## Homework 4

4.1 Write out the Induction argument in the proof of theorem 2.5

4.2 Give an example of a collection of closed sets whose union is neither open nor closed.

4.3 Determine whether the set A = $\left\{1-\frac{1}{n} : n \in N\right\}$ is closed or open or neither

4.4 A point $x \in R$ is said to be a boundary point of A in case every neighborhood $V$ of $x$ contains points in A and points in its complement $A^c$. Show that the sets $A$ and $A^c$ have exactly the same boundary points.

4.5 Show that a set G is open if and only if it does not contain any of its boundary points.

4.6 Give an example of a set $A$ such that $\overset{o}{A} = \varphi$ and $\overline{A} = R$.

4.7 Show that if $G$ is an open nonempty set that is bounded above, then *sup G* does not belong to $G$.

4.8 Find an example of a set $A$ for each of the following:
(i) $A \cap A' = \varphi$ (ii) $A \subset A'$ (iii ) $A' \subset A$ (iv) $A = A'$

## Homework 5

5.1 Exhibit an open cover of the interval (1 , 2] that has no finite subcover.

5.2 Find an infinite collection $\{K_n : n \in N\}$ of compact sets in $R$ such that the union $\bigcup_{n=1}^{\infty} K_n$ is not compact.

5.3 Prove that the intersection of an arbitrary collection of compact sets is compact