The Chain Rule:

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f\big(g(x)\big)$ is differentiable at x and F' is given by the product

$$F'(x) = f'\big(g(x)\big) \,.\, g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example: Find $F'(x)$ if $F(x) = \sqrt{x^2+1}$.

Solution:

Example: Differentiate

$$y = \sin(x^2)$$

Solution:

Example: Differentiate

$$y = \sin^2 x$$

Solution:

Differentiation Rules:

$\frac{d}{dx}[f(x)]^n = n\,[f(x)]^{n-1}.f'(x)$	$\frac{d}{dx}\left(\sqrt{f(x)}\right) = \frac{f'(x)}{2\sqrt{f(x)}}$
$\frac{d}{dx}\left(a^{f(x)}\right) = a^{f(x)}.\ f'(x)\ .\ \ln a$	$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)}\ .\ f'(x)$
$\frac{d}{dx}(\sin u(x)) = \cos u(x)\ .u'(x)$	$\frac{d}{dx}(\csc u(x)) = -\csc u(x)\cot u(x)\,.u'(x)$
$\frac{d}{dx}(\cos u(x)) = -\sin u(x)\,.u'(x)$	$\frac{d}{dx}(\sec u(x)) = \sec u(x)\tan u(x)\,.u'(x)$
$\frac{d}{dx}(\tan u(x)) = \sec^2 u(x)\,.u'(x)$	$\frac{d}{dx}(\cot u(x)) = -\csc^2 u(x)\,.u'(x)$

Example: Differentiate

$$y = (x^3 - 1)^{100}$$

Solution:

Example: Find $f'(x)$ if

$$f(x) = \frac{1}{\sqrt[3]{x^2 + x + 1}}$$

Solution:

Example: Find the derivative of the function

$$g(t) = \left(\frac{t - 2}{2t + 1}\right)^9$$

Solution:

Example: Differentiate

$$y = (2x + 1)^5(x^3 - x + 1)^4$$

Solution:

Example: Differentiate

$$y = e^{\sin x}$$

Solution:

Example: If $f(x) = \sin(\cos(\tan x))$, then find $f'(x)$.

Solution:

Example: Differentiate

$$y = e^{\sec 3\theta}$$

Solution:

Example: Find the derivative of the functions

$$y = \cos(a^3 + x^3)$$

$$y = a^3 + \cos^3 x$$