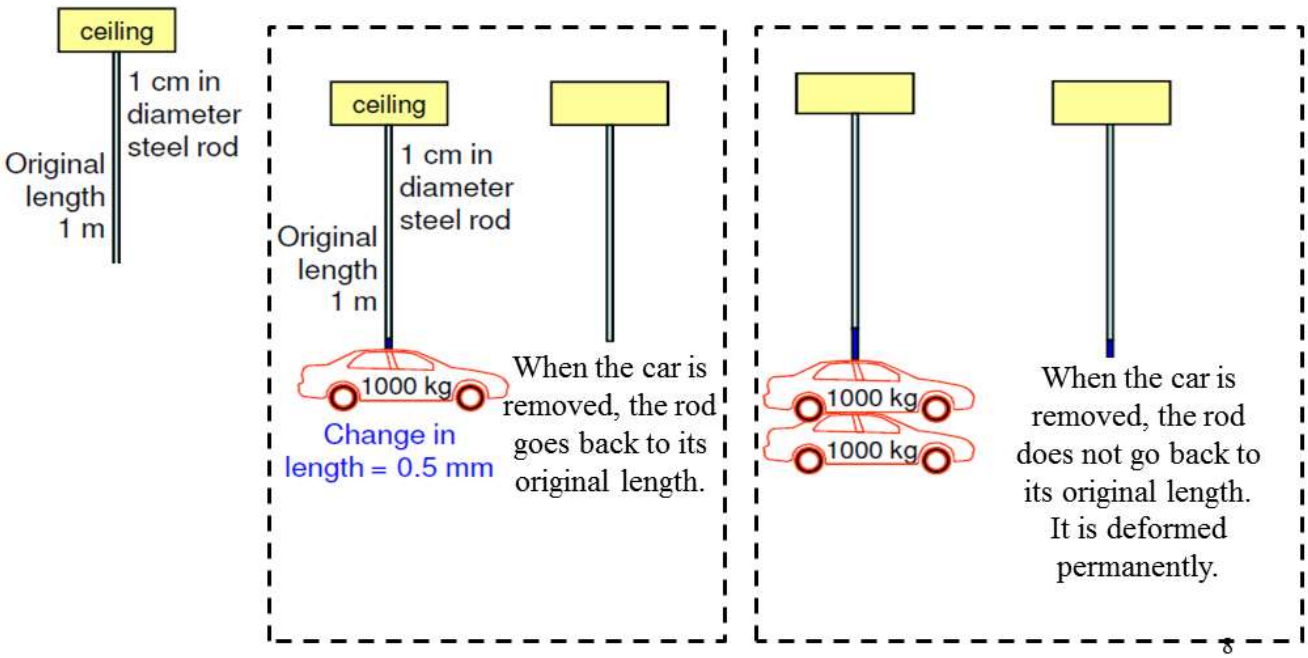
Change in volume without change in shape



Volume change
Hydraulic change

Changing shape

Solids change their shapes when they are pulled, pushed, or twisted.



12-7 *ELASTIC PROPERTIES OF MATERIALS*

Kinds of Bodies

Elastic body

An **elastic body** is one that returns to its original shape after a deformation.

Perfect Elastic body

Golf Ball

Rubber Band

Elastic body

Soccer Ball

Inelastic body

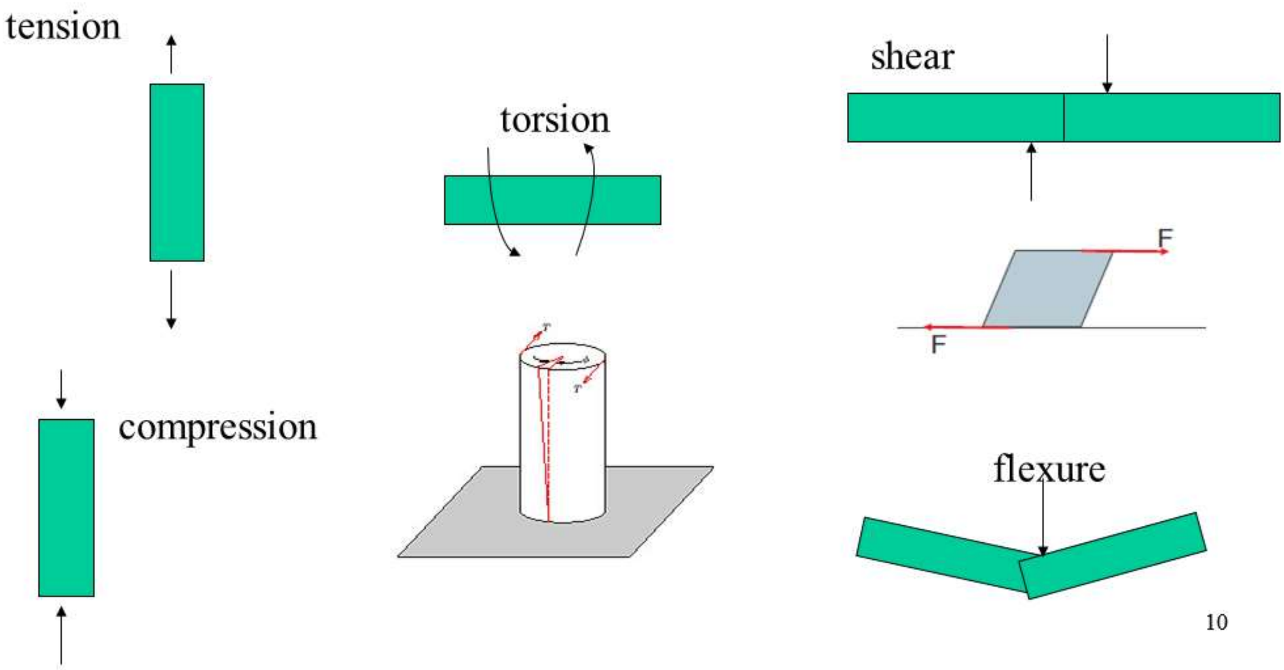
An **inelastic body** is one that does **not** return to its original shape after a deformation.

Dough or Bread

9

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Types of Loading



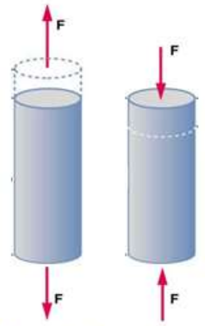
10

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Types of Loading (force)

We will study three ways to change the shape of a solid.

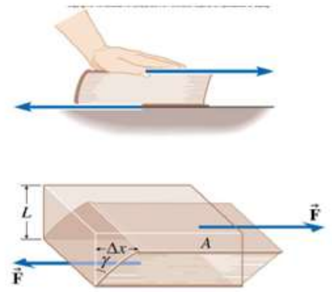
Tensile force
(Tension or compression)



The deforming force is perpendicular to the cross section.

Solid

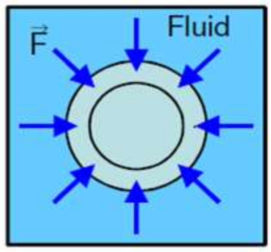
Shearing force



The deforming force is parallel to the cross section and two forces are applied on opposite surfaces .

Solid

Uniform compression force



The deforming force is perpendicular to the surface.

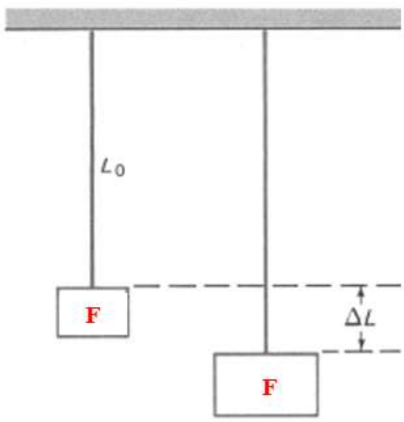
Solid-Liquid-Gas

12-7 *ELASTIC PROPERTIES OF MATERIALS*

In comparing the elasticity of materials there are certain terms we need to define.

Stress الاجهاد　　　**Strain** الانفعال

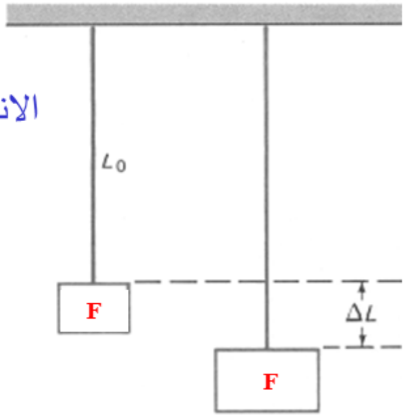
Suppose that we have a steel wire that is held rigidly at the top end and has a load fastened to the lower end (See Figure). The wire is then said to be under *stress*. We also can introduce another term called *strain which is* produced by stress.



12-7 *ELASTIC PROPERTIES OF MATERIALS*

In comparing the elasticity of materials there are certain terms we need to define.

الاجهاد **Stress** — **Strain** الانفعال



Stress refers to the cause of a deformation, and

strain refers to the effect of the deformation.

The downward force F causes the displacement ΔL.

Thus, the stress is the force;
the strain is the change in length (elongation/ contraction).

A stress produces a strain.

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Stress

- **Stress** is the ratio of an applied force F to the area A over which it acts
- **Stress :** Force per unit Area

$$Stress = \frac{F}{A}$$

$$\boldsymbol{\sigma = \frac{F}{A}}$$

F is load applied perpendicular to specimen cross section;

A is cross-sectional area

σ : stress in N/m^2
F : Force applied in Newton
A : cross sectional area in m^2

$$Units: \ \mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2} \ \text{or} \ \frac{\mathrm{lb}}{\mathrm{in.}^2}$$

12-7 *ELASTIC PROPERTIES OF MATERIALS*

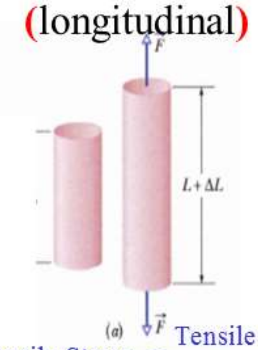
Stress

Stress is not directly measurable. We can calculate it from different formulas for different types of the loading.

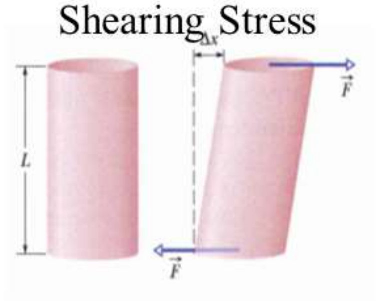
Types of stress

$$\sigma = \frac{F}{A}$$

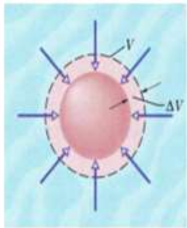
Tensile Stress (longitudinal)



Shearing Stress



Hydraulic Stress



$$\text{Tensile Stress} = \frac{\text{Tensile Force}}{\text{Surface Area}}$$

$$\sigma = \frac{F}{A}$$

$$\text{Shear Stress} = \frac{\text{Shear Force}}{\text{Surface Area}}$$

$$\sigma = \frac{F}{A}$$

$$\text{Hydraulic Stress} = \frac{\text{Hydraulic Force}}{\text{Surface Area}}$$

(pressure) $\sigma = P = \frac{F}{A}$

15

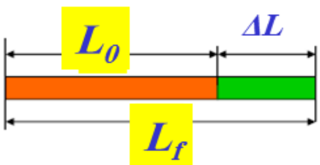
12-7 *ELASTIC PROPERTIES OF MATERIALS*

Strain

- **Strain** is the ratio of the change in length (**elongation**) to the original length
- **Strain** : is the fractional change in length.
- **Strain** : is the measure of the degree of deformation.

$$Strain = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{\Delta L}{L}$$



ε : strain Dimensionless (a unitless quantity)

ΔL : elongation (m) ($\Delta L = L_f - L_0$)

L : original length of a material (m)

12-7 *ELASTIC PROPERTIES OF MATERIALS*

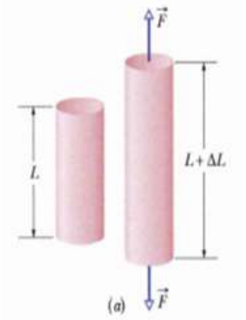
Strain

Strain is not directly measurable. We can calculate it from different formulas for different types of the deformation (force).

Types of strain

$$\varepsilon = \frac{\Delta L}{L}$$

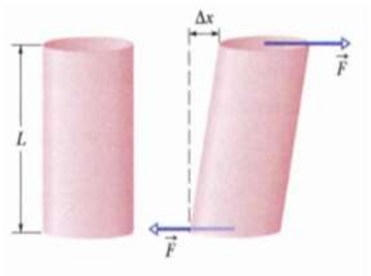
Tensile Strain



$$\text{Tensile Strain} = \frac{\text{Change in length}}{\text{length}}$$

$$\varepsilon = \frac{\Delta L}{L}$$

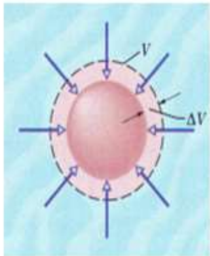
Shearing Strain



$$\text{Shear Strain} = \frac{\text{displacement of surfaces}}{\text{separation of surfaces}}$$

$$\varepsilon = \frac{\Delta x}{L}$$

Hydraulic Strain



$$\text{Hydraulic Strain} = \frac{\text{change in volume}}{\text{volume}}$$

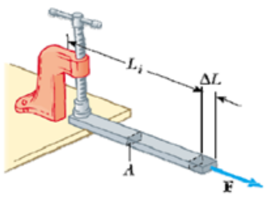
$$\varepsilon = \frac{\Delta V}{V}$$

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Example

A bar has dimensions 1cm by 1cm by 20cm. It is subjected to a 10000N tension force and stretches 0.01cm. Find (a) the stress; (b) the strain;

(c) If the stress-strain graph is straight line, how much does the bar stretch when the applied force is increased to 50000N?



12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **The relation between the stress and the strain for a material under tension can be found experimentally.**
- **A plot of Strain vs. Stress.**
- **The diagram gives us the behavior of the material and material properties.**
- **Each material produces a different stress-strain diagram.**

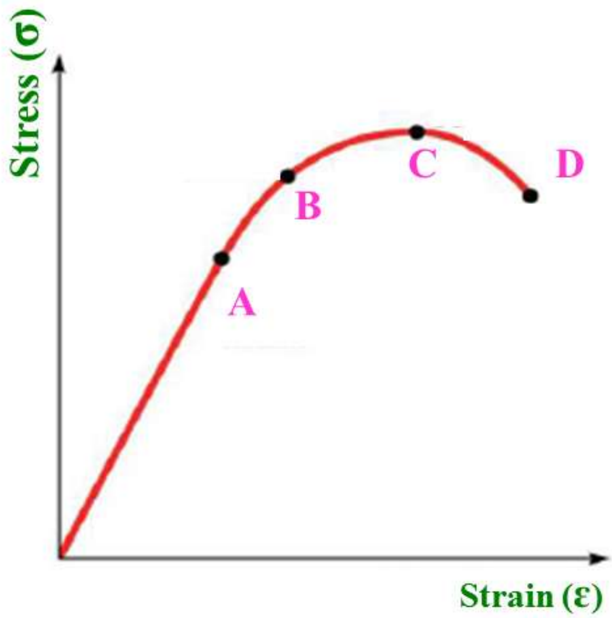
12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

Almost all of the mechanical properties of a material can be obtained from its stress-strain diagram curve.

The method that have been used to measure modulus of elasticity are following: tension (or compression) test

Stress – Strain Diagram



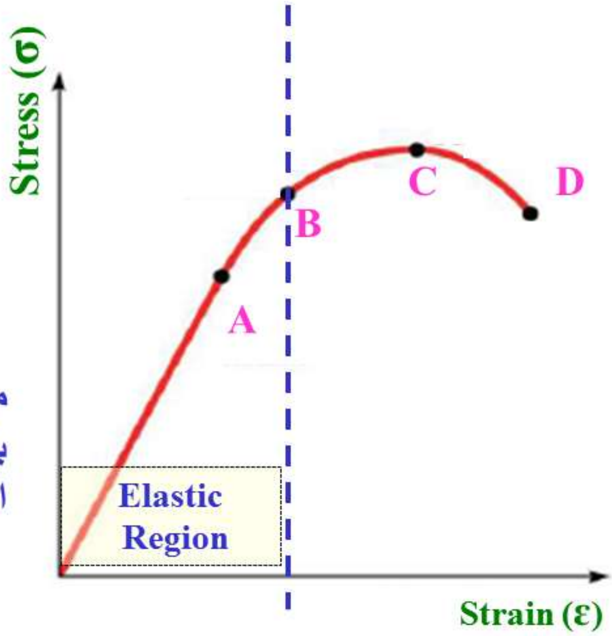
12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Elastic behavior (OB):** the object will returns to its original length (or shape) when the stress acting on it is removed.

Perfect elastic body **Elastic body**

مرحلة المرونة هي التي تحتفظ فيها المادة بمرونتها ويكون لديها القدرة على استرجاع شكلها الأصلى



Stress – Strain Diagram

21

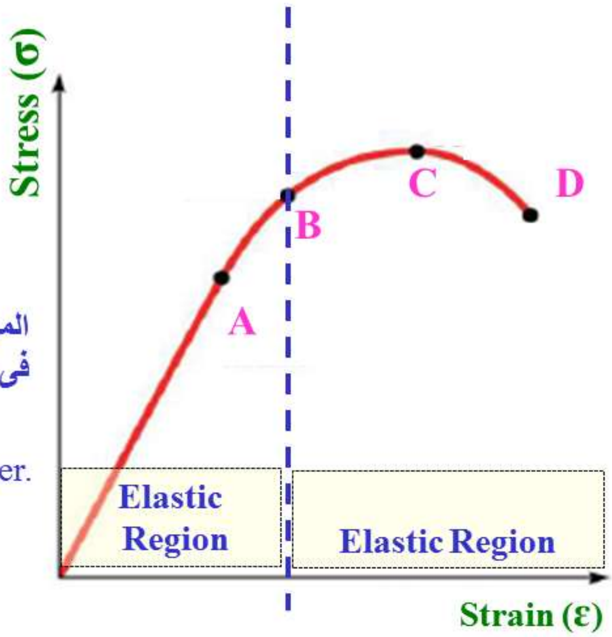
12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Plastic behavior (BD):** the object will NOT returns to its original length (or shape) when the stress acting on it is removed.

المرحلة الغير مرنة وهى المرحلة التي تحدث تشوهات كبيرة فى المادة تؤدى فى النهاية إلى القطع .

- stress and strain are not proportional to each other.
- deformation occurs by breaking and re-arrangement of atomic bonds (in crystalline materials primarily by motion of dislocations



Stress – Strain Diagram

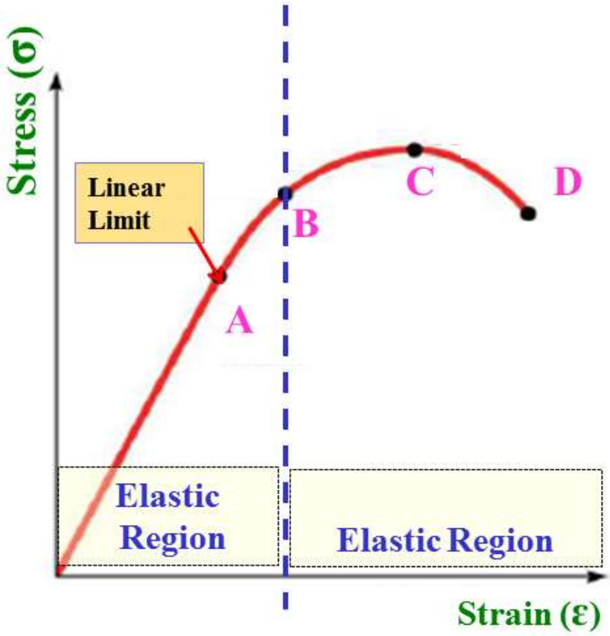
22

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Proportionality limit (A):** the stress above is not longer proportional to strain.

مرحلة المرونة التي يتناسب فيها الانفعال طرديا مع الاجهاد وتحتفظ فيها المادة بكامل مرونتها ويكون لديها القدرة التامة على استرجاع شكلها الأصلى



Stress – Strain Diagram

12-7 *ELASTIC PROPERTIES OF MATERIALS*

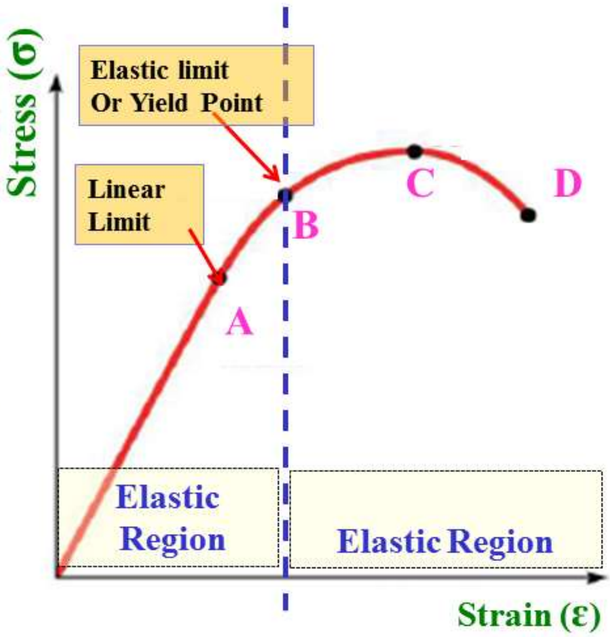
Relation Between Stress and Strain

- **Elastic Limit (B):** The maximum stress that can be applied without resulting permanent deformation.

حد المرونة وفيها لا تستطيع المادة استرجاع كامل شكلها الأصلى ولكن يحدث تشوهات قليلة فى شكل المادة بعد زوال القوة المؤثرة عليها.
و إذا تجاوزت القوى المؤثرة على الجسم الصلب هذا الحد – مهما كان نوعه – أكسبته تشوهاً دائماً

Elastic Region

- Linear Elastic Behavior
- Nonlinear Elastic Behavior



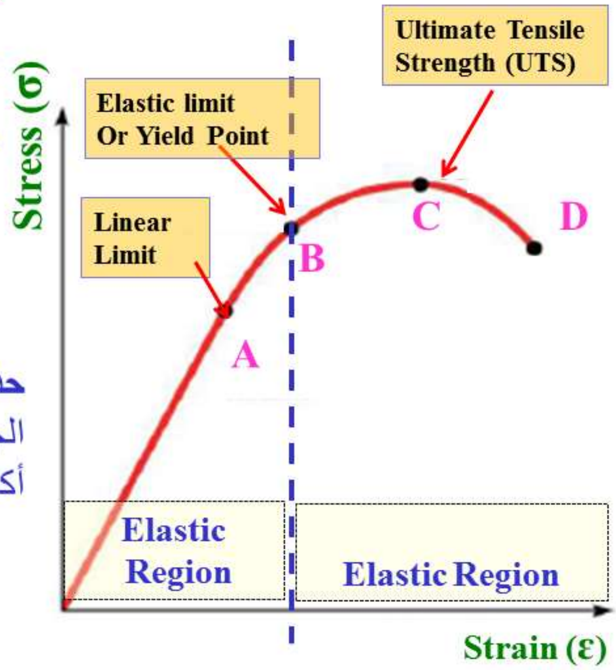
Stress – Strain Diagram

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Ultimate Tensile Strength (C):** the maximum stress the object can withstand before breaking.

حد المرونة : إذا تجاوزت القوى المؤثرة على الجسم الصلب هذا الحد – مهما كان نوعه – أكسبته تشوهاً دائماً



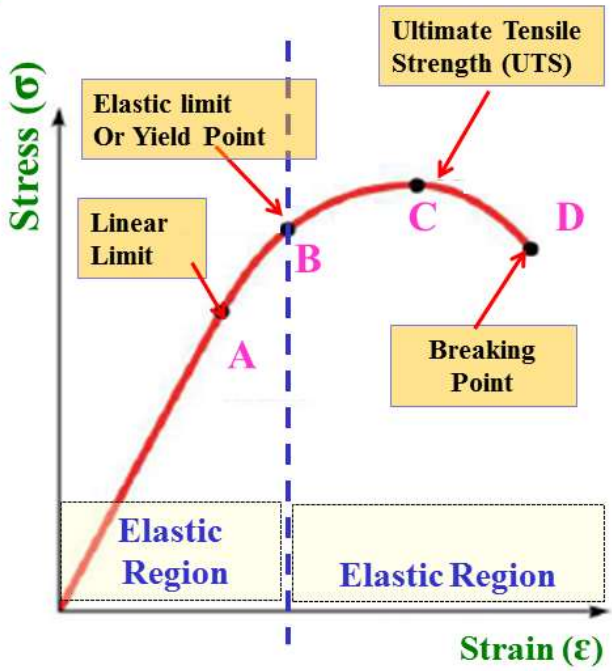
Stress – Strain Diagram

25

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Breaking point (D):** after the ultimate stress, an object will fracture if the stress exceeds the **breaking point**.



Stress – Strain Diagram

26

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Breaking point (D):**

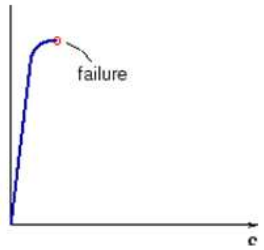
Material Behavior

Behavior of materials can be broadly classified into two categories

- **Brittle**

(Example: glass, ceramics)

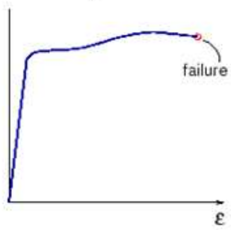
- The ultimate strength and breaking point C and D are close together

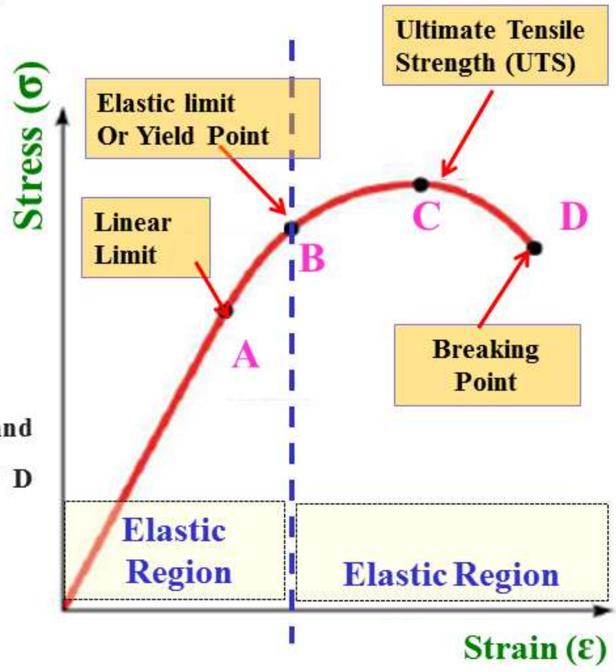


- **Ductile**

(Example: Metals; Gold, silver, copper, iron)

- The ultimate strength and breaking point C and D are far apart





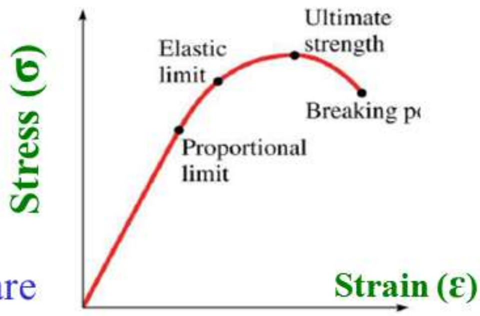
Stress – Strain Diagram

27

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

Stress – Strain Diagram



- **Proportionality (Linear) limit (A):**
- In the linear region of the Strain vs. Stress diagram, stress is directly proportional to strain.
- For deformations, two important characteristics are observed:

I. The object may returns to its original shape when the force is removed.

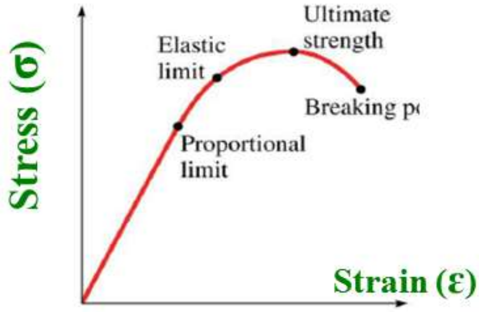
II. The size of the deformation is proportional to the force.

stress $F \propto \Delta L$ strain

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

Stress – Strain Diagram



- **Proportionality (Linear) limit (A):**

stress $F \propto \Delta L$ strain

$$F = -k\,\Delta L$$

In tensile tests, if the deformation is elastic:

- the strain and stress are related by Hooke´s Law
- the stress strain relationship is called Hooke's law
- Coefficient of proportionality between stress and strain is called a modulus of elasticity

Stress = Modulus of elasticity x Strain

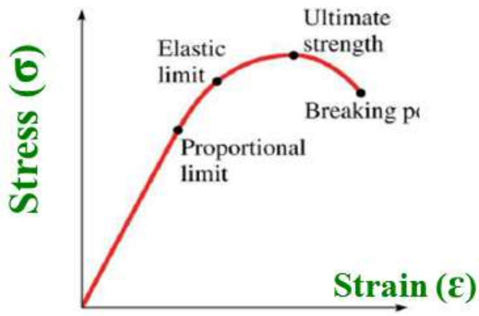
Constant of proportionality

29

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

Stress – Strain Diagram



- **Proportionality (Linear) limit (A):**

Graphically we can define modulus of elasticity as a slope of the linear portion of the stress-strain diagram

An elastic coefficient is defined as the stress divided by strain.

From the Hook's law the modulus of elasticity is defined as the ratio of the stress to the strain :

$$\text{Slope} = \text{Elastic Modulus (E)} \equiv \frac{\text{stress}}{\text{strain}} \qquad E = \frac{\sigma}{\varepsilon}$$

E has the same units as σ, N/m2 or Pa

30

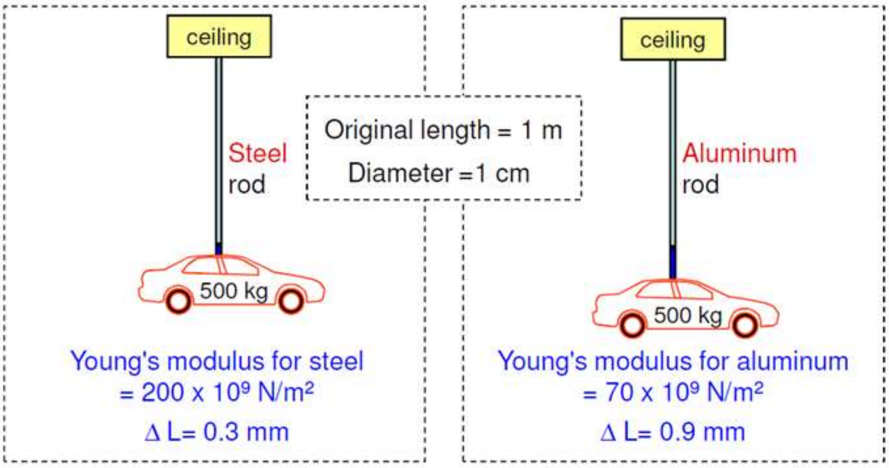
12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Proportionality (Linear) limit (A):**

$$\text{Strain} = \frac{\text{Stress}}{\text{Modulus}}$$

For the same stress, larger modulus ➔ smaller strain.



31

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Relation Between Stress and Strain

- **Proportionality (Linear) limit (A):**

$$\text{Strain} = \frac{\text{Stress}}{\text{Modulus}}$$

For the same stress, larger modulus ➔ smaller strain.

Some Elastic Properties of Selected Materials of Engineering Interest

Material	Density ρ (kg/m^3)	Young's Modulus E (10^9 N/m^2)	Ultimate Strength S_u (10^6 N/m^2)	Yield Strength S_y (10^6 N/m^2)
Steel[a]	7860	200	400	250
Aluminum	2710	70	110	95
Glass	2190	65	50[b]	—
Concrete[c]	2320	30	40[b]	—
Wood[d]	525	13	50[b]	—
Bone	1900	9[b]	170[b]	—
Polystyrene	1050	3	48	—

32

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Deformation types and define an elastic modulus

Types of Modulus of Elasticity (E) $E = \frac{\sigma}{\varepsilon}$

Elasticity in Length	Elasticity in Shape	Elasticity in Volume
Young's Modulus (Y)	**Shear's Modulus (S)**	**Bulk Modulus (B)**
$Y = \frac{\text{tensile } stress}{\text{tensile } strain}$	$S = \frac{\text{shearing } stress}{\text{shearing } strain}$	$B = \frac{\text{volume } stress}{\text{volume } strain}$
$\sigma = \frac{F}{A}$, $\varepsilon = \frac{\Delta L}{L}$	$\sigma = \frac{F}{A}$, $\varepsilon = \frac{\Delta x}{L}$	$\sigma = \frac{F}{A}$, $\varepsilon = \frac{\Delta V}{V}$
$Y = \frac{F/A}{\Delta L/L}$	$S = \frac{F/A}{\Delta x/L}$	$B = -\frac{F/A}{\Delta V/V} = \frac{P\,V}{\Delta V}$

33

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Deformation types and define an elastic modulus

Types of Modulus of Elasticity (E) $E = \frac{\sigma}{\varepsilon}$

Elasticity in Length	Elasticity in Shape	Elasticity in Volume
Young's Modulus (Y)	**Shear's Modulus (S)**	**Bulk Modulus (B)**
$Y = \frac{F/A}{\Delta L/L}$	$S = \frac{F/A}{\Delta x/L}$	$B = -\frac{F/A}{\Delta V/V} = -\frac{P\,V}{\Delta V}$

Stress and strain are positive for tensile loads, negative for compressive loads

negative sign is inserted in this defining equation so that *B is a positive number. This* is necessary because an increase in pressure (positive ΔP*) causes a decrease* in volume (negative ΔV*) and vice versa.*

34

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Deformations summary table

	Tensile or compressive	Shear	Volume
Stress	Force per unit cross-sectional area	Shear force divided by the area of the surface on which it acts	Pressure
Strain	Fractional change in length	Ratio of the relative displacement to the separation of the two parallel surfaces	Fractional change in volume
Constant of proportionality	Young's modulus (Y)	Shear modulus (S)	Bulk Modulus (B)

35

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Example

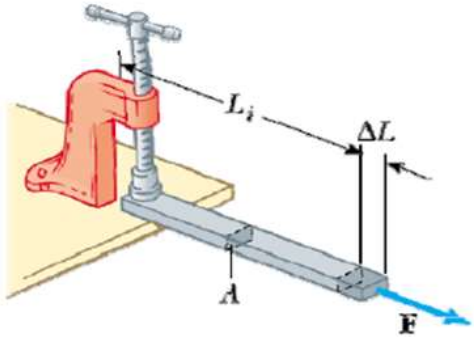
How much pressure is needed to compress the volume of an iron block by 0.10 percent? Express answer in N/m^2. (Hint: *B* for iron = $90x10^{10}$ N/m^2)

36

12-7 *ELASTIC PROPERTIES OF MATERIALS*

Example

In example of bar, a 10^8 N/m^2 stress produces a strain of $5x10^{-4}$. What is Young's modulus for this bar?



12-7 *ELASTIC PROPERTIES OF MATERIALS*

Example

A 15 cm long animal tendon (وتر)was found to stretch 3.7mm by a force of 13.4N. The tendon was approximately round with an average diameter of 8.5 mm.
Calculate the elastic modulus of this tendon.

12-7 *ELASTIC PROPERTIES OF MATERIALS*

problem **H.W.**

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0×10^5 N/m^2 (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2.0×10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

A vertical steel girder ((عارضة خشبية)with a cross sectional area of 0.15 m^2 has a 1550kg sign hanging from its end. (Ignore the mass of the girder itself.)

(a)What is the stress within the girder?

(b)What is the strain on the girder?

(c) If the girder is 9.50 m long, how much is it lengthened?

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

A 200-kg load is hung on a wire having a length of 4.00 m, cross-sectional area 0.200 x 10^{-4} m^2, and Young's modulus 8.00 x10^{10} N/m^2. What is its increase in length?

41

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

The upper surface of a cube of gelatin, 5.0 cm on a side, is displaced by 0.64 cm by a tangential force. If the shear modulus of the gelatin is 940 Pa, what is the magnitude of the tangential force?

42

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

A 0.50 m long guitar string, of cross- sectional area 1.0×10^{-6} m2, has a Young's modulus of 2.0×10^{9} Pa. By how much must you stretch a guitar string to obtain a tension of 20.0 N?

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

A steel wire 10 m long and with a cross-sectional area of 0.01 cm^2 is hung from a support and a mass of 5 kg is hung from its end. Calculate the new length of the wire. The Young modulus for steel = 210 GPa.

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

One end of steel rod of radius R=9.5 mm and length L=81 cm is held in a vise. A force of magnitude F=62 kN is then applied perpendicularly to the end face at the other end. What are the stress on the rod and the elongation ΔL and the strain of the rod?

12-7 *ELASTIC PROPERTIES OF MATERIALS*

H.W.

problem

An anchor, made of cast iron of bulk modulus 60.0 X109 Pa and a volume of 0.230 m3, is lowered over the side of a ship to the bottom of the harbor where the pressure is greater than sea level pressure by 1.75 X 106 Pa.

Find the change in the volume of the anchor.

Chapter (12)

المصطلح العلمي	الترجمة العلمية
Brittle	
Compress	
Compress	
Compression	
Contraction	
Deformation	
Diagram	
Ductile	
Elastic	
Elasticity	
Elongation	
Flexure	
Fluid	
Fracture	
Inelastic	
Linear	
Load(N)	
Longitudinal	
Modulus	

المصطلح العلمي	الترجمة العلمية
Nonlinear	
Perfect	
Permanent	
Pressure	
Proportional	
Shear	
Slope	
Specimen	
Stiffness	
Strain	
Strength	
Stress	
Tensile	
Tension	
Torsion	
Twist	
Ultimate	
Uniform	
Yield	