

Cut here and keep for reference

**ALGEBRA****Arithmetic Operations**

$$a(b + c) = ab + ac$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

**Exponents and Radicals**

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

**Factoring Special Polynomials**

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

**Binomial Theorem**

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2$$

$$+ \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

$$\text{where } \binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdot 3 \cdots k}$$

**Quadratic Formula**

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

**Inequalities and Absolute Value**

If  $a < b$  and  $b < c$ , then  $a < c$ .

If  $a < b$ , then  $a + c < b + c$ .

If  $a < b$  and  $c > 0$ , then  $ca < cb$ .

If  $a < b$  and  $c < 0$ , then  $ca > cb$ .

If  $a > 0$ , then

$|x| = a$  means  $x = a$  or  $x = -a$

$|x| < a$  means  $-a < x < a$

$|x| > a$  means  $x > a$  or  $x < -a$

**GEOMETRY****Geometric Formulas**

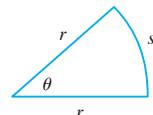
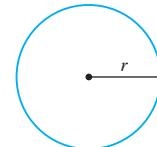
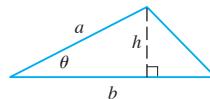
Formulas for area  $A$ , circumference  $C$ , and volume  $V$ :

Triangle

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}ab \sin \theta$$

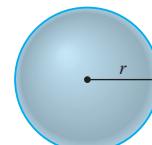
$$\begin{array}{lll} \text{Circle} & A = \pi r^2 & \text{Sector of Circle} \\ & C = 2\pi r & A = \frac{1}{2}r^2\theta \\ & & s = r\theta \quad (\theta \text{ in radians}) \end{array}$$



Sphere

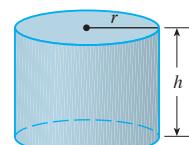
$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$



Cylinder

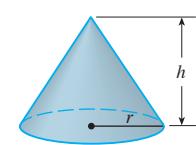
$$V = \pi r^2 h$$



Cone

$$V = \frac{1}{3}\pi r^2 h$$

$$A = \pi r r \sqrt{r^2 + h^2}$$

**Distance and Midpoint Formulas**

Distance between  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint of } \overline{P_1P_2}: \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Lines**

Slope of line through  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through  $P_1(x_1, y_1)$  with slope  $m$ :

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope  $m$  and  $y$ -intercept  $b$ :

$$y = mx + b$$

**Circles**

Equation of the circle with center  $(h, k)$  and radius  $r$ :

$$(x - h)^2 + (y - k)^2 = r^2$$

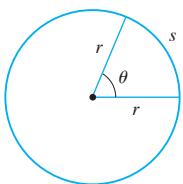
**TRIGONOMETRY****Angle Measurement**

$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \quad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta$$

( $\theta$  in radians)

**Right Angle Trigonometry**

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

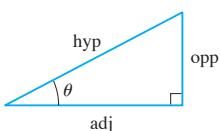
$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

**Trigonometric Functions**

$$\sin \theta = \frac{y}{r}$$

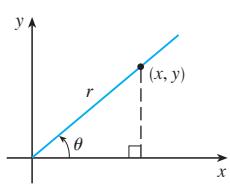
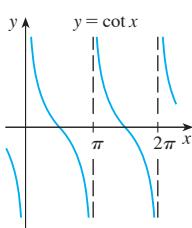
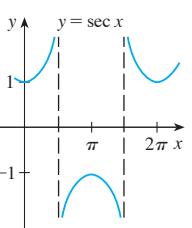
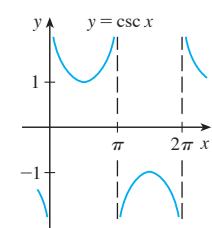
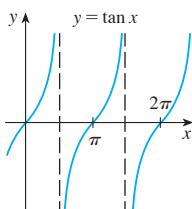
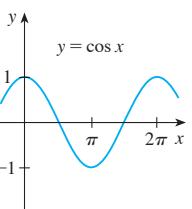
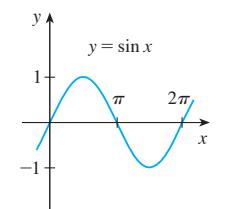
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

**Graphs of Trigonometric Functions****Trigonometric Functions of Important Angles**

$\theta$	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
$0^\circ$	0	0	1	0
$30^\circ$	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$45^\circ$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$60^\circ$	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90^\circ$	$\pi/2$	1	0	—

**Fundamental Identities**

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

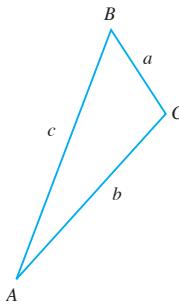
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

**The Law of Sines**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**The Law of Cosines**

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Addition and Subtraction Formulas**

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

**Double-Angle Formulas**

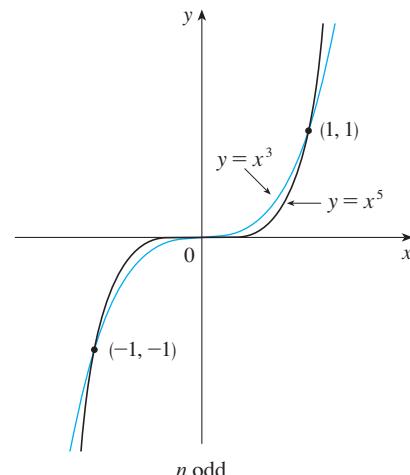
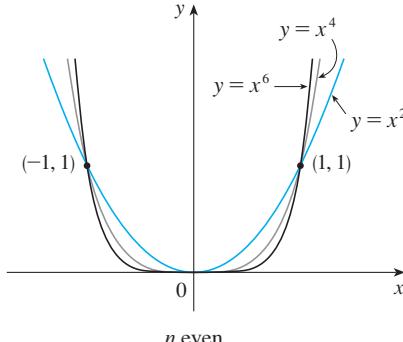
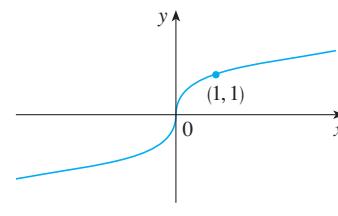
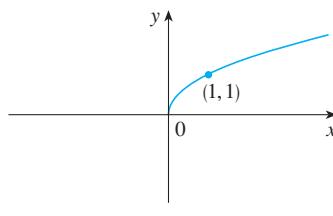
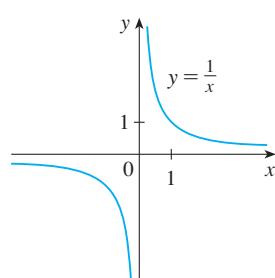
$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

**Half-Angle Formulas**

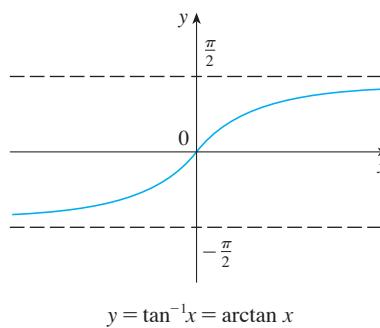
$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Cut here and keep for reference **SPECIAL FUNCTIONS****Power Functions**  $f(x) = x^a$ (i)  $f(x) = x^n, n$  a positive integer(ii)  $f(x) = x^{1/n} = \sqrt[n]{x}, n$  a positive integer(iii)  $f(x) = x^{-1} = \frac{1}{x}$ **Inverse Trigonometric Functions**

$$\arcsin x = \sin^{-1} x = y \iff \sin y = x \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$\arccos x = \cos^{-1} x = y \iff \cos y = x \text{ and } 0 \leq y \leq \pi$$

$$\arctan x = \tan^{-1} x = y \iff \tan y = x \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}$$



$$\lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow \infty} \tan^{-1} x = \frac{\pi}{2}$$

## SPECIAL FUNCTIONS

### Exponential and Logarithmic Functions

$$\log_a x = y \iff a^y = x$$

$\ln x = \log_e x$ , where  $\ln e = 1$

$$\ln x = y \iff e^y = x$$

#### Cancellation Equations

$$\log_a(a^x) = x \quad a^{\log_a x} = x$$

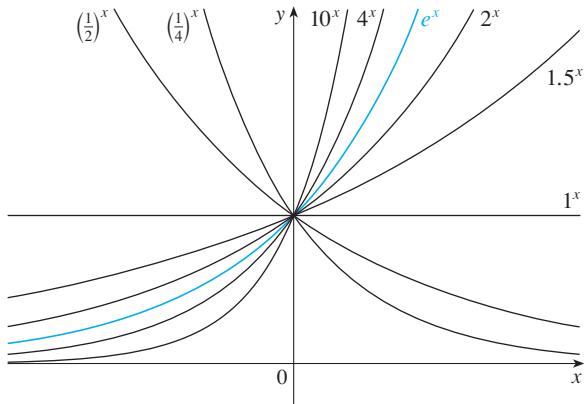
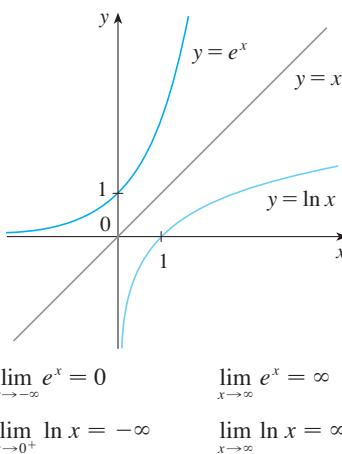
$$\ln(e^x) = x \quad e^{\ln x} = x$$

#### Laws of Logarithms

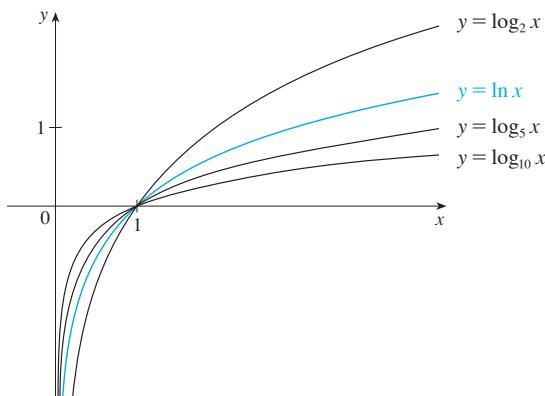
$$1. \log_a(xy) = \log_a x + \log_a y$$

$$2. \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$3. \log_a(x^r) = r \log_a x$$



Exponential functions



Logarithmic functions

### Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

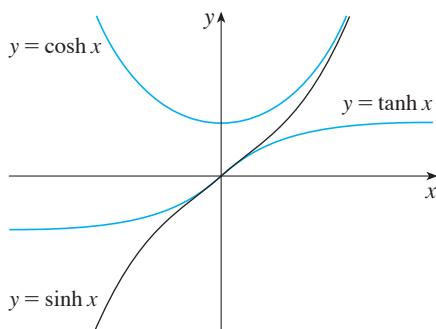
$$\csc x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$



### Inverse Hyperbolic Functions

$$y = \sinh^{-1} x \iff \sinh y = x$$

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$y = \cosh^{-1} x \iff \cosh y = x \text{ and } y \geq 0$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$$

$$y = \tanh^{-1} x \iff \tanh y = x$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

**DIFFERENTIATION RULES****General Formulas**

1.  $\frac{d}{dx}(c) = 0$

3.  $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

5.  $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$  (Product Rule)

7.  $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$  (Chain Rule)

2.  $\frac{d}{dx}[cf(x)] = cf'(x)$

4.  $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

6.  $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$  (Quotient Rule)

8.  $\frac{d}{dx}(x^n) = nx^{n-1}$  (Power Rule)

**Exponential and Logarithmic Functions**

9.  $\frac{d}{dx}(e^x) = e^x$

11.  $\frac{d}{dx}\ln|x| = \frac{1}{x}$

10.  $\frac{d}{dx}(a^x) = a^x \ln a$

12.  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

**Trigonometric Functions**

13.  $\frac{d}{dx}(\sin x) = \cos x$

16.  $\frac{d}{dx}(\csc x) = -\csc x \cot x$

14.  $\frac{d}{dx}(\cos x) = -\sin x$

17.  $\frac{d}{dx}(\sec x) = \sec x \tan x$

15.  $\frac{d}{dx}(\tan x) = \sec^2 x$

18.  $\frac{d}{dx}(\cot x) = -\csc^2 x$

**Inverse Trigonometric Functions**

19.  $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

22.  $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

20.  $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

23.  $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

21.  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

24.  $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

**Hyperbolic Functions**

25.  $\frac{d}{dx}(\sinh x) = \cosh x$

28.  $\frac{d}{dx}(\csch x) = -\csch x \coth x$

26.  $\frac{d}{dx}(\cosh x) = \sinh x$

29.  $\frac{d}{dx}(\sech x) = -\sech x \tanh x$

27.  $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

30.  $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

**Inverse Hyperbolic Functions**

31.  $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

34.  $\frac{d}{dx}(\csch^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$

32.  $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

35.  $\frac{d}{dx}(\sech^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

33.  $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

36.  $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

**TABLE OF INTEGRALS****Basic Forms**

1.  $\int u \, dv = uv - \int v \, du$

2.  $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

3.  $\int \frac{du}{u} = \ln |u| + C$

4.  $\int e^u \, du = e^u + C$

5.  $\int a^u \, du = \frac{a^u}{\ln a} + C$

6.  $\int \sin u \, du = -\cos u + C$

7.  $\int \cos u \, du = \sin u + C$

8.  $\int \sec^2 u \, du = \tan u + C$

9.  $\int \csc^2 u \, du = -\cot u + C$

10.  $\int \sec u \tan u \, du = \sec u + C$

11.  $\int \csc u \cot u \, du = -\csc u + C$

12.  $\int \tan u \, du = \ln |\sec u| + C$

13.  $\int \cot u \, du = \ln |\sin u| + C$

14.  $\int \sec u \, du = \ln |\sec u + \tan u| + C$

15.  $\int \csc u \, du = \ln |\csc u - \cot u| + C$

16.  $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$

17.  $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

18.  $\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$

19.  $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$

20.  $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

**Forms Involving  $\sqrt{a^2 + u^2}$ ,  $a > 0$** 

21.  $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$

22.  $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$

23.  $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$

24.  $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$

25.  $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$

26.  $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$

27.  $\int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$

28.  $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$

29.  $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

**TABLE OF INTEGRALS****Forms Involving  $\sqrt{a^2 - u^2}$ ,  $a > 0$** 

30.  $\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$

31.  $\int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$

32.  $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

33.  $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$

34.  $\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$

35.  $\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

36.  $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$

37.  $\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$

38.  $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$

**Forms Involving  $\sqrt{u^2 - a^2}$ ,  $a > 0$** 

39.  $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$

40.  $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$

41.  $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$

42.  $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$

43.  $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$

44.  $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$

45.  $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46.  $\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$

**TABLE OF INTEGRALS****Forms Involving  $a + bu$** 

47.  $\int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$

48.  $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$

49.  $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$

50.  $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$

51.  $\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$

52.  $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$

53.  $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left( a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$

54.  $\int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$

55.  $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$

56.  $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$

57. 
$$\begin{aligned} \int \frac{du}{u\sqrt{a + bu}} &= \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{if } a > 0 \\ &= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{if } a < 0 \end{aligned}$$

58.  $\int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u\sqrt{a + bu}}$

59.  $\int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a + bu}}$

60.  $\int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[ u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$

61.  $\int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$

62.  $\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$

**TABLE OF INTEGRALS****Trigonometric Forms**

63.  $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

64.  $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

65.  $\int \tan^2 u \, du = \tan u - u + C$

66.  $\int \cot^2 u \, du = -\cot u - u + C$

67.  $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$

68.  $\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$

69.  $\int \tan^3 u \, du = \frac{1}{2}\tan^2 u + \ln |\cos u| + C$

70.  $\int \cot^3 u \, du = -\frac{1}{2}\cot^2 u - \ln |\sin u| + C$

71.  $\int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln |\sec u + \tan u| + C$

72.  $\int \csc^3 u \, du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln |\csc u - \cot u| + C$

73.  $\int \sin^n u \, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$

74.  $\int \cos^n u \, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$

75.  $\int \tan^n u \, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u \, du$

76.  $\int \cot^n u \, du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u \, du$

77.  $\int \sec^n u \, du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$

78.  $\int \csc^n u \, du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$

79.  $\int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$

80.  $\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

81.  $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82.  $\int u \sin u \, du = \sin u - u \cos u + C$

83.  $\int u \cos u \, du = \cos u + u \sin u + C$

84.  $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$

85.  $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

86. 
$$\begin{aligned} \int \sin^n u \cos^m u \, du &= -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du \\ &= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du \end{aligned}$$

**Inverse Trigonometric Forms**

87.  $\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$

88.  $\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$

89.  $\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$

90.  $\int u \sin^{-1} u \, du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

91.  $\int u \cos^{-1} u \, du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$

92.  $\int u \tan^{-1} u \, du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$

93.  $\int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$

94.  $\int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$

95.  $\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$

**TABLE OF INTEGRALS****Exponential and Logarithmic Forms**

**96.**  $\int ue^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$

**97.**  $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$

**98.**  $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$

**99.**  $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$

**100.**  $\int \ln u du = u \ln u - u + C$

**101.**  $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$

**102.**  $\int \frac{1}{u \ln u} du = \ln |\ln u| + C$

**Hyperbolic Forms**

**103.**  $\int \sinh u du = \cosh u + C$

**104.**  $\int \cosh u du = \sinh u + C$

**105.**  $\int \tanh u du = \ln \cosh u + C$

**106.**  $\int \coth u du = \ln |\sinh u| + C$

**107.**  $\int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$

**108.**  $\int \operatorname{csch} u du = \ln |\tanh \frac{1}{2}u| + C$

**109.**  $\int \operatorname{sech}^2 u du = \tanh u + C$

**110.**  $\int \operatorname{csch}^2 u du = -\coth u + C$

**111.**  $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$

**112.**  $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

**Forms Involving  $\sqrt{2au - u^2}$ ,  $a > 0$** 

**113.**  $\int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**114.**  $\int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**115.**  $\int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**116.**  $\int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**117.**  $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**118.**  $\int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**119.**  $\int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + C$

**120.**  $\int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$

## Integration by Parts

$$\int u \, dv = uv - \int v \, du$$

$$\int \frac{P(x)}{Q(x)} dx$$

- اگر در برابری معادل علی  $Q(x)$  و همانکه  $\deg(Q(x)) < \deg(P(x))$  ، آن‌گاه اگر  $Q(x) = 0$  باشد، می‌توانیم از  $P(x)$  برای  $Q(x)$  جزو  $P(x)$  در نظر گیری کنیم.

Case(I)

میراث حضرت مسیح

$$\therefore \frac{P(x)}{Q(x)} = \frac{A_1}{(a_1x+b_1)} + \frac{A_2}{(a_2x+b_2)} + \dots + \frac{A_k}{(a_kx+b_k)}$$

$$\therefore \frac{P(x)}{Q(x)} = \frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_r}{(ax+b)^r}$$

$$Q(x) = (ax + b)^r$$

Case(II)

المرصد لذوق وتحليل

$$\therefore \frac{P(x)}{Q(x)} = \frac{A_1}{(a_1x+b_1)} + \frac{A_2}{(a_2x+b_2)} + \dots + \frac{A_k}{(a_kx+b_k)}$$

اذاكا به لستنا معاشر  $ax^2 + bx + c$  . لذا فالجواب يربع ثالثتنا  
 $\frac{b}{2\sqrt{a}}$

اذا كان  $a \neq 0$  فالنهاية الثانية للمنحنى هي  $y = \frac{c}{a}x^2 + bx + c$

صفحه و نظریه آنکه

$$\left( \frac{b}{2\sqrt{a}} \right)^2$$

\* حملة خاصة لذواجها ت = 0 تكريه الالات لفريم المقدمة

$$x^2 + bx + c = 0$$

case(III)

حاصل مزدوج كثرة حدود من المرجعية المائية وندر تقابلها

$$Q(x) = (a_0 x^2 + b_1 x + c_1)(a_2 x^2 + b_2 x + c_2) \cdots (a_k x^2 + b_k x + c_k)$$

$$\therefore \frac{P(x)}{Q(x)} = \frac{A_1x + B_1}{a_1x^2 + b_1x + c_1} + \frac{A_2x + B_2}{a_2x^2 + b_2x + c_2} + \dots + \frac{A_kx + B_k}{a_kx^2 + b_kx + c_k}$$

النحو المعنى المفهوم المفهوم

$$= \frac{cu}{u^2 + a^2} + \frac{D}{u^2 + a^2}$$

م إذا احتجنا لتقديم الـ  $\omega$  في  $\mathbb{R}^n$  فإنه يكتب على الشكل

$$C + \sum_{i=1}^m a_i x_i^{n_i}$$

حيث  $a_i \in \mathbb{R}$  و  $n_i \geq 1$  و  $x_i \in \mathbb{R}^n$

Case 4

حاصل ضرب كثافة عدد صلادة  $\rho = \frac{1}{(B^2 - 4ac)}$  و مقدار

$$Q(x) = (ax^2 + bx + c)^r$$

$$\therefore \frac{P(x)}{Q(x)} = \frac{A_1 x + B_1}{ax^2 + bx + c} + \frac{A_2 x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_r x + B_r}{(ax^2 + bx + c)^r}$$

م إذا احتجنا لتقديم الـ  $\omega$  في  $\mathbb{R}^n$  فإنه يكتب على الشكل

$$\int_{\Omega} \omega = \int_{\Omega} \sum_{|\alpha| \leq n} a_{\alpha} x^{\alpha} dx = \sum_{|\alpha| \leq n} a_{\alpha} \int_{\Omega} x^{\alpha} dx$$