Homework 1

1.1 Show that, $\sqrt{5}$ is not a rational number.

1.2 Show that $\left(5-\sqrt{3}\right)^{1/3}$ does not represent a rational number.

Homework 2

2.2 (a) Prove that $|a+b+c| \le |a|+|b|+|c|$ for all $a,b,c \in R$. Hint: Apply the triangular inequality twice.

(b) Use mathematical induction to prove

$$|a_1 + a_2 + ...a_n| \le |a_1| + |a_2| + ... + |a_n|$$

for n numbers $a_1, a_2, ...a_n$.

2.2 Let $a,b \in R$. Show that if $a \le b_1$ for every $b_1 > b$, then $a \le b$.

2.3 If $a,b \in R$, show that $|a+b| = |a| + |b|$ if and only if $ab \ge 0$.

2.4 Let ε > 0 and δ > 0, and $a \in R$. Show that $V_\varepsilon(a) \cap V_\delta(a)$ and $V_\varepsilon(a) \cup V_\delta(a)$ are γ-neighborhoods of a for appropriate values of γ.

2.5 Show that if $a,b \in R$, then

$$\max\{a,b\} = \frac{1}{2}(a+b+|a-b|) \text{ and } \min\{a,b\} = \frac{1}{2}(a+b-|a-b|)$$

Homework 3

3.1 For each set below that is bounded above, list three upper bounds for the set. Otherwise write 'NOT BOUNDED ABOVE' or 'NBA'

(e) $\left\{\frac{1}{n} : n \in N\right\}$

(f) $\{0\}$

(g) $[0,1] \cup [2,3]$

(h) $\bigcup_{n=1}^{\infty} [2n, 2n+1]$

(j) $\left\{1-\frac{1}{3^n} : n \in N\right\}$

(n) $\{r \in Q : r^2 < 2\}$

(t) $\{x \in R : x^3 < 8\}$

3.2 Let *S* be a nonempty subset of *R* that is bounded above. Prove that if *sup S* belongs to *S*, then *sup S* = *max S* . Hint: Your proof should be very short.

3.3 Let *S* and *T* be nonempty bounded subsets of R.

(a) Prove that if $S \subseteq T$, then $\inf T \leq \inf S \leq \sup S \leq \sup T$.

(b) Prove that $\sup(S \cup T) = \max\{\sup S, \sup T\}$.

Note: In part (b) do not assume $S \subseteq T$

3.4 Let *S* and *T* be nonempty bounded subsets of R with the following property: $s \leq t$ for all $s \in S$ and $t \in T$.

(a) Observe that *S* is bounded above and that *T* is bounded below.

(b) Prove that $\sup S \leq \inf T$.

(c) Give an example of such sets *S* and *T* where $S \cap T$ is nonempty.

(d) Give an example of sets *S* and *T* where $\sup S = \inf T$ and $S \cap T = \varphi$.

3.5 Let *A* and *B* be nonempty bounded subsets of *R* and let *S* be the set of all sums $a + b$ where $a \in A$ and $b \in B$. Prove that *inf S* = *inf A* + *inf B.*

3.6 Let $S_4 = \{1 - \frac{(-1)^n}{n} : n \in N\}$. Find *inf* S_4 and *sup* S_4.

3.7 If a set $S \subseteq R$ contains one of its upper bounds, show that this upper bound is the supremum of *S*.