Homework 13

13.1 Use the definition of the derivative to calculate the derivative of the following functions at the indicated point.

(a) $f(x) = x^3$ at $x = 2$

(b) $g(x) = x + 2$ at $x = 2$

(c) $f(x) = x^2 \cos x$ at $x = 0$

13.2 Prove that $|\cos x - \cos y| \le |x - y|$ fo all $x, y \in R$.

13.3 Suppose that f is differentiable on R and that $f(0) = 0$ and $f(2) = 1$. Show that $f'(x) = \frac{1}{2}$ for some $x \in (0\ ,\ 2)$.

13.4 Lat f and g be two differentiable functions on an open interval I. Suppose $a, b \in I$, $a < b$ and that $f(a) = f(b) = 0$. Show that $f'(x) + f(x)g'(x) = 0$ for some $x \in (a, b)$.

(Hint: Consider $h(x) = f(x)e^{g(x)}$).

13.5 (a) Show that $x < \tan x$ for all $x \in (0, \frac{\pi}{2})$.

(b) Show that $\frac{x}{\sin x}$ is strictly increasing function on $(0, \frac{\pi}{2})$.

(c) Show that $x \le \frac{\pi}{2} \sin x$ for $x \in [0, \frac{\pi}{2}]$

13.6 Use theorem 5.17 to obtain the derivative of the inverse $g = \tan^{-1}$ of f where $f(x) = \tan x$ for $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$.