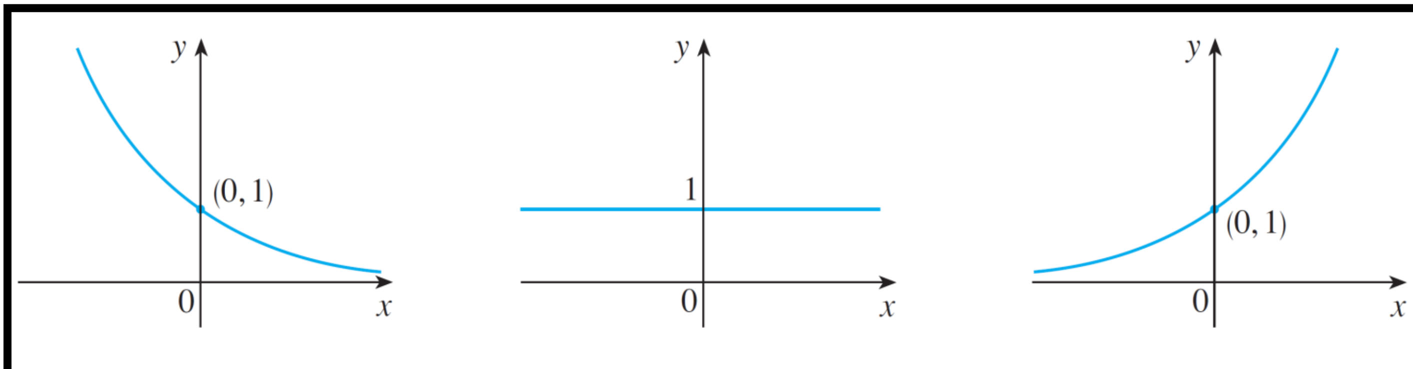
## Exponential Functions

The function $\boldsymbol{f(x) = a^x,\ a > 0,\ a \neq 1}$ is called an **exponential function** because the variable, $\boldsymbol{x}$, is the exponent.



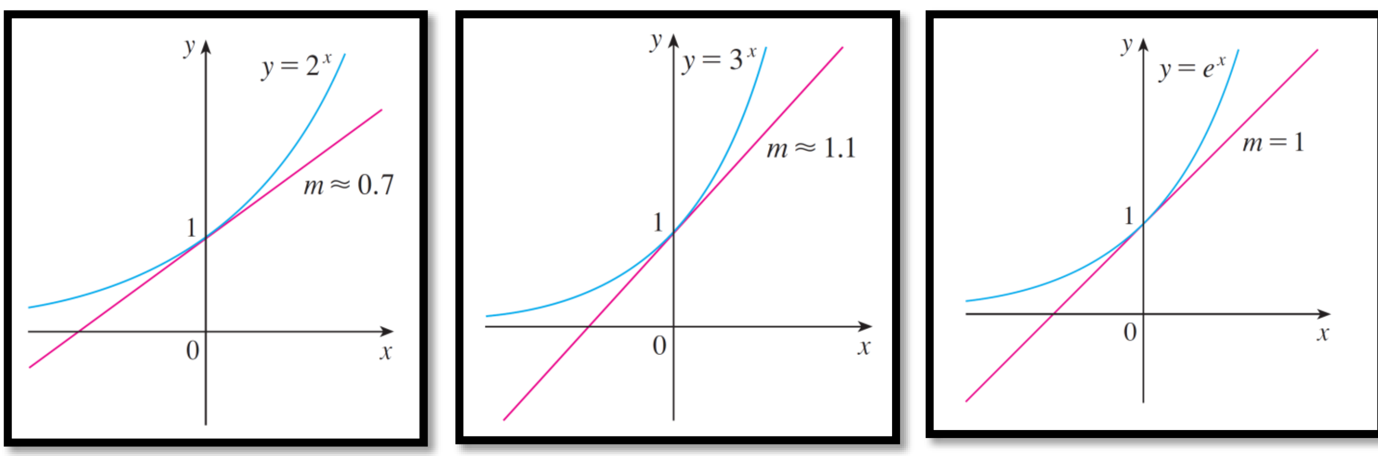

(a) $y = a^x,\ 0 < a < 1$ (b) $y = 1^x$ (c) $y = a^x,\ a > 1$

**If $a \neq 1$, then $D_f = \mathbb{R} = (-\infty, \infty)$, $R_f = (0, \infty)$**

**Remarks:**

1. $f(x) = \left(\dfrac{1}{a}\right)^x = \dfrac{1}{a^x} = a^{-x}$
2. The graphs of all exponential functions pass through the point $(0,1)$.

**The Number $e$:**




$D_f = \mathbb{R} = (-\infty, \infty)$, $R_f = (0, \infty)$

To find the domain and range of translated exponential functions:

(1) If $f(x) = a^{\pm x} \pm k \implies D_f = \mathbb{R} = (-\infty, \infty)$, $R_f = (\pm k, \infty)$

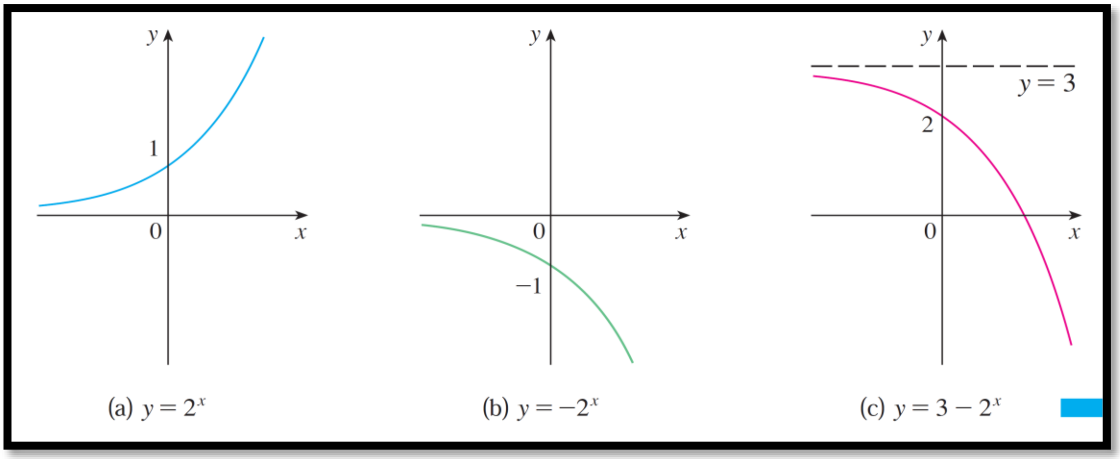
(2) If $f(x) = -a^{\pm x} \pm k \implies D_f = \mathbb{R} = (-\infty, \infty)$, $R_f = (-\infty, \pm k)$

(3) If $f(x) = e^{\pm x} \pm k \implies D_f = \mathbb{R} = (-\infty, \infty)$, $R_f = (\pm k, \infty)$

(4) If $f(x) = -e^{\pm x} \pm k \implies D_f = \mathbb{R} = (-\infty, \infty)$, $R_f = (-\infty, \pm k)$

**Example:** Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.

**Solution:**




$D_f =$ $R_f =$

**Example:** State the domain and range of the following functions

| | | |
|---|---|---|
| (1) $y = 4^x - 3$ | (2) $y = 5 - 3^x$ | (3) $y = 6 - 3e^x$ |
| (4) $y = 2^{-x} + 1$ | (5) $y = -3^x - 7$ | (6) $y = \frac{1}{2}e^{-x} - 1$ |
| (7) $y = 2(1 + e^x)$ | (8) $y = -5e^{-x}$ | (9) $y = e^x + 1$ |

**Example:** Find the domain of the following functions

| | |
|---|---|
| (1) $f(x) = \dfrac{1}{1 + e^x}$ | (2) $f(x) = \dfrac{1}{1 - e^x}$ |

(3) $f(x) = \sqrt{1 + 2^x}$

(4) $f(x) = \dfrac{1}{2 - e^x}$

(5) $f(x) = \sqrt{e^x + 3}$

**Laws of Exponents:**

If $a$ and $b$ are positive numbers and $x$ and $x$ are any real numbers, then

(1) $a^{x+y} = a^x .\ a^y$

(2) $a^{x-y} = \dfrac{a^x}{a^y}$

(3) $(a^x)^y = a^{xy}$

(4) $(ab)^x = a^x .\ b^x$

(5) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

(6) $e^{x+y} = e^x .\ e^y$

(7) $e^{x-y} = \dfrac{e^x}{e^y}$

(8) $(e^x)^y = e^{xy}$

(9) $(ae)^x = a^x .\ e^x$

(10) $\left(\dfrac{a}{e}\right)^x = \dfrac{a^x}{e^x}$

**Remark:**

If the bases in an equation are equal, then the exponents are equal. That is, if

$$x^a = x^b \quad \Leftrightarrow \quad a = b$$

**Example:** Find $x$ if $2^{x+1} = 16$.

**Solution:**

$$2^{x+1} = 16$$

**Example:** Solve the equation if $9^{2x-1} = 81$.

**Solution:**

$$9^{2x-1} = 81$$

**Example:** Solve $25^{x+2} = 125$.

**Solution:**

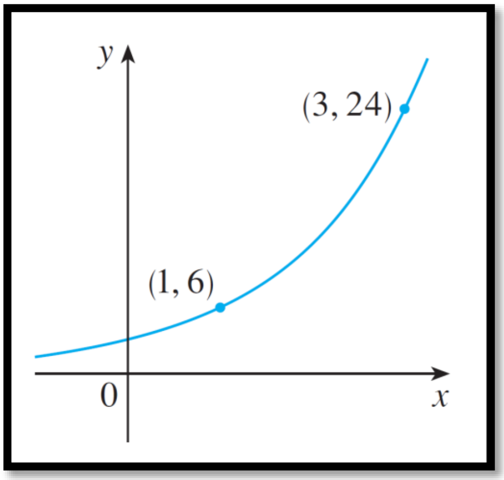
$$25^{x+2} = 125$$

**Example:** Find $x$ if $6^{2(x+1)} = 36$.

**Solution:**

$$6^{2(x+1)} = 36$$

**Example:** Find the exponential function $f(x) = Ca^x$ whose graph is given:



# Sections 1.5. Exercises

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**Homework:** Page 57

**17.** Starting with the graph of $y = e^x$, write the equation of the graph that results from

$(a)$ shifting $2$ units downward

$(b)$ shifting $2$ units to the right

$(c)$ reflecting about the $x$ -axis

$(d)$ reflecting about the $y$ -axis

$(e)$ reflecting about the $x$ -axis and then about the $y$ -axis

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Find the domain of each function.

**19.** $(a)$ $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$

**20.** $(b)$ $g(t) = \sqrt{1 - 2^t}$

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**23.** If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h}\right)$$

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**18.** Starting with the graph of $y = e^x$, find the equation of the of the graph that results from

$(a)$ reflecting about the line $y = 4$

$(b)$ reflecting about the line $x = 2$

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Find the domain of each function.

**19.** $(b)$ $f(x) = \dfrac{1 + x}{e^{\cos x}}$

**20.** $(a)$ $g(t) = \sin(e^{-t})$