

If an electron makes a transition from the $n = 4$ to the $n = 1$ Bohr orbital in a hydrogen atom, determine the wavelength of the light emitted and the recoil speed of the atom

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

With $R = 1.097 \times 10^7 \text{ m}^{-1}$, $n_i = 4$, and $n_f = 1$, we find $\lambda = 97.2 \text{ nm}$. This photon carries away momentum given by

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{97.2 \times 10^{-9} \text{ m}} = 6.62 \times 10^{-27} \left| \frac{\text{kg} \cdot \text{m}}{\text{s}} \right|.$$

which by momentum conservation must be the recoil momentum of the atom. Using the mass of the hydrogen atom as $1.67 \times 10^{-27} \text{ kg}$, we find its speed must be

$$v = \frac{p}{m} = \frac{6.62 \times 10^{-27} \text{ kg} \cdot \text{m} / \text{s}}{1.67 \times 10^{-27} \text{ kg}} = 4.1 \frac{\text{m}}{\text{s}}.$$

Thus, the wavelength of the emitted photon is 97.2 nm, while the recoil speed of the hydrogen atom is 4.1 m/s.

A photon incident on a hydrogen atom causes the electron to make a transition from the $n = 1$ orbital to the $n = 3$ orbital. What is the wavelength of the photon, and what are the possible

Solution:

We assume the photon gives up all of its energy to the electron. In making a transition from $n = 1$ to $n = 3$, the electron gains an energy

$$E = -13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = -13.6 \text{ eV} \left(\frac{1}{3^2} - \frac{1}{1^2} \right) = 12.1 \text{ eV},$$

which by energy conservation must be equal to the energy of the incident photon. The associated wavelength is then found using

$$\lambda = \frac{hc}{E} = \frac{1243 \text{ eV} \cdot \text{nm}}{12.1 \text{ eV}} = 102.8 \text{ nm}.$$

9.39 What percentage of incident X-ray radiation passes through 5.0 mm of material whose linear absorption coefficient is 0.07 mm^{-1} ?

Ans.
$$\frac{I}{I_0} = e^{-\mu x} = e^{-(0.07 \text{ mm}^{-1})(5.0 \text{ mm})} = 0.705 = 70.5\%$$

11.1. Determine, in angstroms, the shortest and longest wavelengths of the Lyman series of hydrogen.

Ans. Wavelengths in the Lyman series are given by $n_l = 1$:

$$\frac{1}{\lambda} = (1.097 \times 10^{-3} \text{ \AA}^{-1}) \left(\frac{1}{1^2} - \frac{1}{n_u^2} \right) \quad n_u = 2, 3, 4, \dots$$

The longest wavelength corresponds to $n_u = 2$:

$$\frac{1}{\lambda_{\max}} = (1.097 \times 10^{-3} \text{ \AA}^{-1}) \left(1 - \frac{1}{2^2} \right) \quad \text{or} \quad \lambda_{\max} = 1215 \text{ \AA}$$

The shortest wavelength corresponds to $n_u = \infty$:

$$\frac{1}{\lambda_{\min}} = 1.097 \times 10^{-3} \text{ \AA}^{-1} \left(1 - \frac{1}{\infty^2} \right) \quad \text{or} \quad \lambda_{\min} = 912 \text{ \AA}$$

11.6. Find the wavelength of the photon that is emitted when a hydrogen atom undergoes a transition from $n_u = 5$ to $n_l = 2$.

Ans. From the Bohr model, the energy levels are $E_n = (-13.6 \text{ eV})/n^2$. Hence,

$$E_2 = -\frac{13.6 \text{ eV}}{2^2} = -3.40 \text{ eV} \quad E_5 = -\frac{13.6 \text{ eV}}{5^2} = -0.544 \text{ eV}$$

From the Bohr postulates the energy of the emitted photon is

$$E_\gamma = -0.544 \text{ eV} - (-3.40 \text{ eV}) = 2.86 \text{ eV}$$

The wavelength of this photon is given by

$$\lambda = \frac{hc}{E_\gamma} = \frac{12.4 \times 10^3 \text{ eV} \cdot \text{\AA}}{2.86 \text{ eV}} = 4340 \text{ \AA}$$