

Transformations of Functions

Vertical and Horizontal Shifts:

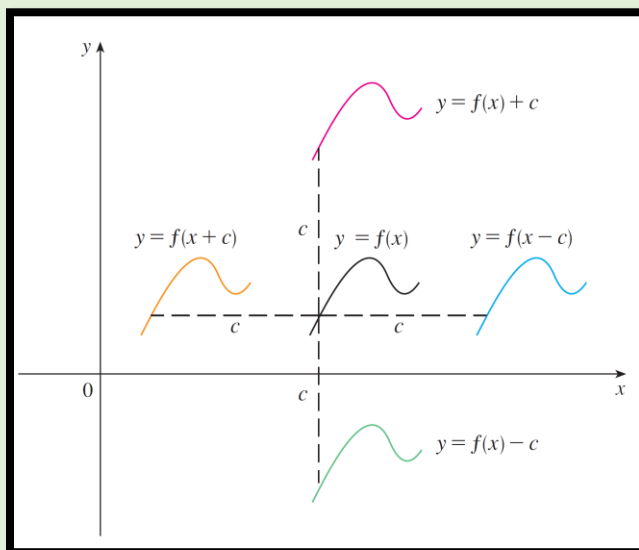
Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left



Vertical and Horizontal Stretching and Reflecting:

Suppose $c > 1$. To obtain the graph of

$y = c f(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c

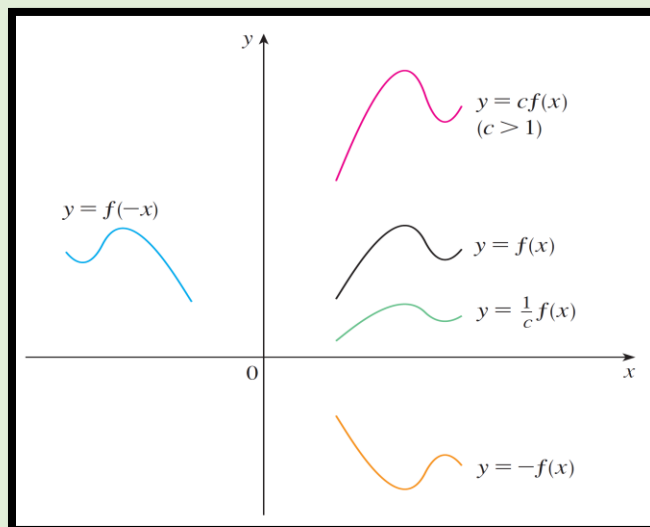
$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c

$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c

$y = -f(x)$, reflect the graph of $y = f(x)$ about the x -axis

$y = f(-x)$, reflect the graph of $y = f(x)$ about the y -axis



Example: If $f(x) = 2x + \sin x + e^x$, then

a) shift $f(x)$ 5 units upward.

Solution:

Example: If $f(x) = 2x + \sin x + e^x$, then

b) shift $f(x)$ 7 units downward.

Solution:

Example: If $f(x) = 2x + \sin x + e^x$, then

c) shift $f(x)$ 4 units to the left.

Solution:

Example: If $f(x) = 2x + \sin x + e^x$, then

d) shift $f(x)$ 6 units to the right.

Solution:

Example: If $y = \cos x$, then

a) stretch the graph of y vertically by a factor of 2 units

Solution:

Example: If $y = \cos x$, then

b) shrink the graph of y vertically by a factor of 2 units

Solution:

Example: If $y = \cos x$, then

c) stretch the graph of y horizontally by a factor of 2 units

Solution:

Example: If $y = \cos x$, then

d) shrink the graph of y horizontally by a factor of 2 units

Solution:

Example: If $y = \cos x$, then

e) reflect the graph of y about the x -axis

Solution:

Example: If $y = \cos x$, then

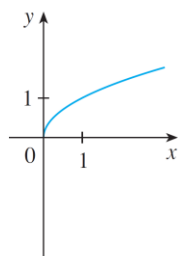
f) reflect the graph of y about the y -axis

Solution:

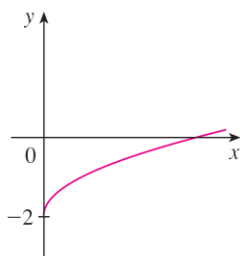
Example 1: Given the graph of $y = \sqrt{x}$, use transformations to graph

$$y = \sqrt{x} - 2, \quad y = \sqrt{x-2}, \quad y = -\sqrt{x}, \quad y = 2\sqrt{x} \quad \text{and} \quad y = \sqrt{-x}.$$

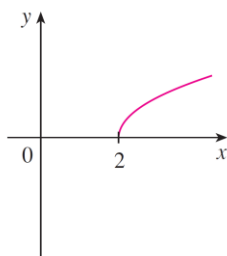
Solution:



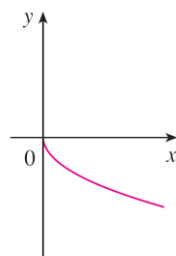
(a) $y = \sqrt{x}$



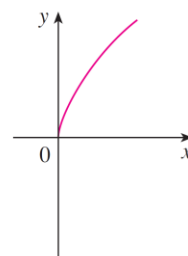
(b) $y = \sqrt{x} - 2$



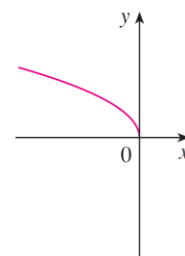
(c) $y = \sqrt{x-2}$



(d) $y = -\sqrt{x}$



(e) $y = 2\sqrt{x}$



(f) $y = \sqrt{-x}$

Combinations of Functions

Definition:

Given two function f & g , we define the following:

- (1) $(f + g)(x) = f(x) + g(x)$
- (2) $(f - g)(x) = f(x) - g(x)$
- (3) $(f \cdot g)(x) = f(x) \cdot g(x)$
- (4) $(f/g)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

Remarks:

- $D_{f+g} = D_{f-g} = D_{fg} = D_f \cap D_g$
- $D_{f/g} = D_f \cap D_g \setminus \{x: g(x) = 0\} = D_f \cap D_g \setminus \{\text{أصفار المقام}\}$

Example: If $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$, then find the domain of $f + g$, $f - g$, fg , f/g and g/f .

Solution:

Example: If $f(x) = x^3 - 4x$, $g(x) = x - 2$, then find the domain of f/g and g/f .

Solution:

Composite Functions**Definition:**

Given two function f & g , the composite function $f \circ g$ (also called the composition of f & g) is defined by

$$(f \circ g)(x) = f(g(x))$$

Remarks:

- $D_{f \circ g} = D_{f(g(x))} \cap D_{g(x)}$
- $D_{g \circ f} = D_{g(f(x))} \cap D_{f(x)}$
- $f \circ g \neq g \circ f$

Example: If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$.

Solution:

Example: If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find each function and its domain.

- | | |
|-----------------|-----------------|
| (1) $f \circ g$ | (2) $g \circ f$ |
| (3) $f \circ f$ | (4) $g \circ g$ |

Solution:

Example: Find $f \circ g \circ h$ if $f(x) = \frac{x}{x+1}$ and $g(x) = x^{10}$, and $h(x) = x + 3$.

Solution:

Example: If $f(x) = x^2$ and $g(x) = \cos x$, find the composite functions $f \circ g$ and $g \circ f$.

Solution:

Example: If $f(x) = x^2$ and $g(x) = \sqrt{1-x}$, find the domain of the composite functions $f \circ g$, $g \circ f$ and $f \circ f$.

Solution:

Sections 1.3. Exercises

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Homework: Page 42-43

1. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.

- | | |
|---|---|
| <p>(a) Shift 3 units upward.</p> <p>(c) Shift 3 units to the right.</p> <p>(e) Reflect about the x-axis.</p> <p>(g) Stretch vertically by a factor of 3.</p> | <p>(b) Shift 3 units downward.</p> <p>(d) Shift 3 units to the left.</p> <p>(f) Reflect about the y-axis.</p> <p>(h) Shrink vertically by a factor of 3.</p> |
|---|---|

Find (a) $f + g$, (b) $f - g$, (c) fg and (d) f/g and state their domains.

29. $f(x) = x^3 + 2x^2$, $g(x) = 3x^2 - 1$

Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$ and (d) $g \circ g$ and state their domains.

31. $f(x) = x^2 - 1$, $g(x) = 2x + 1$

32. $f(x) = x - 2$, $g(x) = x^2 + 3x + 4$

34. $f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{1-x}$

Find $f \circ g \circ h$.

38. $f(x) = |x - 4|$, $g(x) = 2^x$, $h(x) = \sqrt{x}$

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Find (a) $f + g$, (b) $f - g$, (c) fg and (d) f/g and state their domains.

30. $f(x) = \sqrt{3-x}$, $g(x) = \sqrt{x^2 - 1}$

Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$ and (d) $g \circ g$ and state their domains.

33. $f(x) = 1 - 3x$, $g(x) = \cos x$

32. $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+1}{x+2}$

Find $f \circ g \circ h$.

37. $f(x) = 3x - 2$, $g(x) = \sin x$, $h(x) = x^2$