In the preceding section, we considered the derivative of a function $\boldsymbol{f}$ at a fixed number $\boldsymbol{a}$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Here we change our point of view and let the number $\boldsymbol{a}$ vary. If we replace $\boldsymbol{a}$ in the above Equation by a variable $\boldsymbol{x}$, we obtain

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example: (1) Find the derivative of the given function

$$f(x) = \sqrt{x}$$

Solution:

(2) Find the derivative of the given function

$$f(x) = \frac{1-x}{2+x}$$

Solution:

Other Notations:

Notations:

If $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, then

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$$

Definition:

A function $\boldsymbol{f}$ is **differentiable at** $\boldsymbol{a}$ if $\boldsymbol{f}'(\boldsymbol{a})$ exists. It is **differentiable on an open interval** $(\boldsymbol{a}, \boldsymbol{b})$ [or $(\boldsymbol{a}, \infty)$ or $(-\infty, \boldsymbol{a})$ or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

Theorem:

If $\boldsymbol{f}$ is **differentiable at** $\boldsymbol{a}$, then $\boldsymbol{f}$ is continuous at $\boldsymbol{a}$.

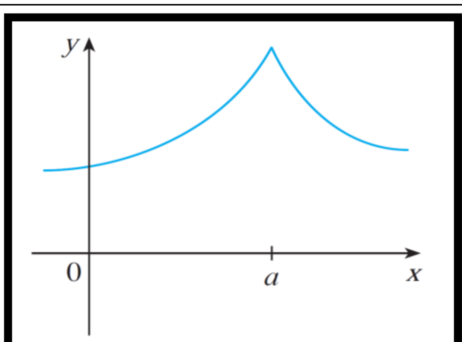
The converse is not true.

Example: Where is the function $f(x) = |x|$ differentiable?

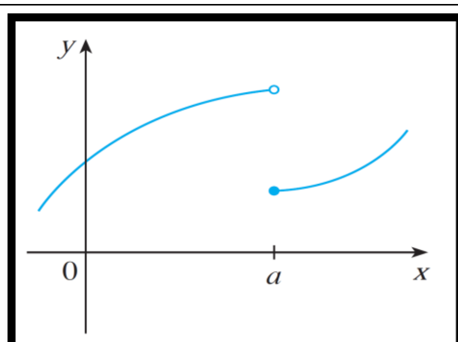
Solution:

How Can a Function Fail to Be Differentiable?

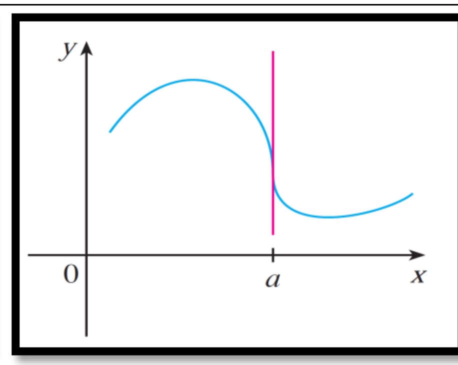
There are **three ways** for f not to be differentiable at a.



(a) A corner



(b) A discontinuity



(c) A vertical tangent

Higher Derivatives:

Notations:

If $\boldsymbol{y}' = \boldsymbol{f}'(\boldsymbol{x})$, then

$$f''(x) = y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{df}{dx}\right) = \frac{d^2}{dx^2}f(x) = D^2 f(x) = D_x^2 f(x)$$

$$f'''(x) = y''' = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2f}{dx^2}\right) = \frac{d^3}{dx^3}f(x) = D^3 f(x) = D_x^3 f(x)$$

$$f^{(n)}(x) = y^{(n)} = \frac{d^n y}{dx^n} = \frac{d^n}{dx^n}f(x) = D^n f(x) = D_x^n f(x)$$

At substituting $x = a$, we have

$$y'(a) = f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \left.\frac{dy}{dx}\right]_{x=a}$$

Example: If $f(x) = x^3 - x$, find $f''(x)$.

Solution:

Example: If $f(x) = x^3 - x$, find $f'''(x)$ and $f^{(4)}(x)$.

Solution: