Increasing / Decreasing Test:

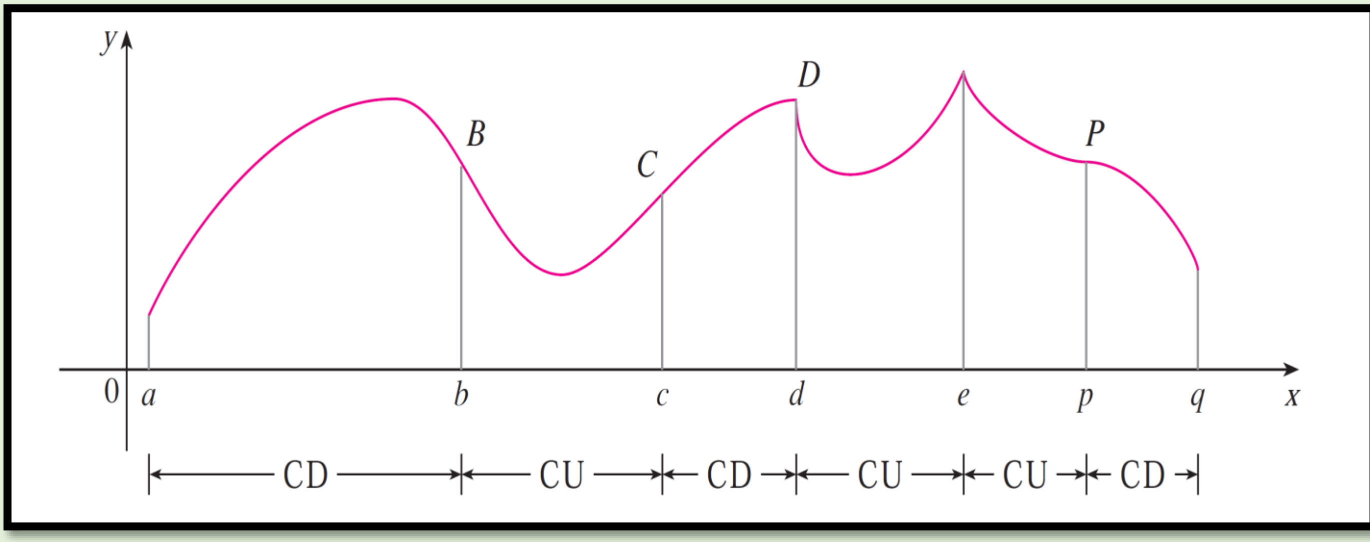
(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.
(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

The First Derivative Test:

Suppose that c is a critical number of a continuous function f.
(a) If f' changes from positive to negative at c, then f has a local maximum at c.
(b) If f' changes from negative to positive at c, then f has a local minimum at c.
(c) If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then has no local maximum or minimum at c.

Definition:

If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.



Concavity Test:

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.
(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition: (Inflection Point)

A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

The Second Derivative Test:

Suppose f'' is continuous near c.
(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

Example: Find the intervals of increasing and decreasing, local extreme values, intervals of concavity and inflection point of

$$f(x) = x^3 - 6x^2 - 36x$$

Solution:

Example: Find the intervals of increasing and decreasing, local extreme values, intervals of concavity and inflection point of

$$f(x) = -x^3 - 6x^2 - 9x + 1$$

Solution:

Example: Find the intervals of increasing and decreasing, local extreme values, intervals of concavity and inflection point of

$$f(x) = x^4 - 4x^3$$

Solution: