King Abdulaziz University
Mathematics Department

Real Analysis (Math 311)
First Exam

Student's Name: Computer Number:

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(1) Prove that every nonempty subset of R that has a lower bound also has an infimum. (3 marks)

(2) Let $a, b \in R$. Prove that $\left||a| - |b|\right| \leq |a - b|$. (2 marks)

(3) Prove that a subset of **R** is closed if and only if its complement is open. (3 marks)

(4) Let S and T be nonempty bounded subsets of R with the following property: $s \le t$ for all $s \in S$ and $t \in T$. Prove that $\sup S \le \inf T$. (2 marks)

(5) Prove that if F is a closed subset of a compact set K, the F is compact. (1 mark).

(6) For each of the following sets find *min*, *max*, *inf* and *sup* if they exist. Then decide whether the set is countable or not and whether it is open or closed or neither: (3 marks)

	min	*max*	*inf*	*sup*	*countable*	*open*	*closed*	*neither*
1. $\{n \in Z : n < 0\}$								
2. $\bigcap_{n=1}^{\infty}\left(1-\frac{1}{n}, 1+\frac{1}{n}\right)$								

(7) State whether each of the f0llowing is true or false and justify your answer: (3 marks)

(a) $\{r \in Q : 0 \le r \le 2\} = [0,2]$

(b)If the set S has a supremum then it must have a maximum

(c) Every infinite set is uncountable

(d) If the set A is not closed then it must be open

(e) A set is open if it contains all of its interior points

(f) A set A is compact if every open cover has a subcover

(8)Explain why: (3 marks)

(a) Every rational number is an algebraic number

(b) Z is not an open set

(c) The set $\{\frac{1}{n} : n \in N\}$ is not compact

(d) The set $\{n^{(-1)^n} : n \in N\}$ is unbounded

(e) Every finite set is closed

(f) Q is neither open nor closed