Math 311
The Second Exam

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Name: Computer number:

(I) Answer the following questions:

(1) Let (x_n) be a sequence of nonnegative real numbers and suppose that $\lim\limits_{n\to\infty} x_n = x > 0$. Prove that $\lim\limits_{n\to\infty} \sqrt{x_n} = \sqrt{x}$. (2.5 marks)

(2) Prove that the sequence $\frac{(-1)^n}{4}$ is divergent. (2 marks)

(3) Prove that if (x_n) converges to x and (y_n) converges to y, then $(x_n\, y_n)$ converges to $x\, y$. That is, $\lim\limits_{n\to\infty}(x_n y_n) = \lim\limits_{n\to\infty} x_n \lim\limits_{n\to\infty} y_n = xy$. (3 marks)

(4) Prove that if (x_n) is an unbounded nonincreasing sequence, then $\lim\limits_{n\to\infty} x_n = -\infty$.

(2.5 marks)

(5) Let $A \subseteq R$ and let $f : A \to R$, $x_o \in R$. Prove that (ii) $\Rightarrow$ (i) where,

(i) $\lim\limits_{x\to x_o} f(x) = L$

(ii) For every sequence (x_n) in A that converges to x_o such that $x_n \neq x_o \ \forall n \in N$, the sequence $f(x_n)$ converges to L. (i.e., $(x_n) \to x_o \Rightarrow (f(x_n)) \to L$). (3 marks)

(6) Let (t_n) be a bounded sequence and let (s_n) be a sequence such that $\lim\limits_{n\to\infty} s_n = 0$. Prove that $\lim\limits_{n\to\infty}(s_n t_n) = 0$. (2 marks)

(II) State whether each of the following is true or false and justify your answer:

(2.5 marks)

(1) Monotone sequences are convergent.

(2) Convergent sequences are Cauchy.

(3) If $\lim\limits_{n\to\infty} x_n = +\infty$ and $\lim\limits_{n\to\infty} y_n = -\infty$, then $\lim\limits_{n\to\infty}(x_n + y_n) = 0$.

(4) If (x_n) has a convergent subsequence, then (x_n) must converge.

(5) Every unbounded monotone sequence is not Cauchy.

<u>(III) Complete the following sentences:</u> (2.5 marks)

(1) If $\lim_{x \to x_o} f(x) = L_1$ and $\lim_{x \to x_o} f = L_2$, then ---------------------------

(2) Convergent sequences are ----------------------------------

(3) A monotone sequence must have a ---------------------------------

(4) If $\lim\inf x_n = \lim\sup x_n$, then-------------------------------

(5) Every bounded sequence has a -----------------------------------