King Abdulaziz University

Mechanical Engineering Department

MEP 365

Ch. 10 Flow measurements

April 2017

Ch.10 Flow measurements

Volume flow rate through velocity determination

Pressure Differential meters (Obstruction meters)

- Orifice
- Venturi
- Nozzle
- Laminar flow elements

Insertion meters

- 1- Electromagnetic flow meter
- 2- Vortex shedding meters
- 3- Rotameters
- 4- Turbine meter
- 5-Transient time and Doppler flow meters
- 6-Positive displacement flow meter

Mass flow meters

Introduction

Mass flow rate $\dot{m} = \rho * U * A$ kg/s

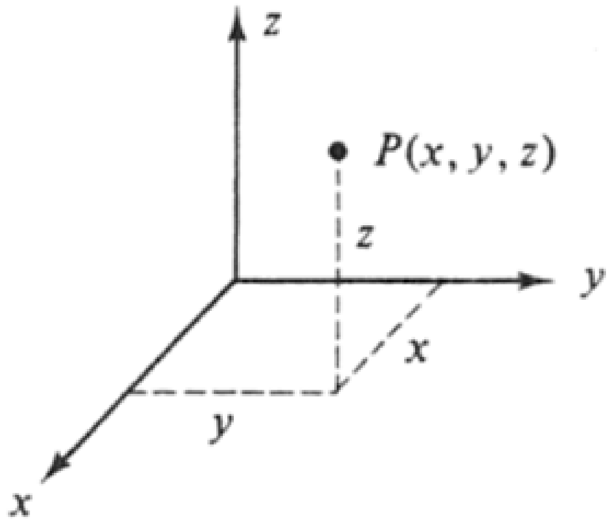
Volume flow rate $Q = U * A$ m^3/s

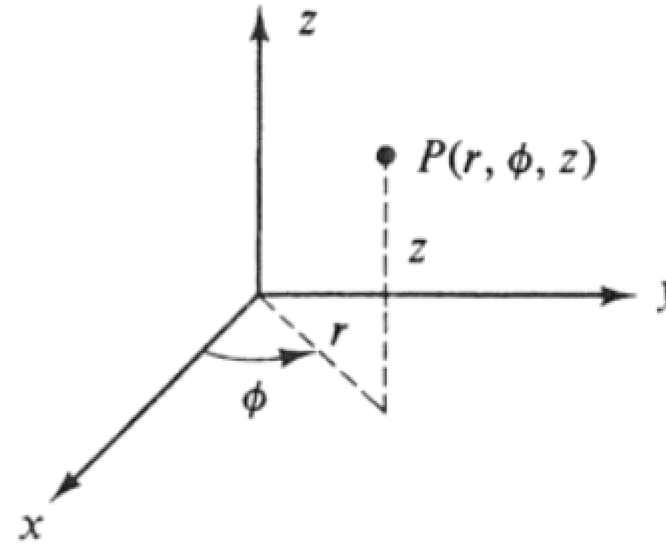
Application Examples

Consumption of household water
Fuel (Gas) stations
Industrial & Engineering applications (power plants, factories, chemical plants etc.

Introduction

Velocity vector





in rectangular coordinate system

in circular cylinder

$$\vec{U} = u\vec{e}_x + v\vec{e}_y + w\vec{e}_z \qquad \vec{U} = u\vec{e}_x + v\vec{e}_r + w\vec{e}_\theta$$

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Introduction

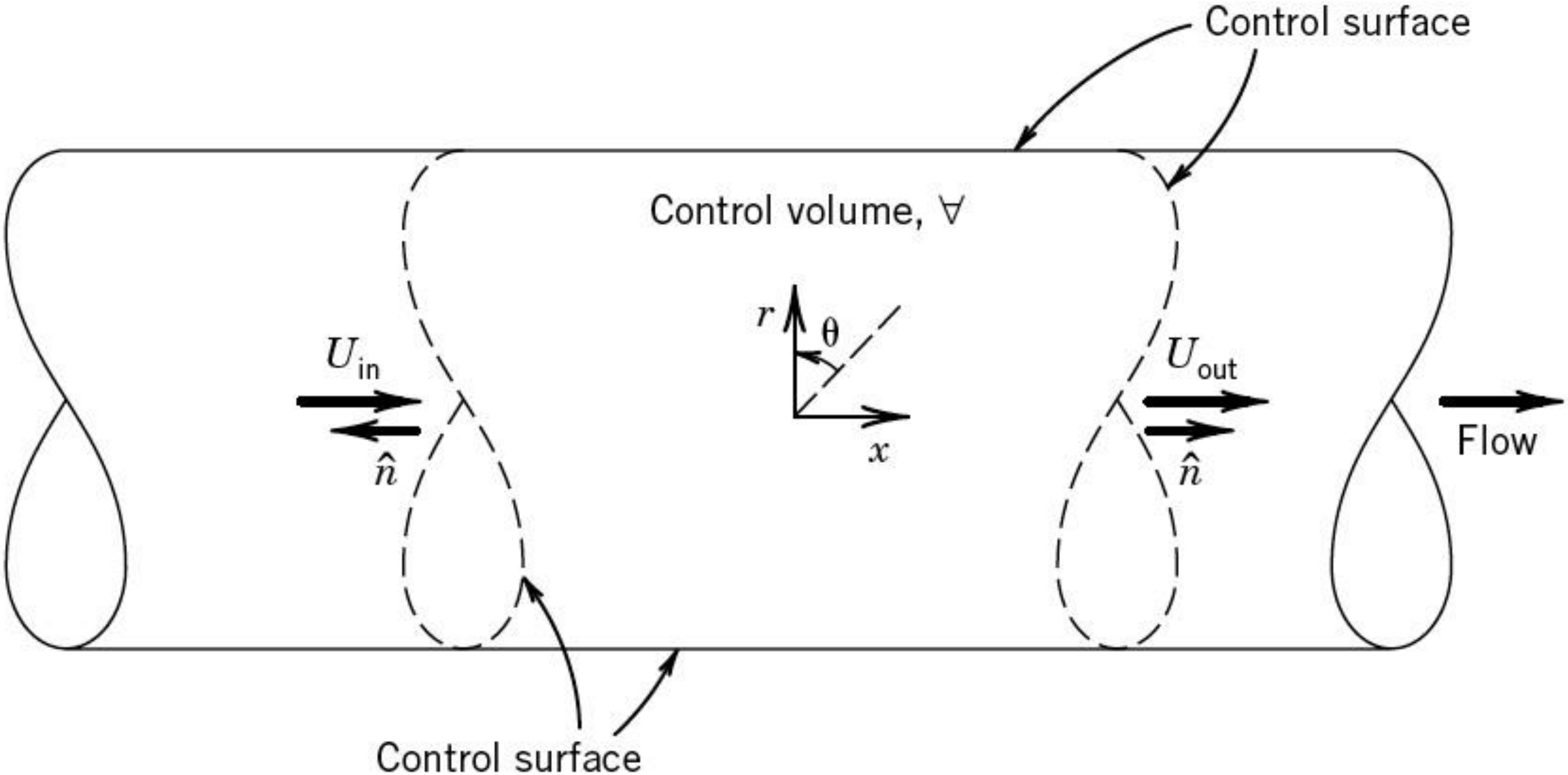
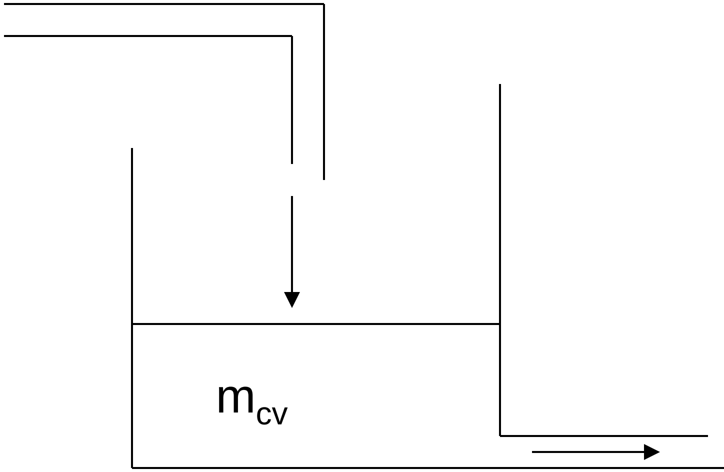


Figure 10.1 Control volume concept as applied to flow through a pipe.

Mass conservation

$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\dot{m} = \rho \overline{U} A$$ kg/s



$\overline{U}$ is the average velocity

Volume flow rate=Q= $\overline{U}A$ m^3/s

When the velocity is not uniform

$$\dot{m} = \iint_A \rho U dA$$

where A is the control surface area

If the density is constant, the volume flow rate

$$Q = \iint_A U dA \qquad \bar{U} = \frac{\iint_A U dA}{A} \qquad Q = \bar{U} A$$

$$\mathrm{Re}_{d_1} = \frac{\rho \bar{U} d_1}{\mu} = \frac{\bar{U} d_1}{\nu} = \frac{4Q}{\pi d_1 \nu}$$

For non circular tubes

Hydraulic diameter D_H

$$D_H = \frac{4(\mathit{wetted}\ \mathrm{area})}{(\mathit{wetted}\ \mathrm{Perimeter})}$$

Volume flow rate through velocity measurements

Velocity at any location can be measured using for example a Pitot tube

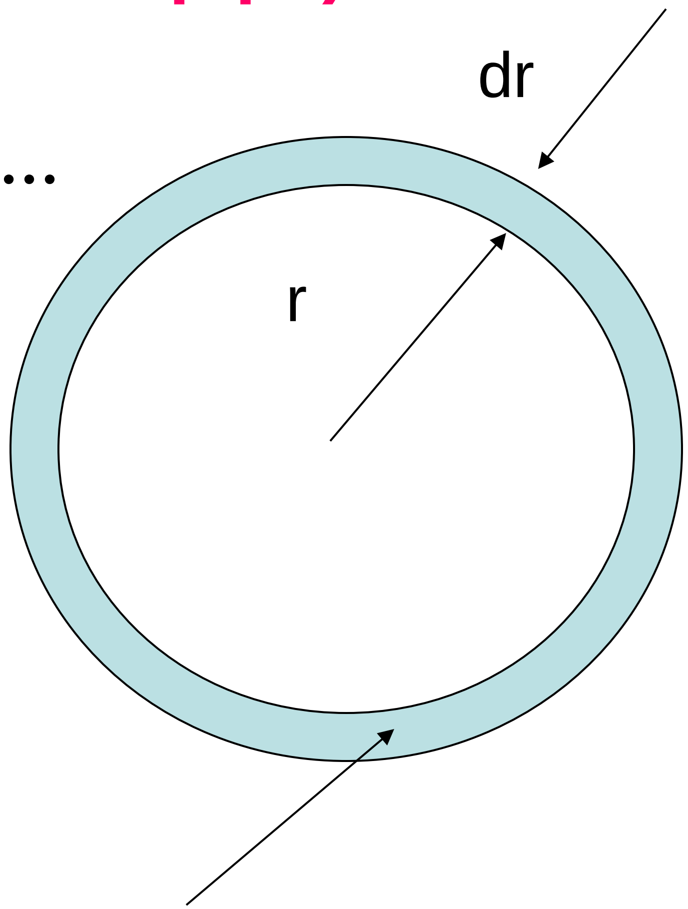
For circular pipe
$$Q = \int U(2\pi r dr)$$

$$Q = 2\pi \sum U_i r \Delta r$$

The volume flow rate is the sum of small elements flow rates

Volume flow rate through velocity measurements (circular pipe)

$$Q = 2\pi \int Urdr = \sum A_1 U_1 + A_2 U_2 + \ldots.$$



For equal areas and 4 positions

$$Q = 2\pi \int Urdr = \frac{A}{4}\left[U_1 + U_2 + U_3 + U_4\right]$$

Area= $2\pi r dr$

Volume flow rate through velocity measurements

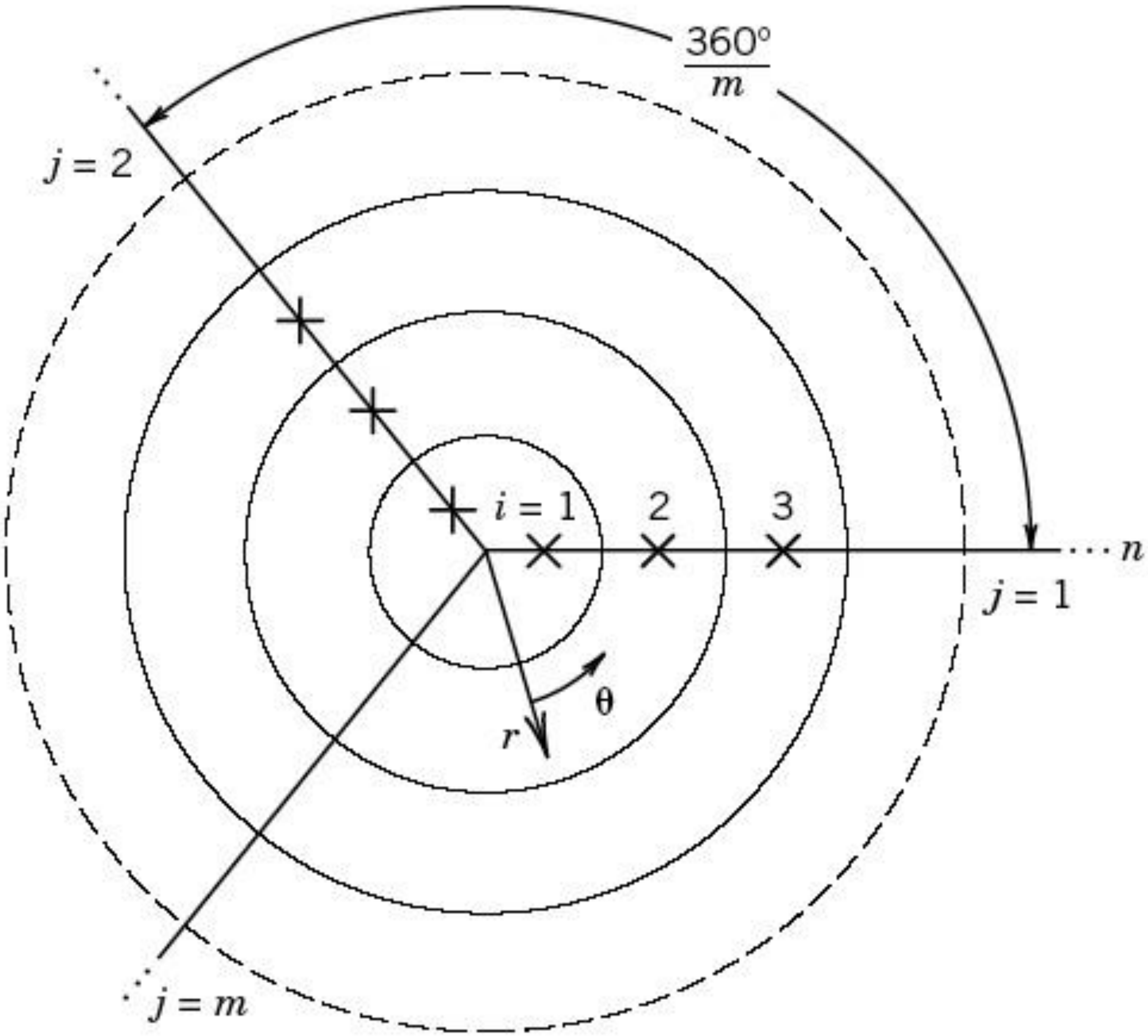


Figure 10.2 Location of n measurements along m radial lines in a pipe.

Volume flow rate through velocity measurements
(square or rectangular cross section)

Flow of air in a square or rectangular duct

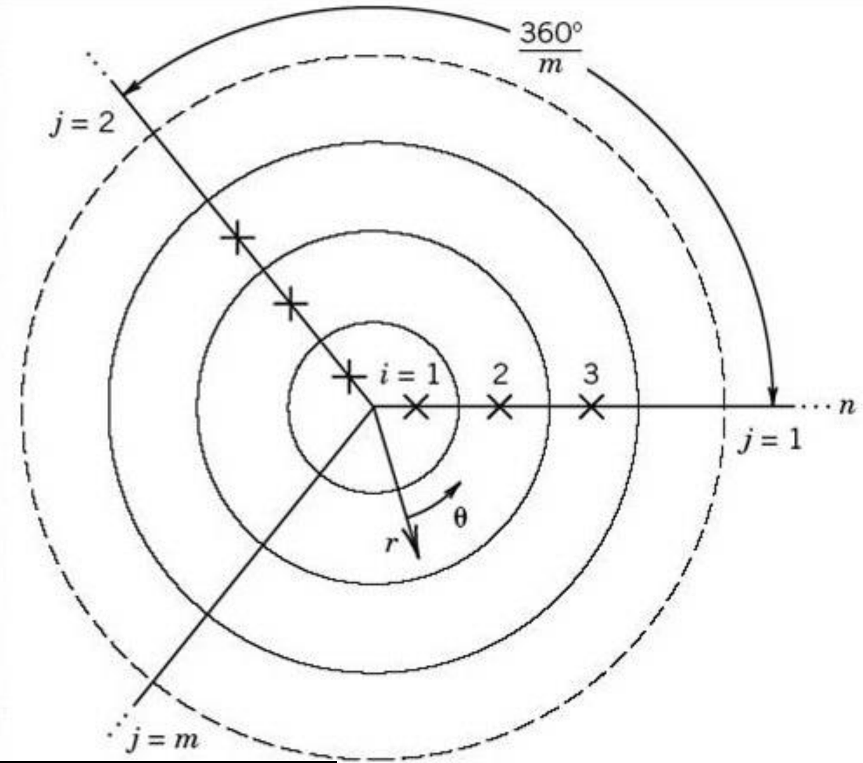
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Use Pitot tube or other devices to measure velocity at these locations and then find the volume flow rate by summing all individual elemental flow rates.

Example 10.1

air flows in d=25.4 cm circular pipe. Equally spaced areas

velocity measurement as shown below. j=1,2,3, and i=1,2,3,4. Determine the volume flow rate of air



Measured velocities

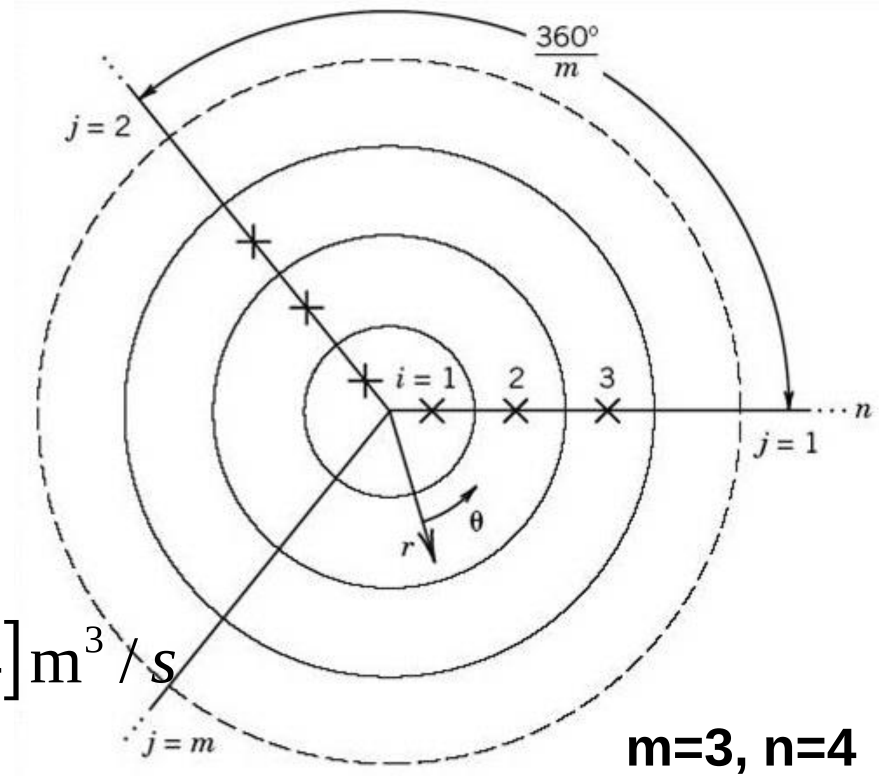
Radial Position	r/r_1	Line 1	Line 2	Line 3
1	0.3536	8.71	8.62	8.78
2	0.6124	6.26	6.31	6.20
3	0.7906	3.69	3.74	3.79
4	0.9354	1.24	1.20	1.28

m=3, n=4

Example 10.1

Along each radial line

$$Q_j = \frac{A}{4}\sum U_{ij} =$$



$$Q_1 = \frac{0.051}{4}\left[8.71 + 6.26 + 3.69 + 1.24\right] \mathrm{m}^3/s$$

m=3, n=4

$$Q_1 = 0.252\,\mathrm{m}^3/s \qquad Q_2 = 0.252\,\mathrm{m}^3/s \qquad Q_3 = 0.254\,\mathrm{m}^3/s$$

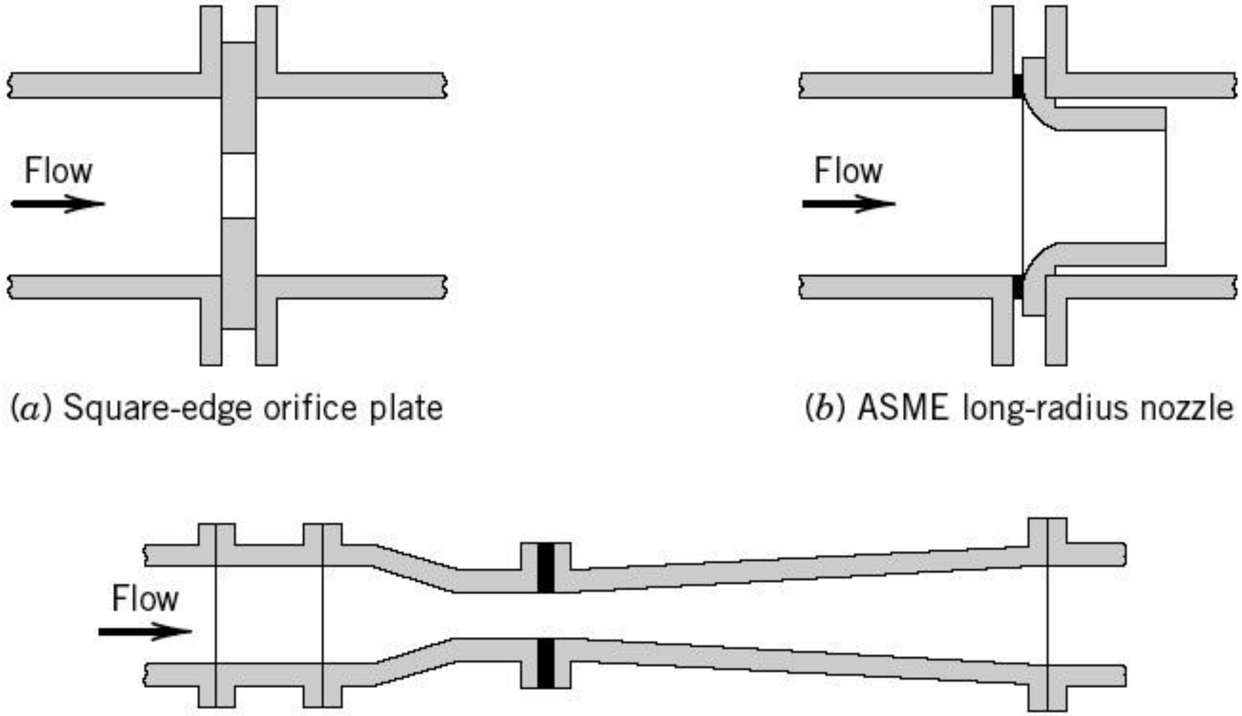
$$\langle Q \rangle = \frac{1}{3}\sum Q_j = 0.252\, m^3/s$$

Pressure differential meters

Obstruction meters

Orifice meter
Flow nozzle
Venturi meter

Obstruction meters



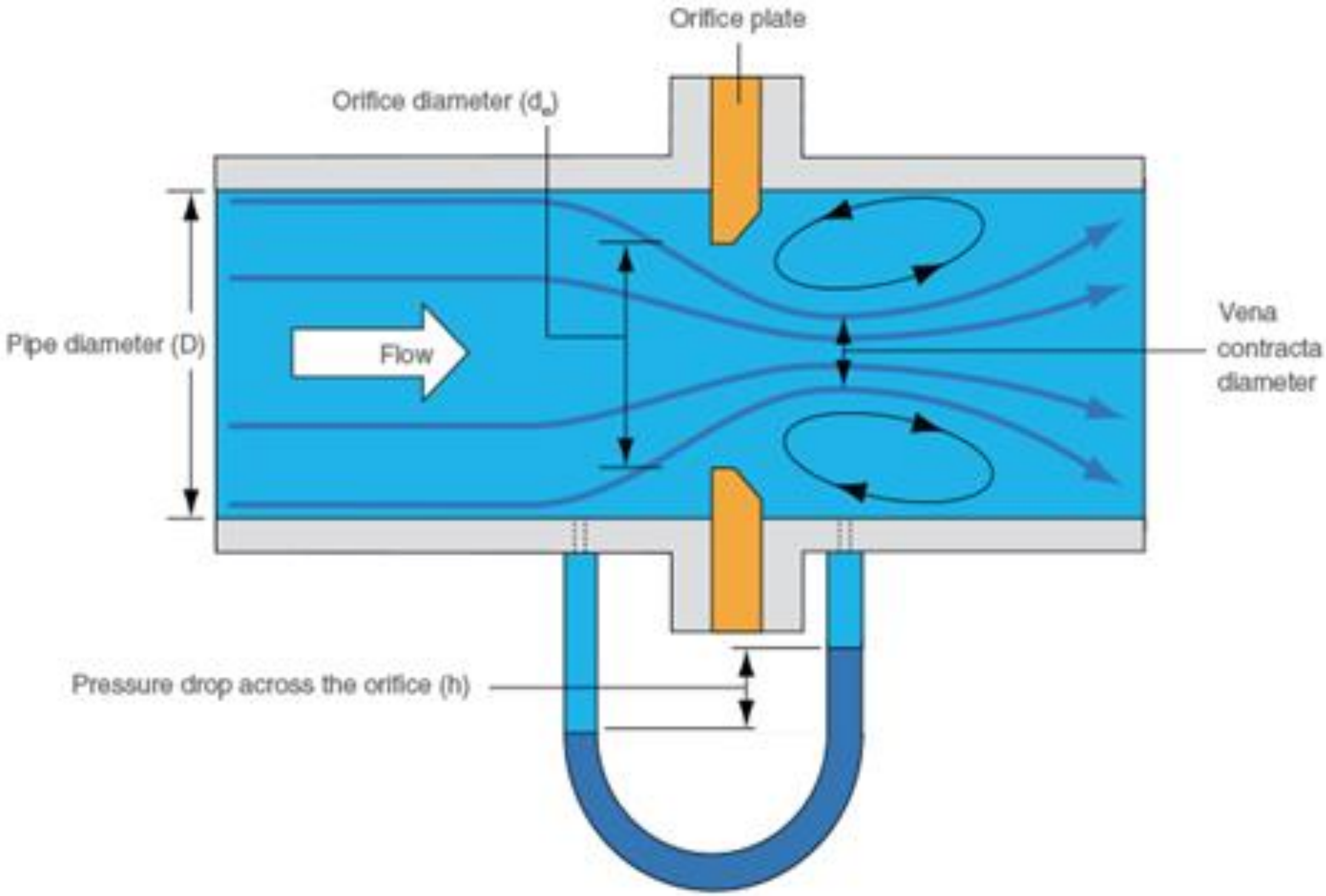
(*a*) Square-edge orifice plate

(*b*) ASME long-radius nozzle

(*c*) ASME Herschel venturi

Figure 10.3 Flow area profiles of common obstruction meters. (a) Square-edged orifice plate meter. (b) ASME long radius nozzle. (c) ASME Herschel venturi meter.

Obstruction meters

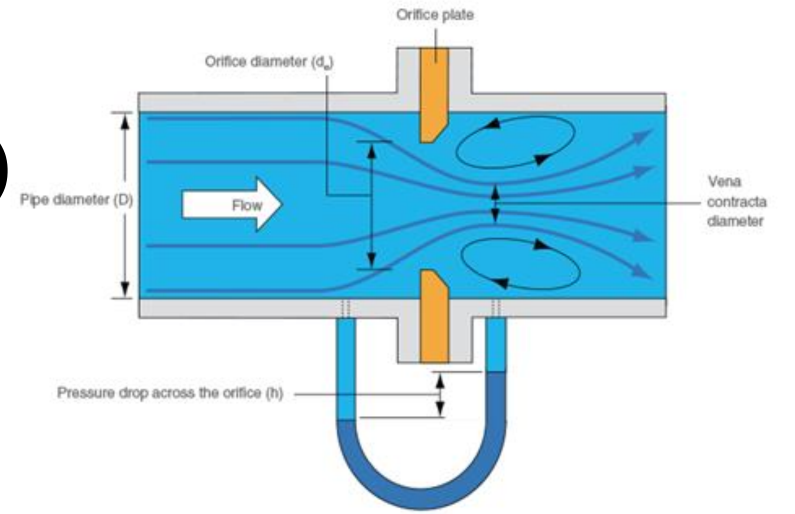


Flow through a nozzle

Obstruction meters

Energy equation

(or modified Bernoulli's equation)



$$\frac{p_1}{\gamma}+\frac{\overline{U}_1^{\,2}}{2g}=\frac{p_2}{\gamma}+\frac{\overline{U}_2^{\,2}}{2g}+h_{L1-2}$$

mass conservation

$$\overline{U}_1 A_1=\overline{U}_2 A_2 \qquad\qquad \overline{U}_1=\overline{U}_2\frac{A_2}{A_1}$$

$$Q_I=\overline{U}_2 A_2=\frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{\frac{2(p_1-p_2)}{\rho}-2gh_{L_{1-2}}}$$

Obstruction meters

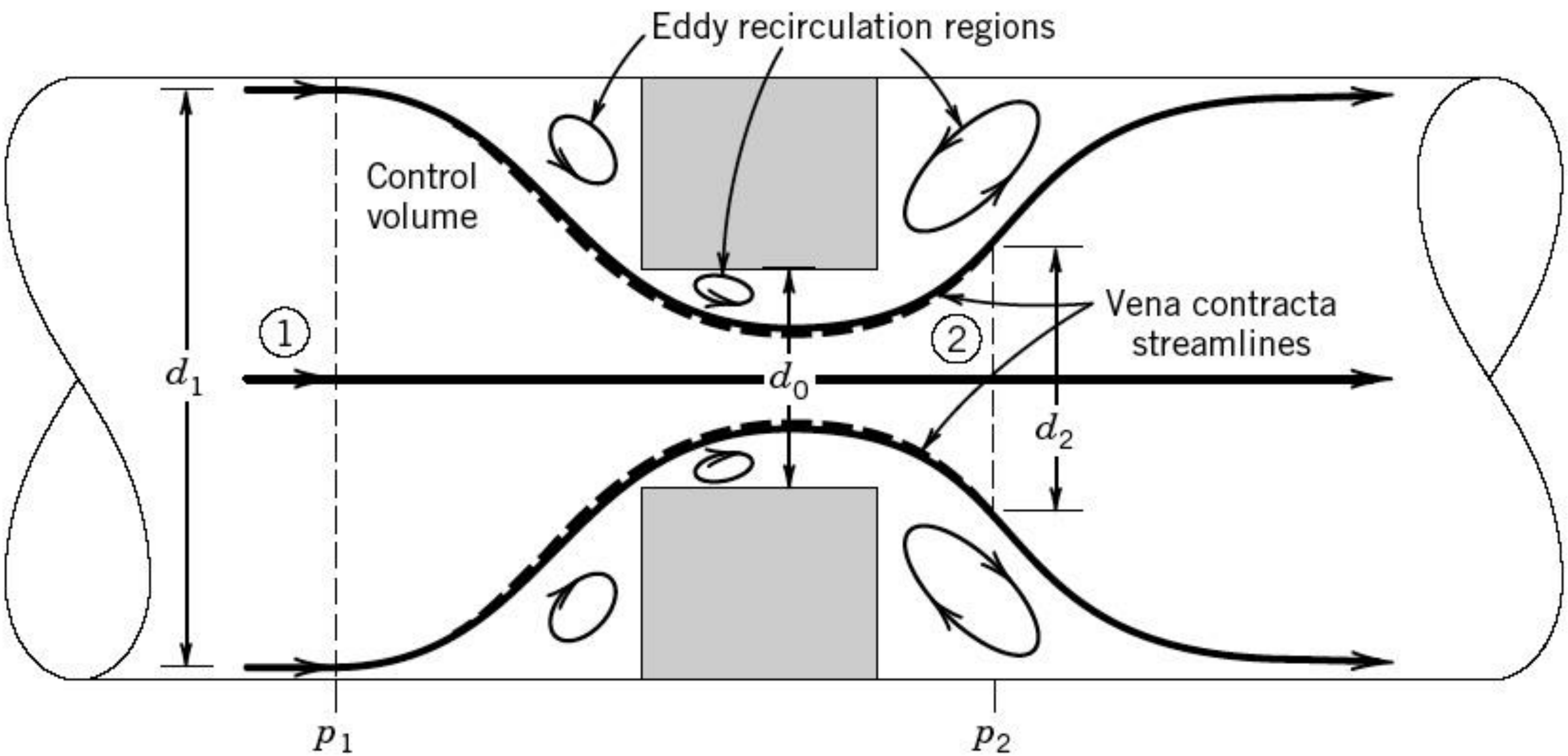


Figure 10.4 Control volume concept as applied between two streamlines for flow through an obstruction meter.

$$\beta = \frac{d_0}{d_1}$$

Obstruction meters

$$Q_I = \bar{U}_2 A_2 = \frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{\frac{2(p_1-p_2)}{\rho} - 2gh_{L_{1-2}}}$$

Introduce **contracting coefficient** $C_c = A_2/A_0$

$$Q_I = \frac{C_c A_0}{\sqrt{1-(C_c A_0/A_1)^2}}\sqrt{\frac{2(p_1-p_2)}{\rho} - 2gh_{L_{1-2}}}$$

$$Q_I = \frac{C_f C_c A_0}{\sqrt{1-(C_c A_0/A_1)^2}}\sqrt{\frac{2(p_1-p_2)}{\rho}}$$

C_f = **friction coefficient** to take care of friction head losses

Obstruction meters

Discharge coefficient C

$$C = \frac{Q_{act}}{Q_{ideal}}$$

$$Q_I = \frac{CA_0}{\sqrt{1-(A_0/A_1)^2}}\sqrt{\frac{2(p_1-p_2)}{\rho}} = CEA_0\sqrt{\frac{2\Delta p}{\rho}}$$

where E is called the **velocity approach factor**

$$E = \frac{1}{\sqrt{1-(A_0/A_1)^2}} = \frac{1}{\sqrt{1-\beta^4}}$$

$$\beta = d_o / d_1$$

$$Q_I = CEA_0\sqrt{\frac{2\Delta p}{\rho}} = K_0 A_0\sqrt{\frac{2\Delta p}{\rho}}$$

K_o=CE

Flow coefficient

Obstruction meters

K_0 is called flow coefficient

$$C = f(\mathrm{Re}_{d1}, \beta)$$

$$K_0 = g(\mathrm{Re}_{d1}, \beta)$$

Curves or equations for K_0 as a function of Re_{d1} and β

Standard Square-edged orifice meter

Notice:

- **Pressure tabs locations**
- **Permanent Pressure loss**
- **Flange type and others**

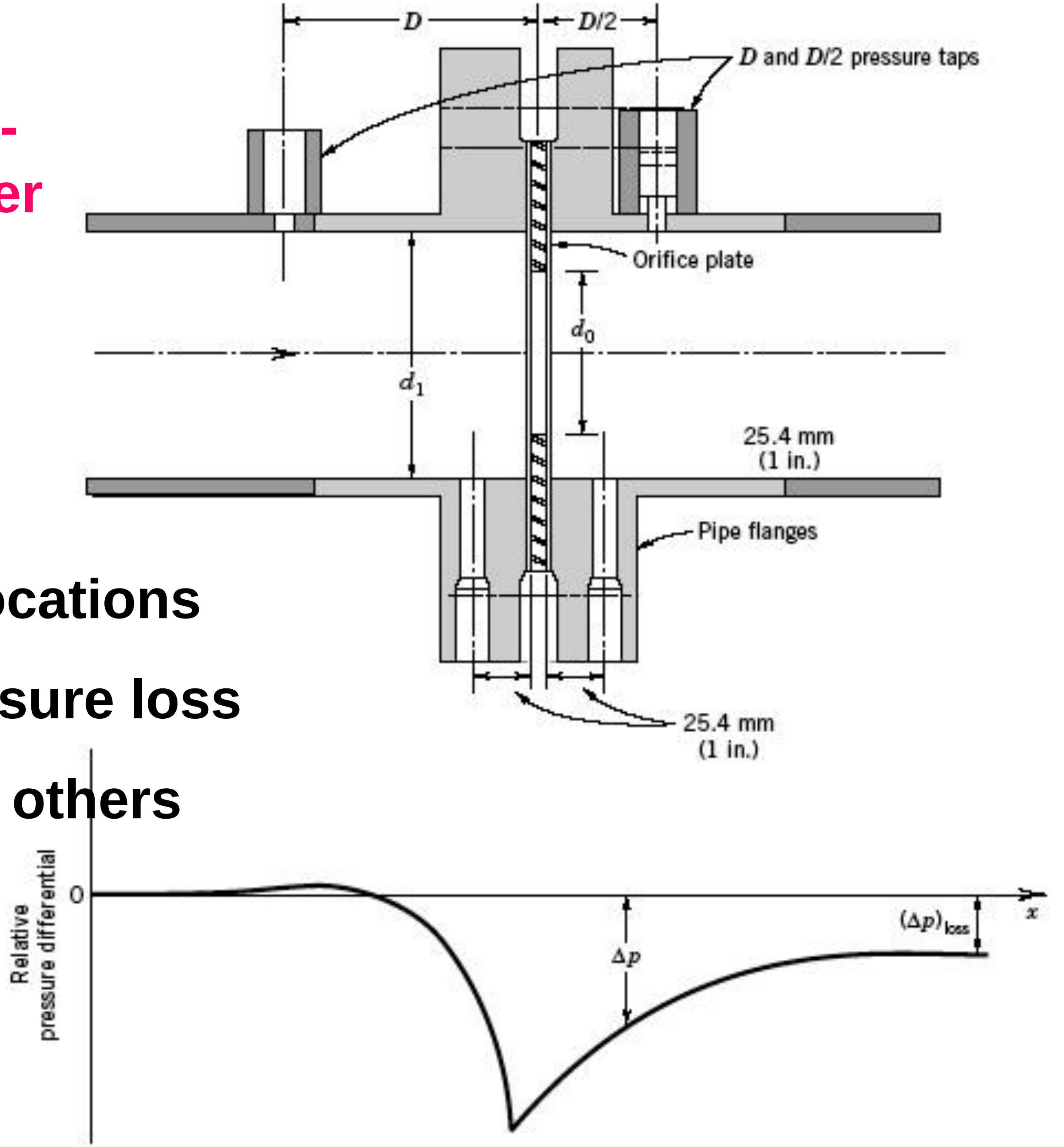
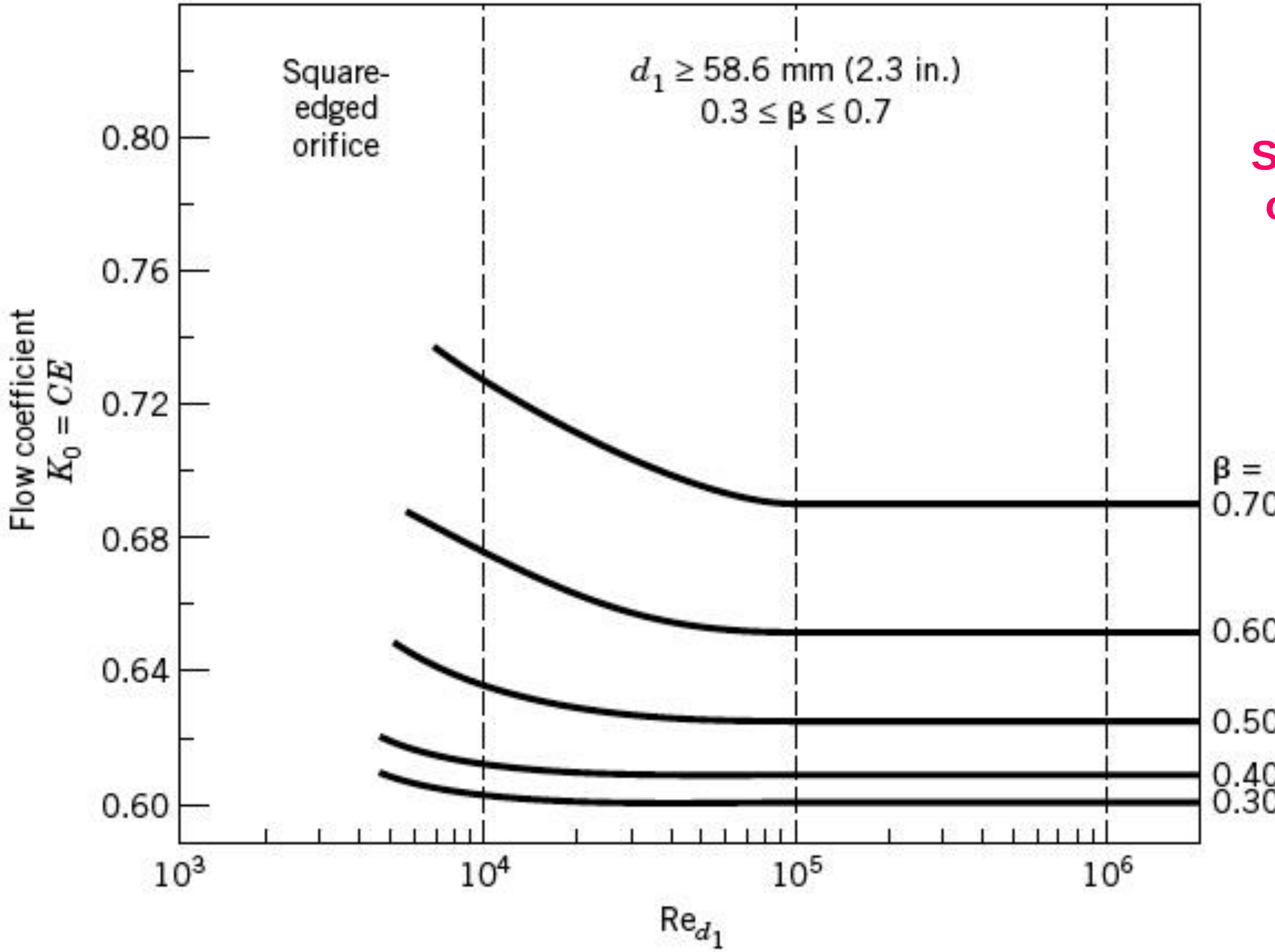


Figure 10.5 Square-edged orifice meter installed in a pipeline with optional 1 D and ½ D, and flange pressure taps shown. Relative flow pressure drop along pipe axis is shown.



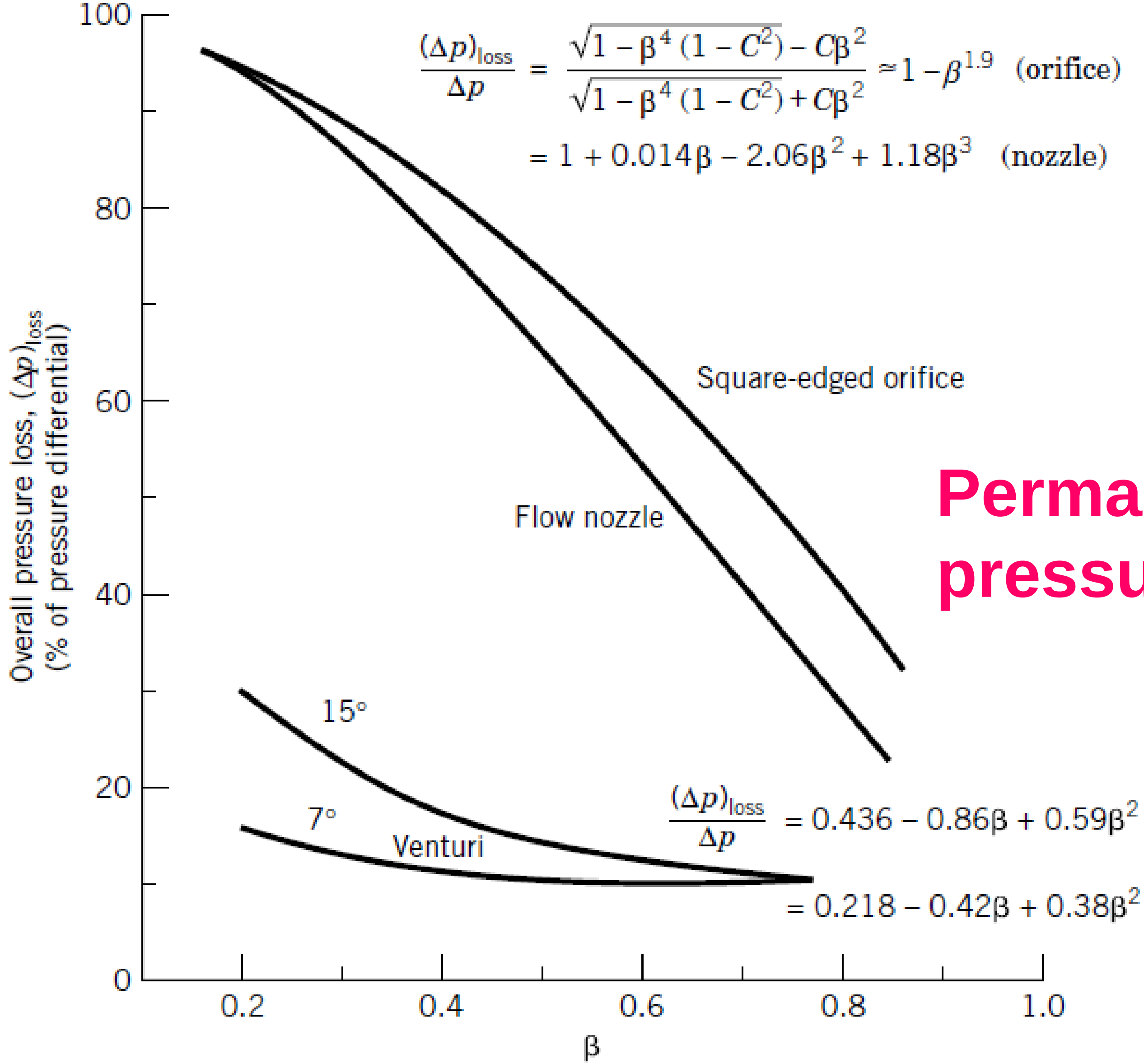
Standard Square-edged orifice meter

$$\beta = d_o / d_1$$

$$K_0 = \frac{1}{(1-\beta^4)^{1/2}} \left(0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 91.71\beta^{2.5} Re_{d_1}{}^{-0.75}\right)$$

Figure 10.6 Flow coefficients for a square-edged orifice meter having flange pressure taps. (Compiled from data in [2]).

Obstruction meters

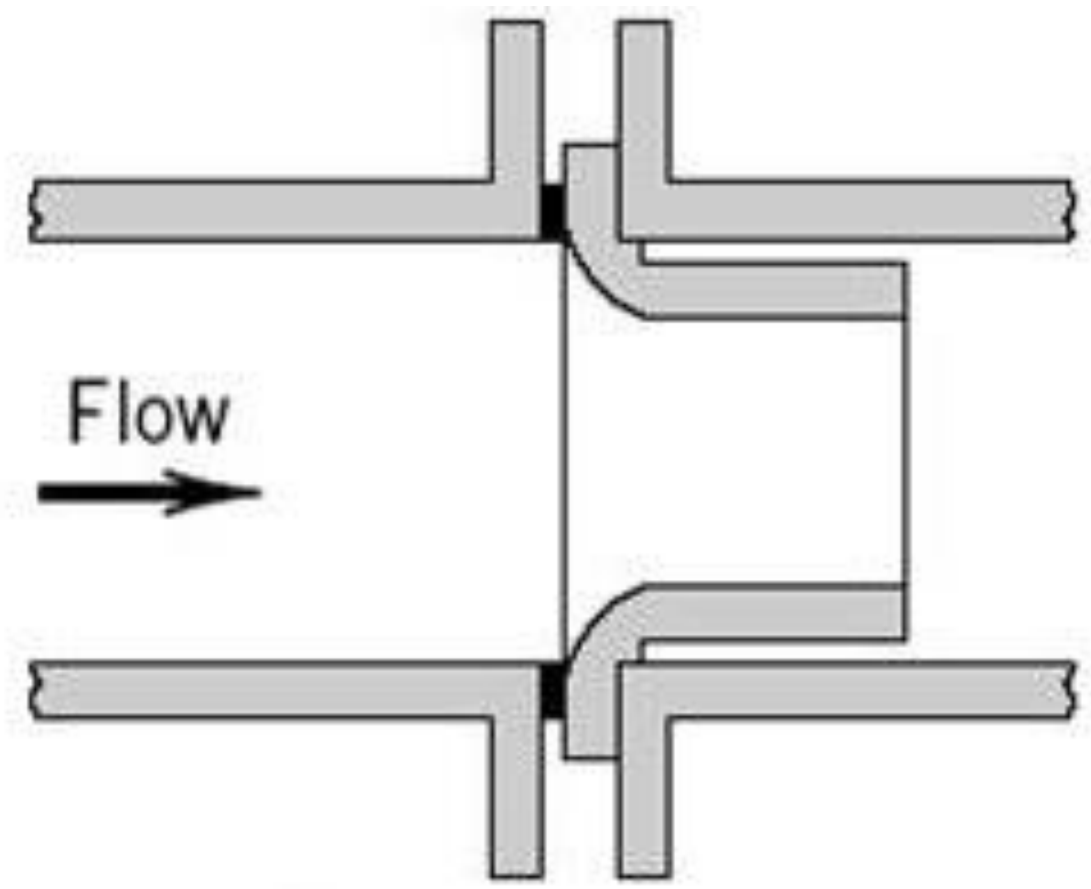


Permanent pressure loss

Figure 10.7 The permanent pressure loss associated with

Obstruction meters

Nozzle

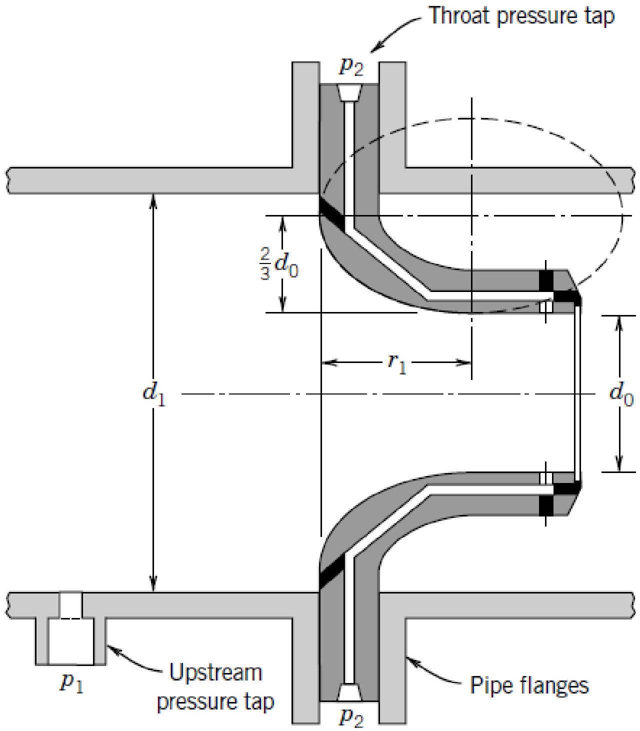


(*b*) ASME long-radius nozzle

Obstruction meters

Nozzle

pressure taps



pressure losses

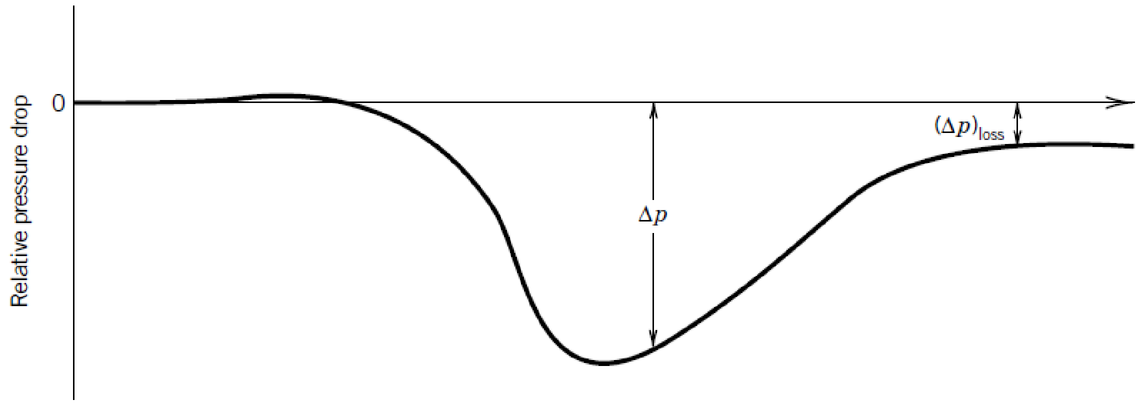


Figure 10.9 The ASME long-radius nozzle with the associated flow pressure drop along its axis.

Obstruction meters

Long radius nozzle

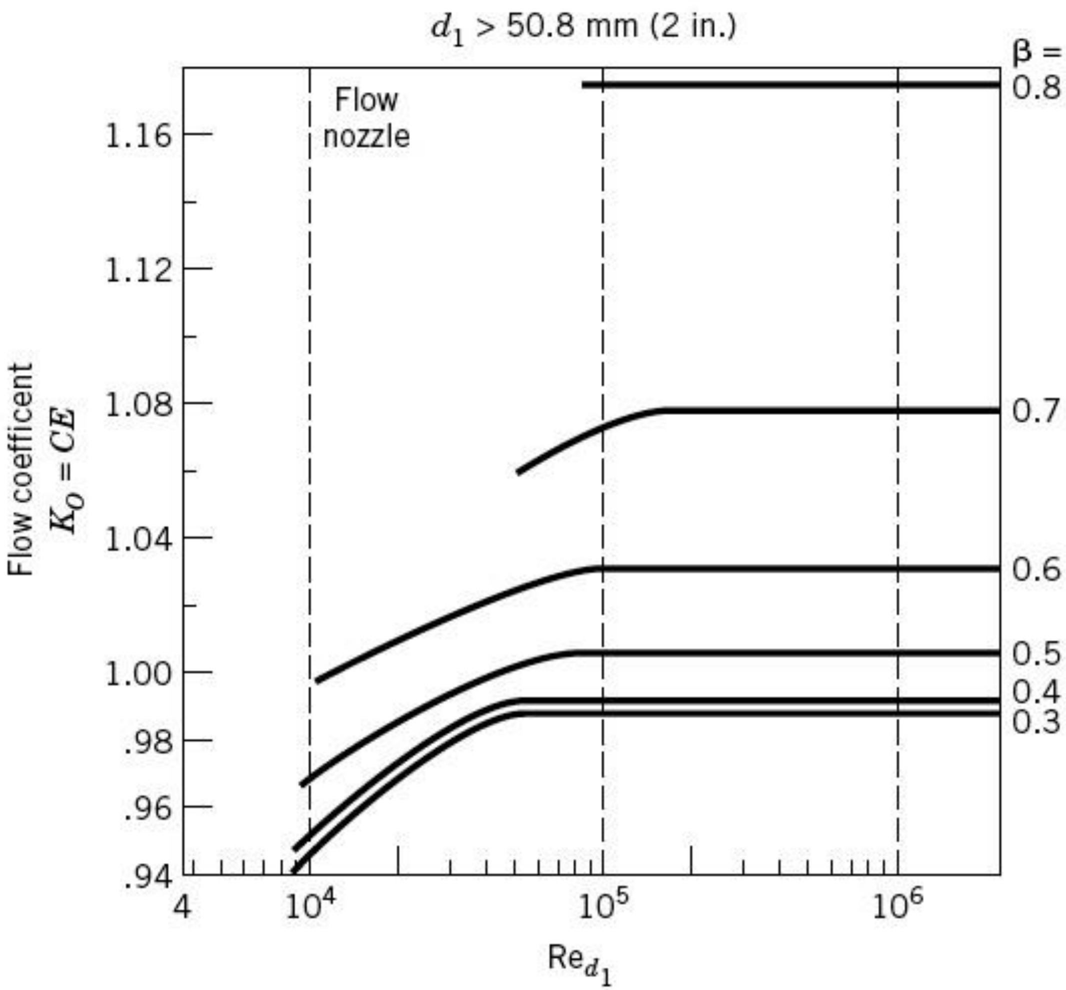
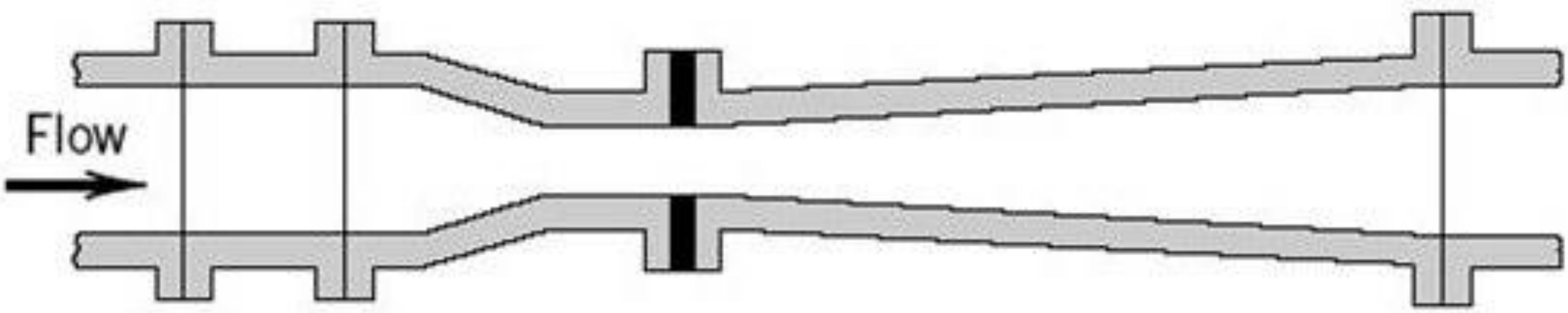


Figure 10.11 Flow coefficients for an ASME long-radius nozzle with a throat pressure tap. (Compiled from [2].)

Obstruction meters

Venturi meter



(*c*) ASME Herschel venturi

Obstruction meters

Venturi meter

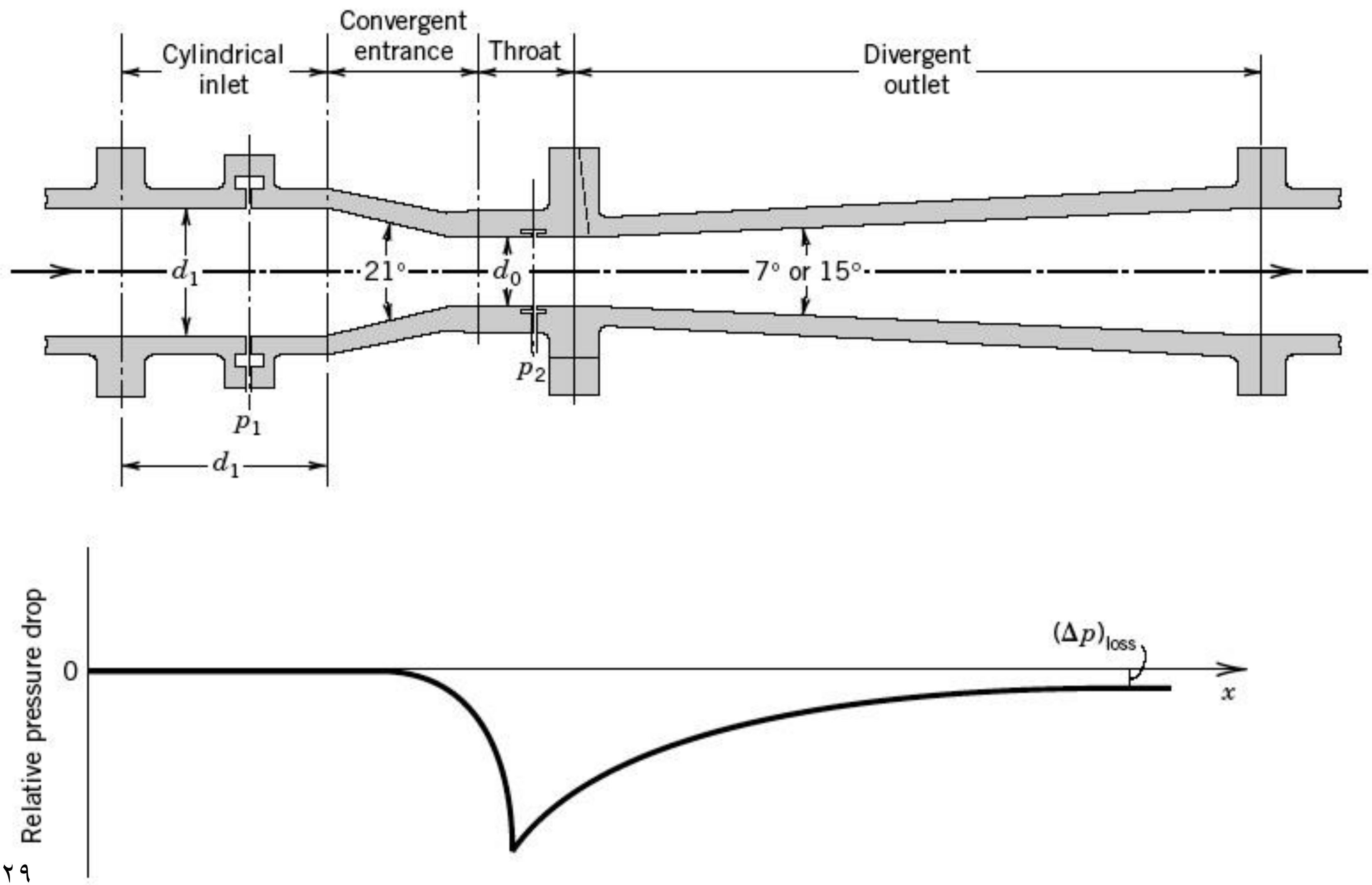


Figure 10.9 The Herschel venturi meter with the associated flow pressure drop along its axis.

Obstruction meters

Venturi meter

$$2*10^5 \leq \mathrm{Re}_{d1} \leq 2*10^6$$

$$0.4 \leq \beta \leq 0.75$$

Cast units $$C = 0.984$$

Machine units $$C = 0.995$$

$$Q = CEA_0\sqrt{\frac{2\Delta p}{\rho}} = K_0 A_0\sqrt{\frac{2\Delta p}{\rho}}$$

Obstruction meters

Compressibility effects

For gases at high pressure, compressibility must be accounted for

$$Q = YQ_I = CEA_0Y\sqrt{\frac{2\Delta p}{\rho}}$$

Q_I Volume flow rate assuming incompressible flow

Y is called compressibility adiabatic expansion factor

Y=Actual compressible flow rate/Assumed incompressible flow rate

Obstruction meters

Y expansion factor

Y factor for orifice, nozzle, and venturi meters

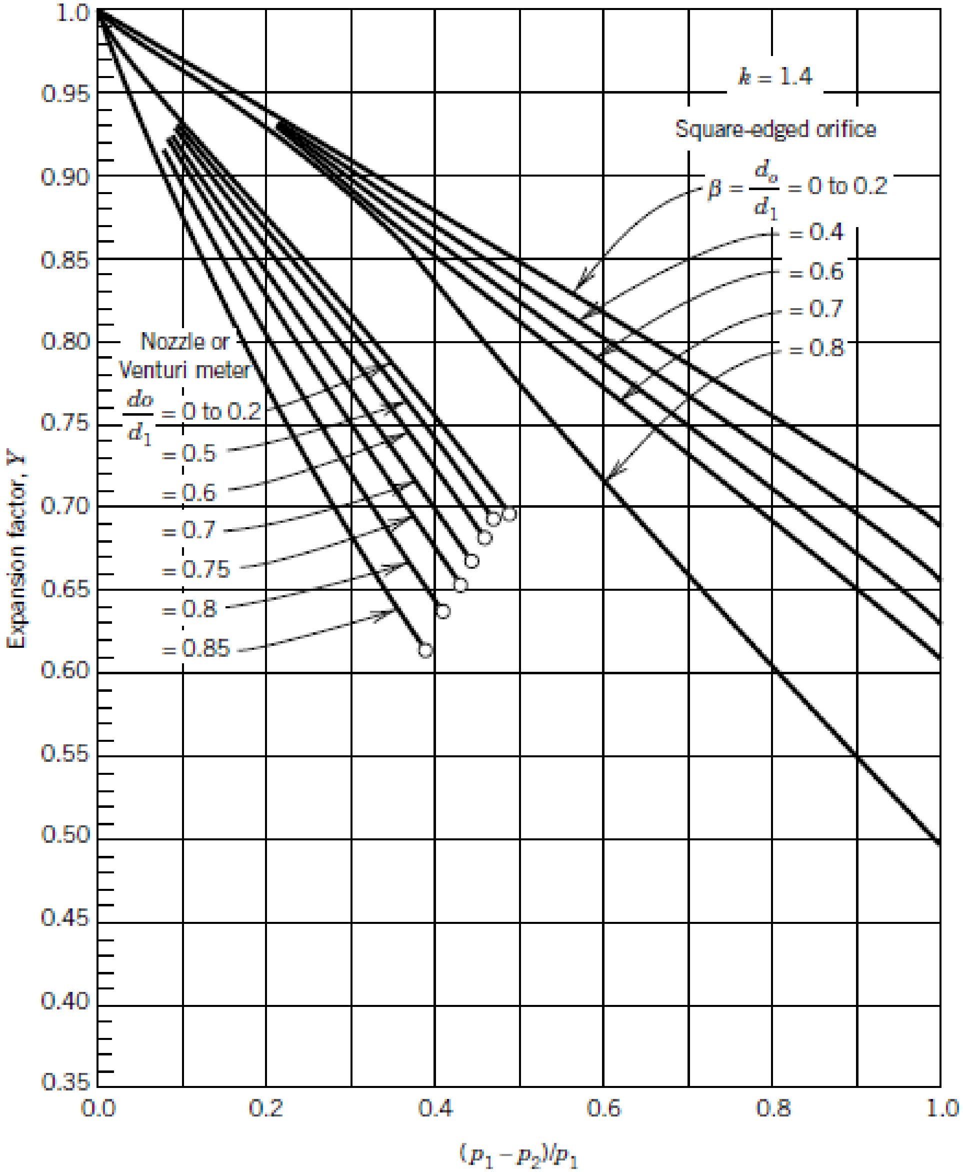
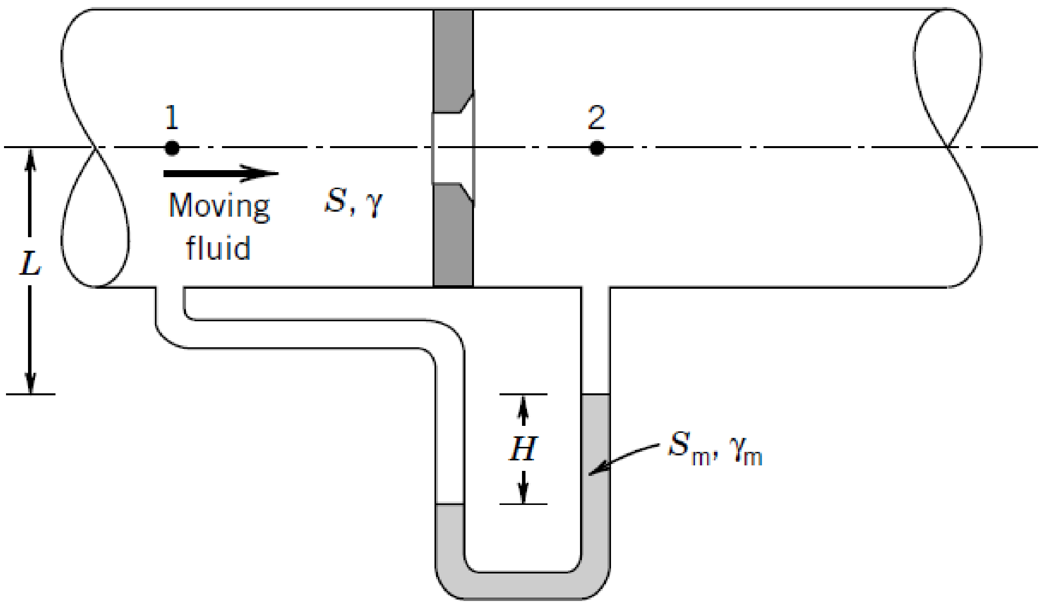


Figure 10.6 Expansion factors for common obstruction meters with $k = c_p/c_v = 1.4$. (Courtesy of American Society of Mechanical Engineers, New York, NY; compiled and reprinted from reference 1.)

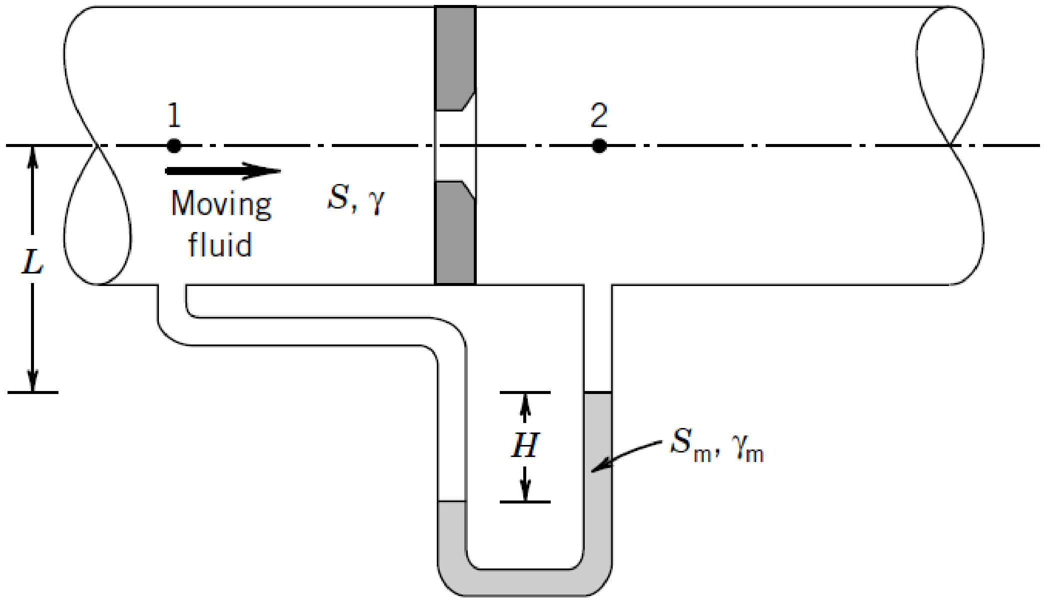
Using the manometer for Pressure difference measurement



S=SG=Specific Gravity=ρ/ρ_w

Figure 10.11 Manometer and flow meter of Example 10.3. A taper must be added to the downstream side of the orifice hole when the plate thickness exceeds $0.02d_1$ (1).

Example 10.3 Find the relation between the volume flow rate and the manometer reading H



$$\Delta p = p_1 - p_2 = (\gamma_m - \gamma)H = \gamma H\left[(S_m / S) - 1\right]$$

$$Q = CEA_0Y\sqrt{2gH\left[(S_m / S) - 1\right]}$$

$$S = SG = Specific\ Gravity = \frac{\rho}{\rho_w} \qquad Specific\ weight\ \ \gamma = \rho g$$

Example 10.4

Flange type orifice meter. d_1=20 cm, d_0=10 cm, H=50 cm Hg, S_m=13.5, water at 16 °C is flowing through the orifice. Find the volumetric flow rate, Q

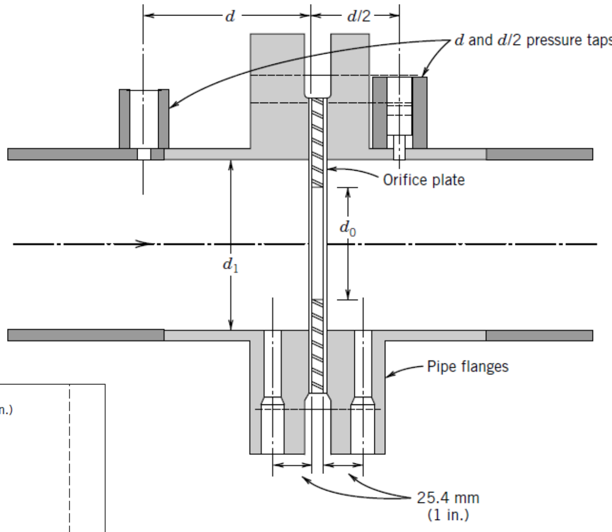
$$\mu = 1.08 * 10^{-3} Pa.s$$

Water properties from appendix B, μ, and ρ

$$\rho = 999 kg/m^3$$

$$Q = CEA_0Y\sqrt{2gH\left[(S_m/S)-1\right]}$$

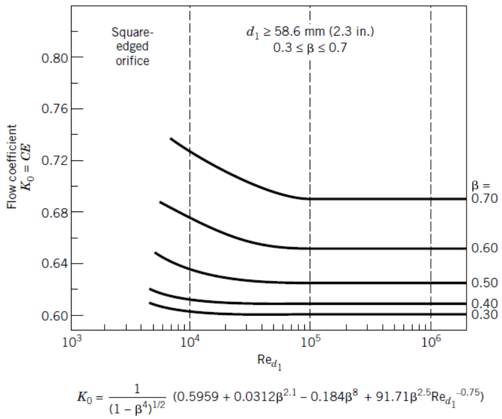
Y=1.0



$$E = \frac{1}{\sqrt{1-\beta^4}} = \frac{1}{\sqrt{1-0.5^4}} = 1.0328$$

K_0=function of Re, β

Trial value of K_0(=CE) from Fig. 10.6



K_0=0.625

Example 10.4 Continue

$$Q = K_0 A_0 \sqrt{2gH\left[(S_m / S) - 1\right]}$$

$$Q = 0.054\, m^3 / s$$

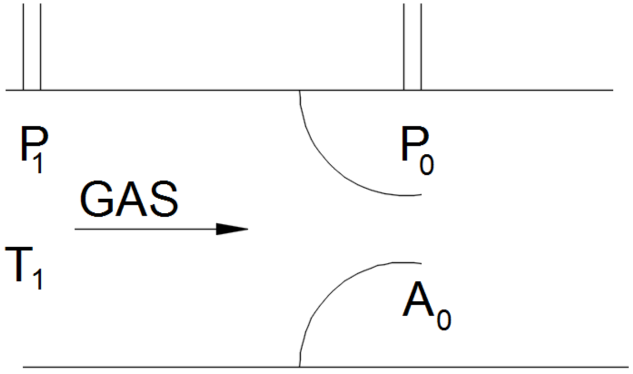
$$\mathrm{Re}_{d_1} = \frac{\rho \bar{U} d_1}{\mu} = \frac{\bar{U} d_1}{\nu} = \frac{4Q}{\pi d_1 \nu}$$

$$\mathrm{Re}_{d1} = 3.2 * 10^5$$

Using Fig. 10.6 again with This value of Re_{d1} and β=0.5, and get K_0=0.625 (Solution converges)

K_0=0.625

Sonic Nozzle



- Used to meter the flow of compressible gas
- It may take any of the forms of obstruction meters (Orifice, Nozzle or venturi)
- At the sonic condition, the gas velocity at the throat equals the speed of sound of the gas (i.e. Mach number=1)
- The throat is said to be chocked. The mass flow rate is the maximum
- Any further increase in pressure drop downstream will not increase the flow rate
- At the critical condition for isentropic process

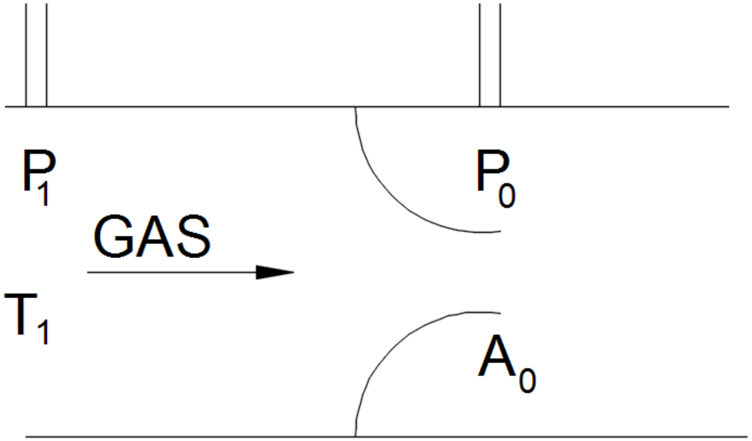
$$\left(\frac{p_0}{p_1}\right)_{critical} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad k = \frac{C_p}{C_v}$$

P_0 is the pressure at the throat. P_1 is the pressure upstream

Sonic Nozzle

if $(p_0/p_1) \leq (p_0/p_1)_{critical}$

The meter throat is chocked and the gas flows at the sonic condition mass flow rate



The energy equation is given by

$$C_p T_1 + \frac{U_1^2}{2} = C_p T_0 + \frac{U_0^2}{2}$$

The mass flow rate for sonic nozzle is given

$$\dot{m}_{\max} = \rho_1 A_o \sqrt{2RT_1} \sqrt{\frac{k}{k+1}\left(\frac{2}{k+1}\right)^{2/(k-1)}}$$

Sonic Nozzle

The mass flow rate through chocked nozzle is given by

$$\dot{m}_{\max} = \rho_1 A_o \sqrt{2RT_1}\sqrt{\frac{k}{k+1}\left(\frac{2}{k+1}\right)^{2/(k-1)}}$$

Where k=specific heat ratio (C_p/C_v)

R is the gas constant in J/kg.K

A_o is the throat area

C=0.99 ±2% (95%)

Sonic nozzle is very convenient method to meter and regulate gas flow

Example 10.7

Chocked flow through a nozzle. mass flow rate =1.3 kg/s, d_1= 6 cm, P_1=690kPa, Nitrogen gas at 20° C, R=297 J/kg.K. Find the maximum β=?

$$\dot{m}_{\max} = \rho_1 A_o \sqrt{2RT_1} \sqrt{\frac{k}{k+1}\left(\frac{2}{k+1}\right)^{2/(k-1)}}$$

k=1.4, T_1=293 K, ρ_1=7.929 kg/m^3, R_{N2}=297 J/kg.K

$$A_{\max} = \frac{\dot{m}}{\rho_1 \sqrt{2RT} \sqrt{[[k/(k+1]][2/(k+1)]^{2/(k-1)}]}}$$

$$A_{\max} = 8.41*10^{-4} m^2 = \pi \frac{d_0^2}{4}$$

d_0=3.27 cm

i.e. $\beta \leq 0.545$

Obstruction meter selection

Considerations

- Meter placement
- Overall pressure loss
- Accuracy
- Overall cost

Meter placement

Must provide sufficient upstream and down stream pipe lengths on both sides of the meter for the flow to develop

Flow development dissipates swirl, promotes a symmetric velocity distribution and allows proper pressure recovery down stream of the meter

Follow the recommendations given by Fig. 10.13 on the next slide

Venture and nozzle needs considerable length due to inherent design

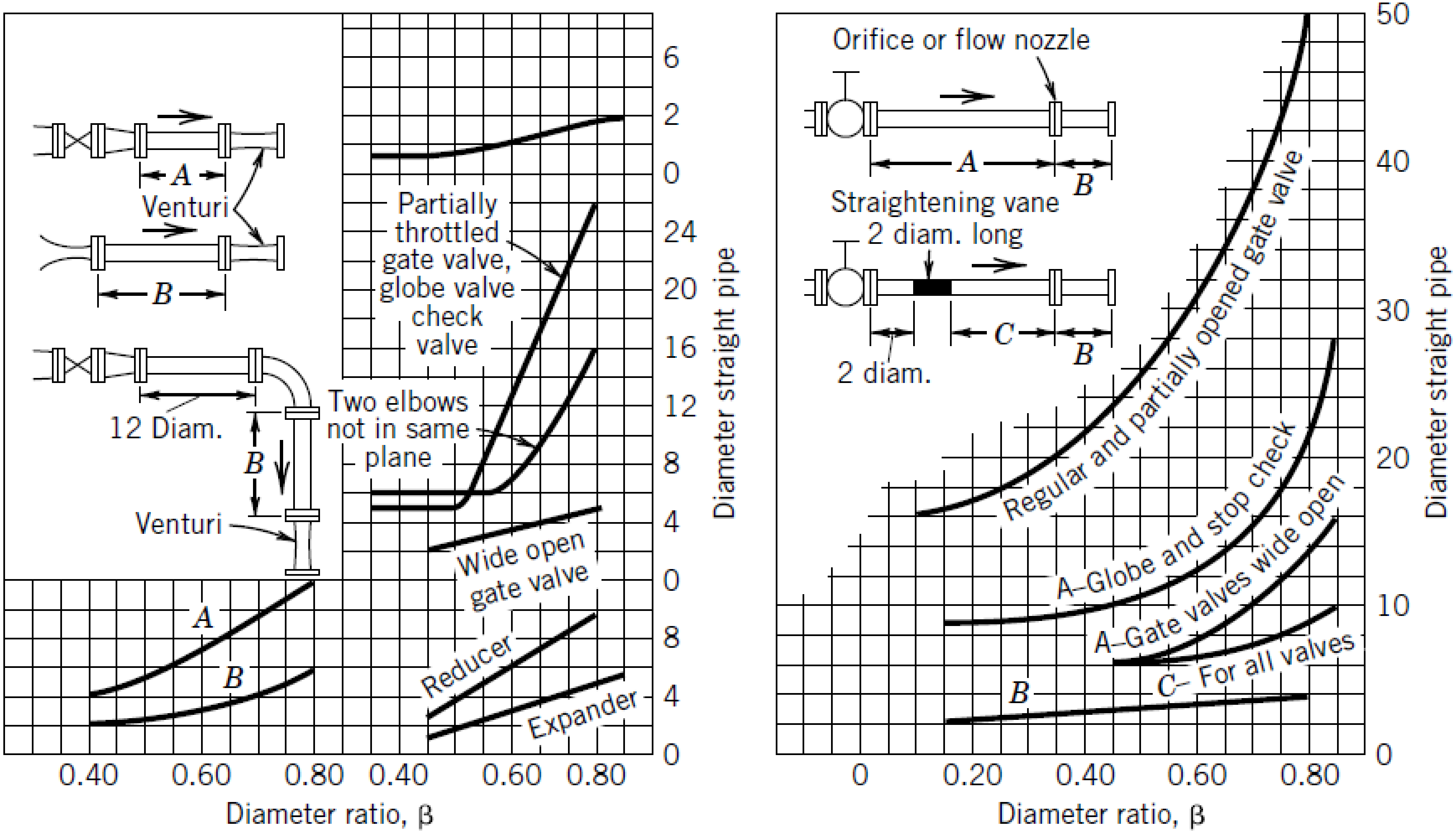


Figure 10.13 Recommended placements for flow meters in a pipeline. (Courtesy of American Society of Mechanical Engineers, New York, NY; reprinted from reference 1.)

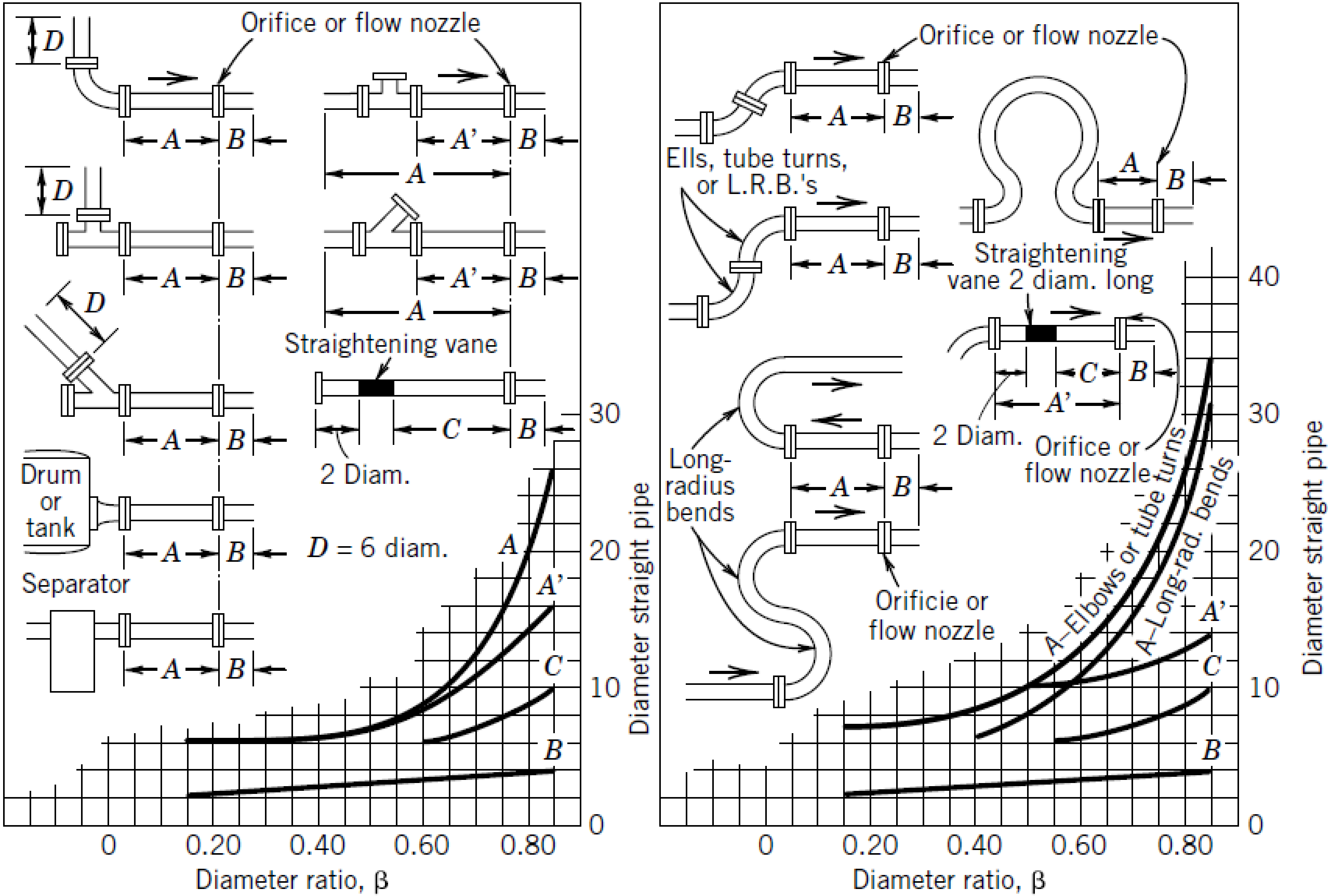


Figure 10.13 continued

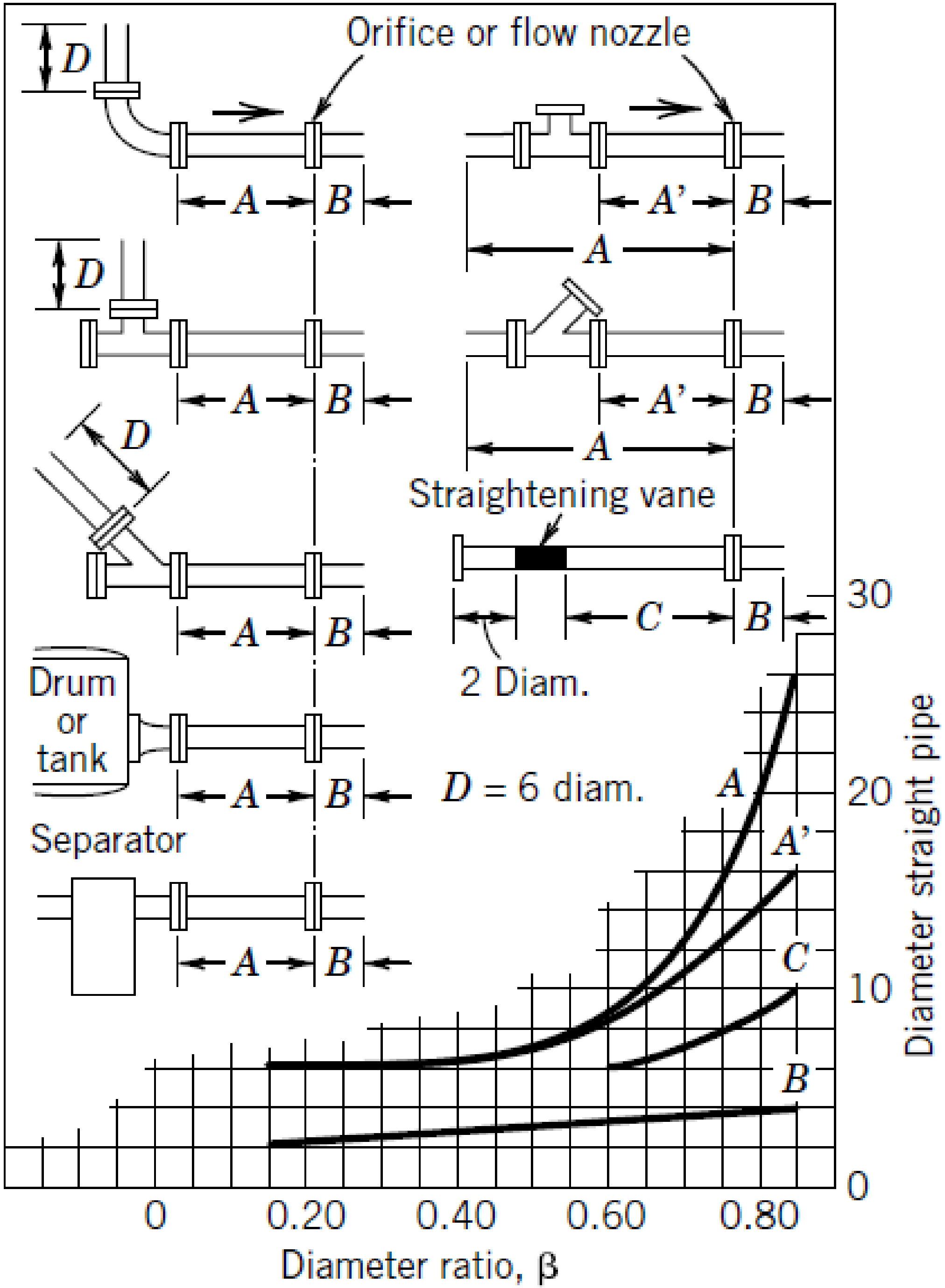


Figure 10.13 Recommended placement of disturbance flow meter in pipe line

Overall pressure loss

The prime mover (fan or pump) must be selected to overcome the pressure loss

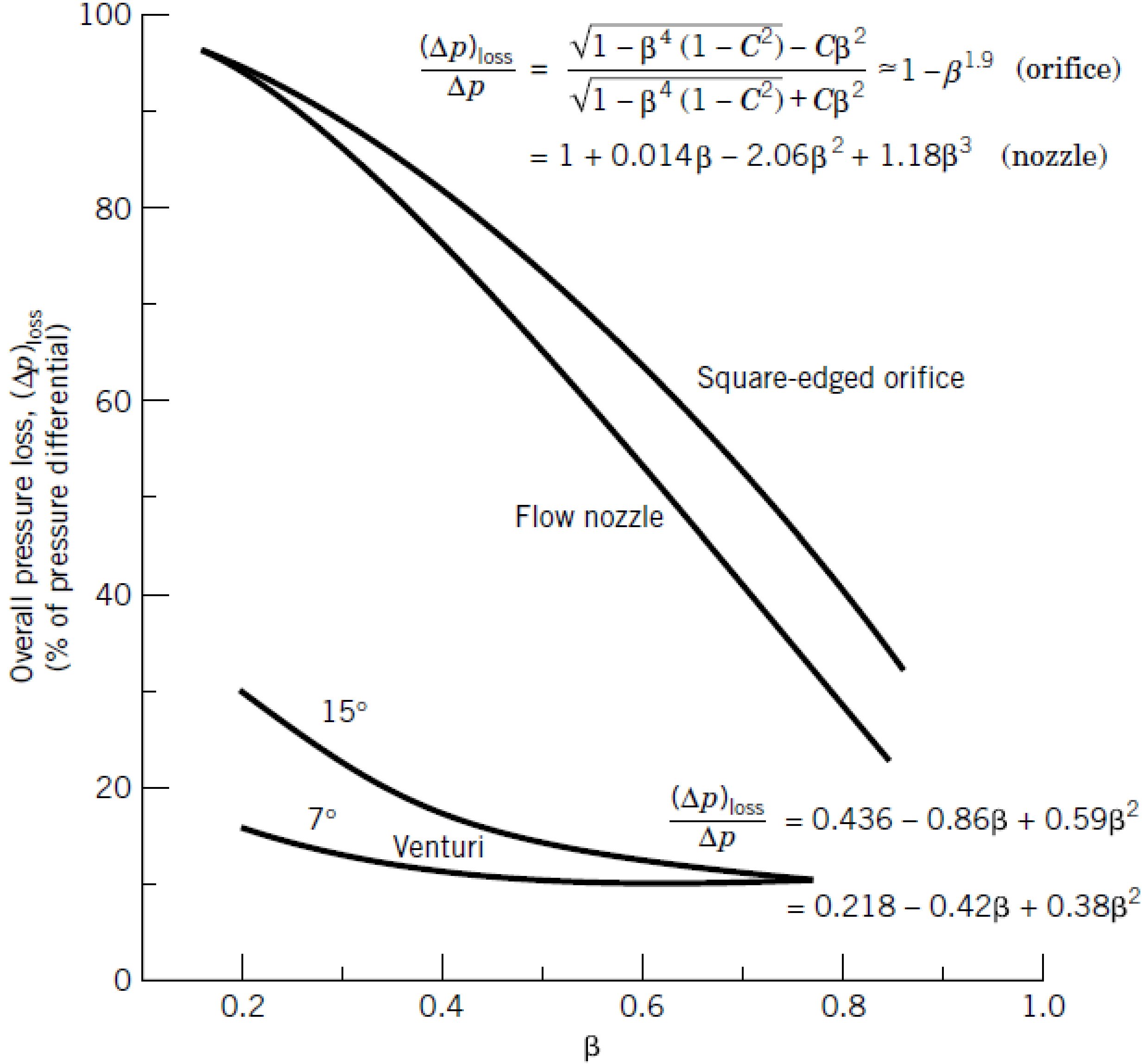
$$\dot{W} = Q\frac{\Delta p_{loss}}{\eta}$$

Overall cost

Overall cost=Initial cost of the meter + cost of installation + Calibration cost + operation cost

System shut down cost must be also considered

Obstruction meters



Permanent pressure loss

Figure 10.7 The permanent pressure loss associated with flow through common obstruction meters. (Courtesy of American Society of Mechanical Engineers, New York, NY; compiled and reprinted from reference 1.)

Meter accuracy

Factors that contribute to the meter accuracy

1) Actual β ratio error and pipe eccentricity

2) Pressure tap position

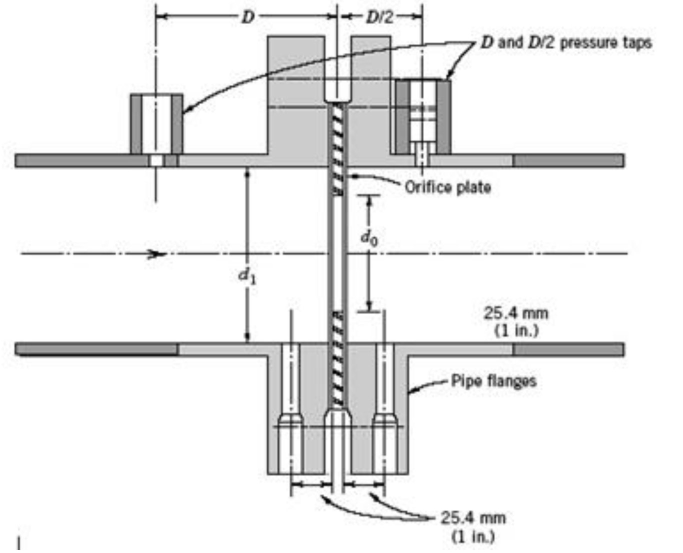
3) Temperature effects leading to relative expansion of components

4) Actual upstream flow profile

In addition to error in density evaluation, and in the coefficients

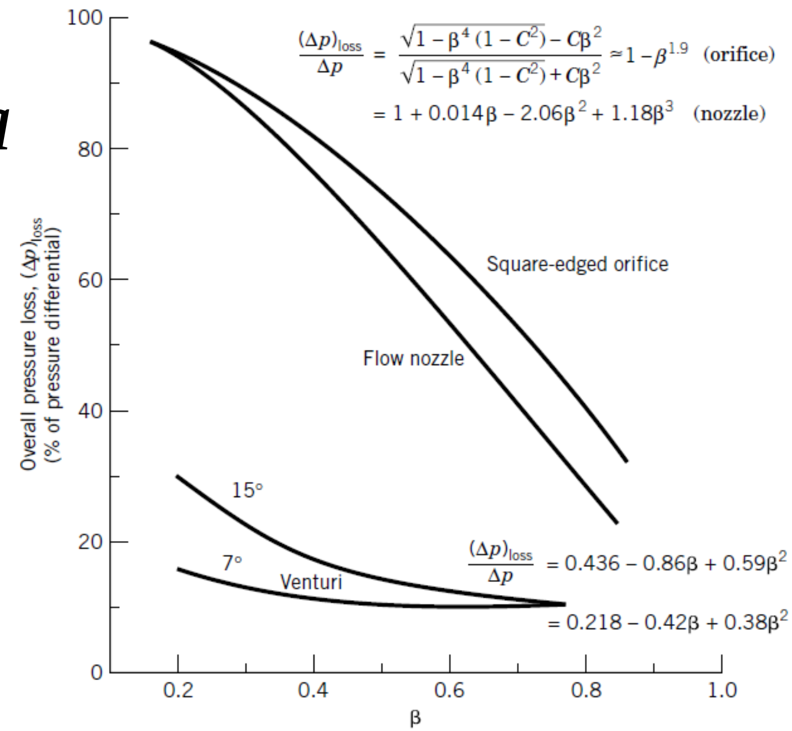
Example 10.8

β=0.5 for orifice meter, Δp=50 cm Hg, Calculate the permanent pressure drop that must be overcome by the pump, From Example 10.4→ Q=0.054 m^3/s



From Fig. 10.7 Δp_{loss} =0.75*Δp

$$\Delta p_{loss} = 0.75\gamma_{Hg} H = 49.9\, kPa$$



Just for comparison a venturi will have Δp=19.6 cm Hg or Δp_{loss} =0.16*19.6=3.1 cm Hg

Or =4.12 kPa

Example 10.9

Calculate the electric cost for the pump in example 10.8. Assume the electric charge is $0.08/kW-hr. Assume the pump efficiency to be 0.6. operation time is 6000 hr/year

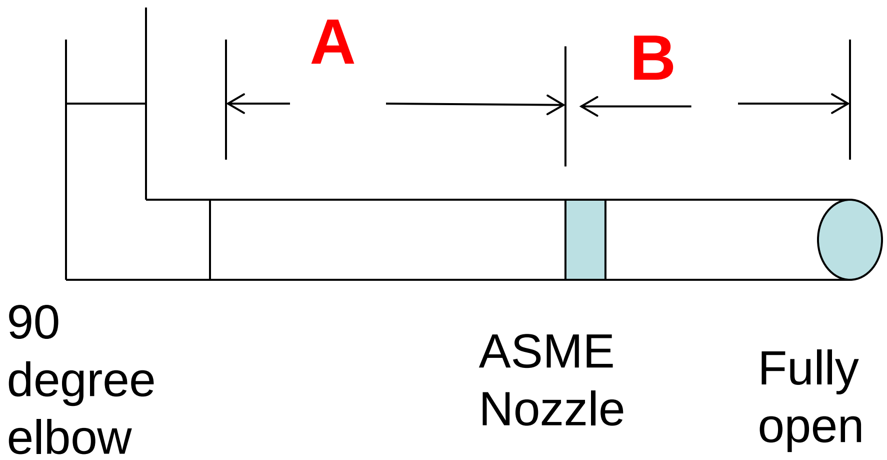
$$\dot{W} = \frac{Q \Delta p_{loss}}{\eta} = \frac{0.054 * 49.9}{0.6} = 4.491\,\mathrm{kW}$$

cost=4.491*6000*0.08=**$ 2156 per year**

if Venturi meter is used the cost will be
0.054*4.12*6000*0.08/0.6=**$ 178 per year**

Example 10.10

Layout for installing ASME Nozzle in 12 in pipe (ANSI schedule 40), D=304.8 mm. There is a gate valve fully open downstream, and 90 elbow upstream. Determine the minimum recommended lengths of straight pipe needed. Assume β=0.5



From Fig. 10.13

A=8D

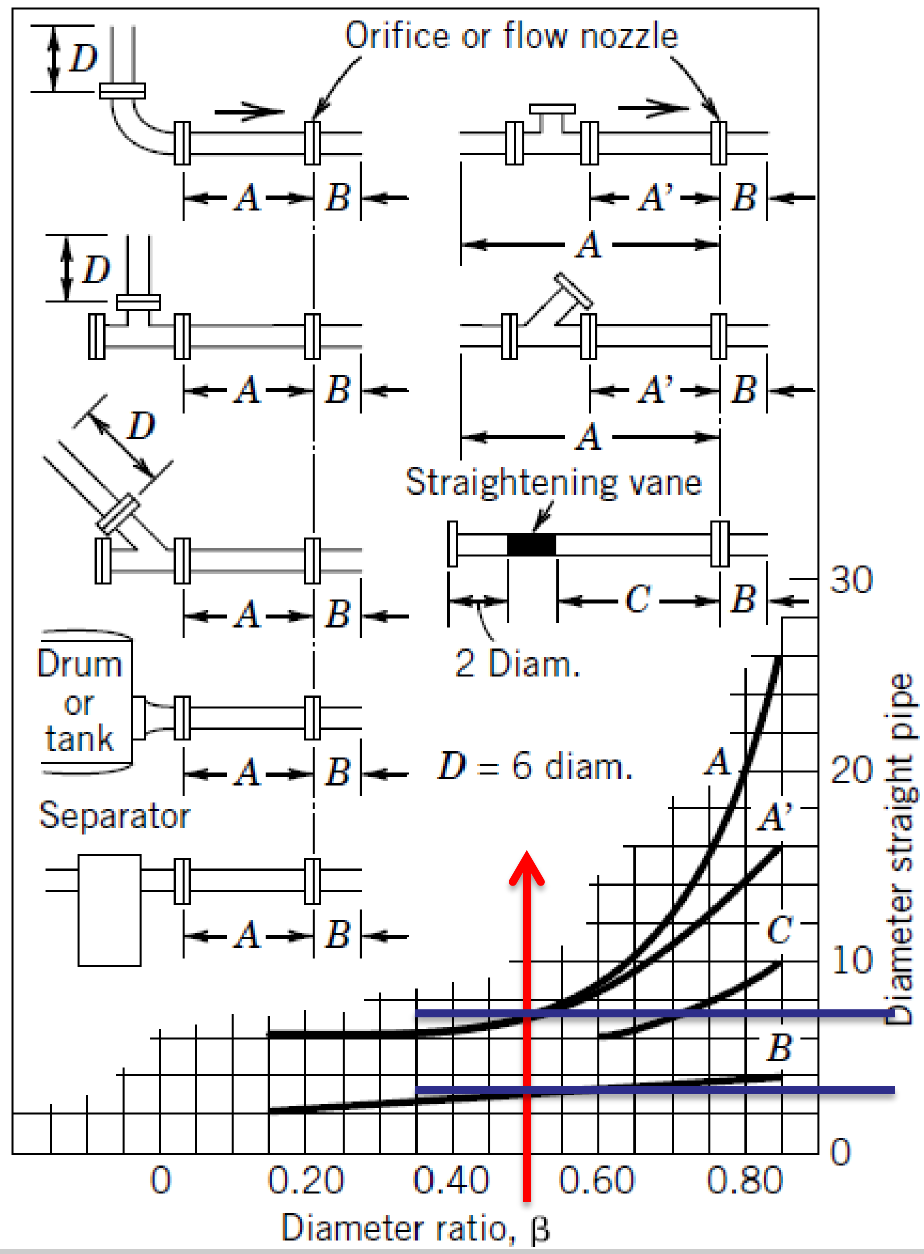
B=3D

For A typical Nozzle one needs 1.5D for the nozzle

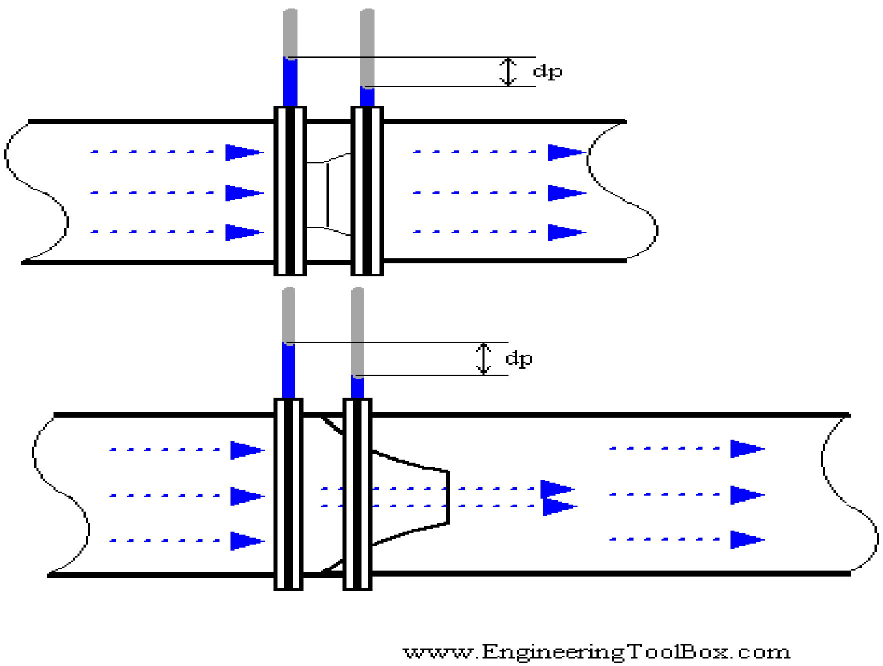
Therefore straight section of the pipe length is 8+3+1.5=12.5 D

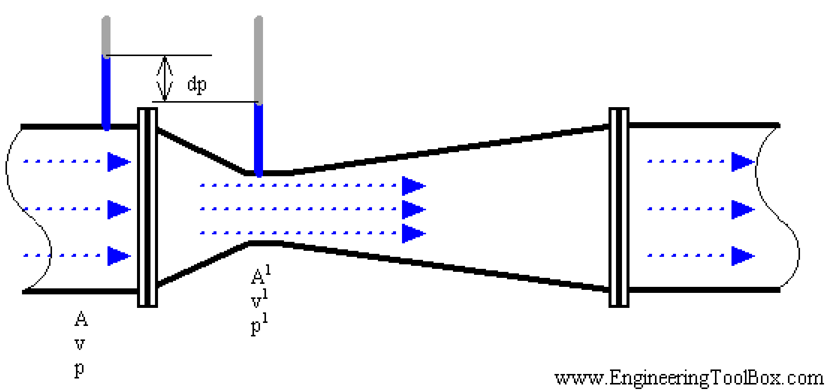
Example 10.10

Continue



Obstruction Meters Merits





- **Orifice Meters**
  - Inexpensive
  - Easy to install
  - Large pressure losses
- **Flow Nozzles**
  - Difficult to install properly
  - High accuracy
  - Good pressure recovery
- **Venturi Meters**
  - High accuracy
  - Expensive to construct
  - Good pressure recovery

Laminar flow elements

For laminar flow, (Re<2000) there is a simple relation for the flow rate

$$p_1 - p_2 = f\frac{L}{d}\rho\,(\frac{V^2}{2})$$

$$f = \frac{64}{Re} \qquad\qquad Q = VA = V\frac{\pi d^2}{4}$$

$$Q = \frac{\pi d^4}{128\mu}\frac{p_1 - p_2}{L}$$

To make the flow laminar, one may use laminar flow elements as shown in the next slide

Laminar flow elements

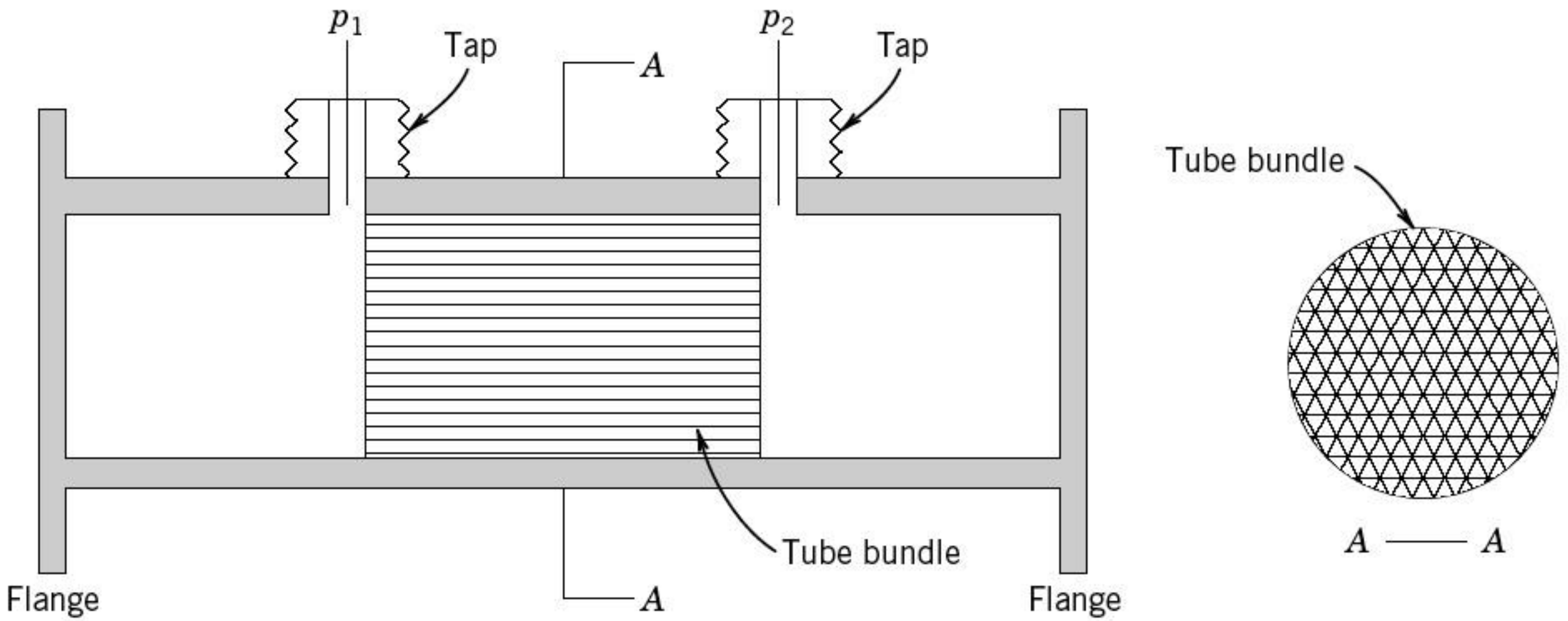


Figure 10.14 Laminar flow element flow meter.

Reynolds number in each pipe must be < 2000

Advantages of laminar flow elements

1-High sensitivity even at extremely low flow rates

2-An ability to measure flow rate in either direction

3-Wide useable flow range

4-Ability to indicate an average flow rate in pulsating flows

Instrument uncertainty is as low as ± 0.25 %
of the flow rate

This method is used only for clean fluids

Insertion volume flow meters

1-Electromagnetic flow meters

2-Vortex shedding meters

3-Rotameters

4-Turbine flow meters

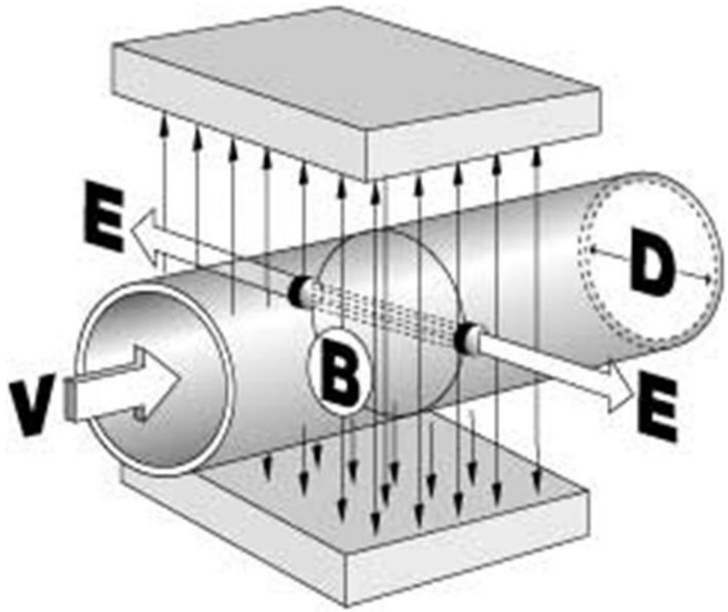
5-Transient time and Doppler (ultrasonic) flow meter

6-Positive displacement meters

1-Electromagnetic flow meters

Principle idea

An electromotive for (emf) of electric potential E is induced in a conductor of length L which moves with velocity V through a magnetic field flux B

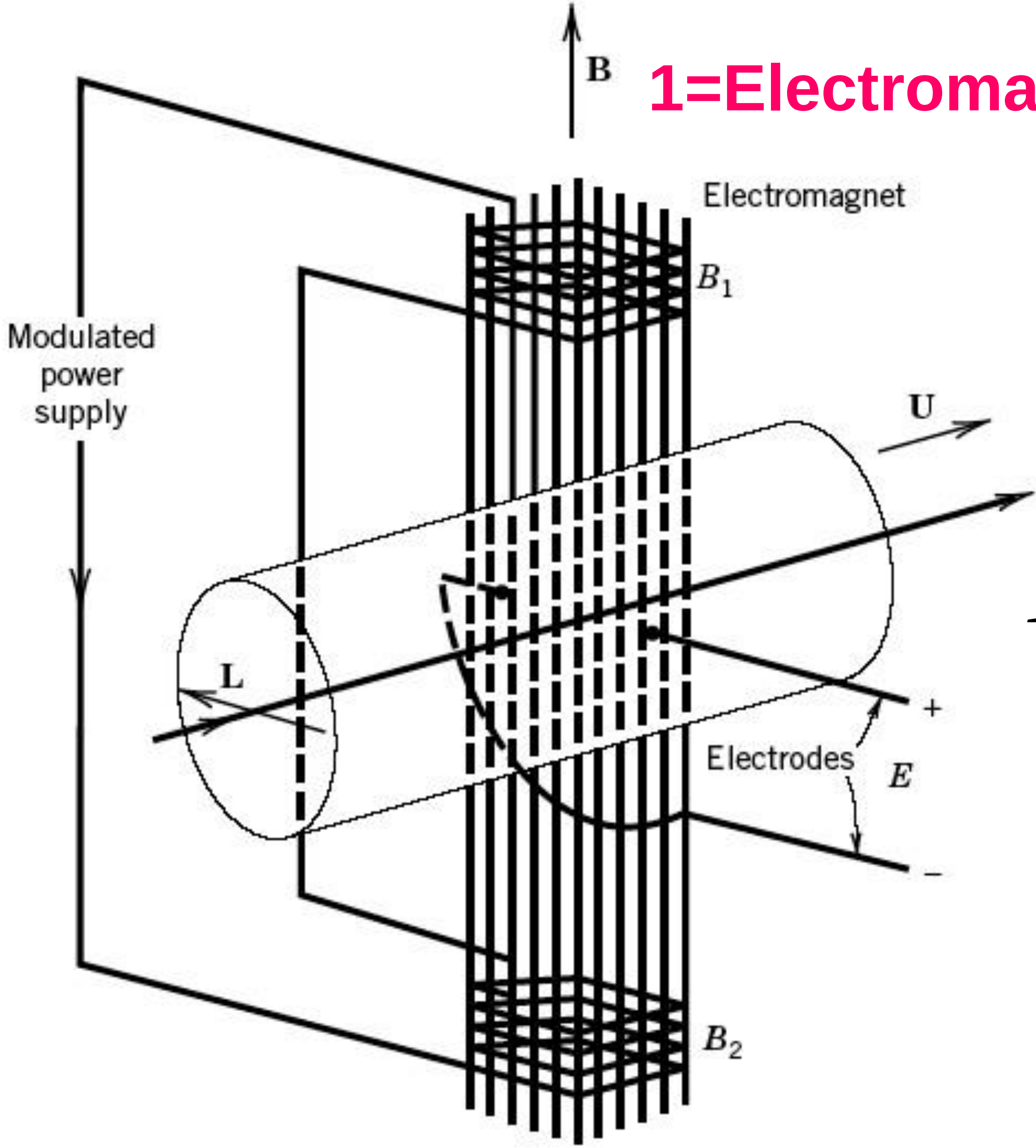


$$E = \overline{V} \times B.L$$

$$E = \overline{V} BL \sin(\alpha)$$

α is the angle between V and B (usually 90 °)

1=Electromagnetic flow meters

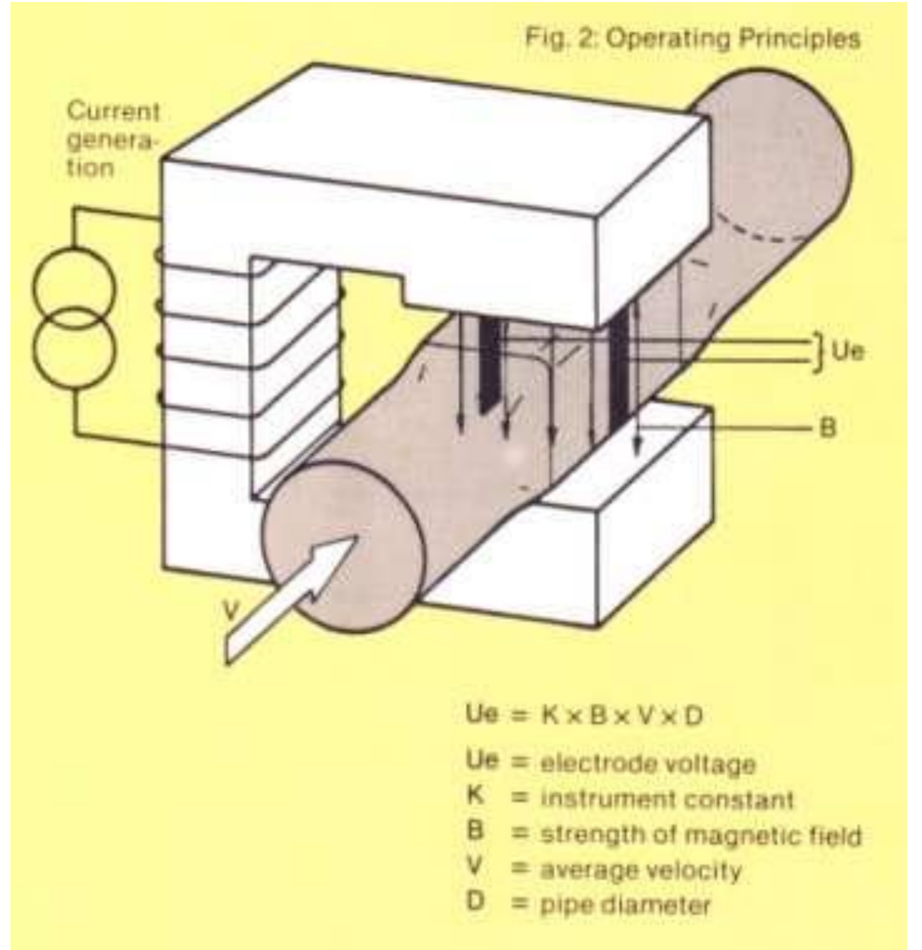


$$E = \bar{U} B L \sin(\alpha)$$

Figure 10.15 Electromagnetic principle as applied to a working flow meter.

1-Electromagnetic flow meters

$$E = \overline{U} B L$$



The volume flow rate

$$Q = \overline{U} \frac{\pi d_1^2}{4} = \frac{E}{BL} \frac{\pi d_1^2}{4} = K_1 E$$

L is of order of pipe diameter D

K_1 is the meter constant and usually supplied by the manufacturer

1-Electromagnetic flow meters

Some of the characteristics of electromagnetic meters

Very low pressure loss

Very useful for small pressure loss applications

Special designs for non-magnetic pipes such as metering blood circulation

Operation idea is independent of fluid properties such as density and viscosity

It can be used in steady or pulsating flow

It is limited to fluids having specified electrical conductivity. Salt may be added to increase the fluid electrical conductivity

It can be used for metering corrosive fluid since no internal part that is contact with the fluid

2-Vortex Shedding Meters

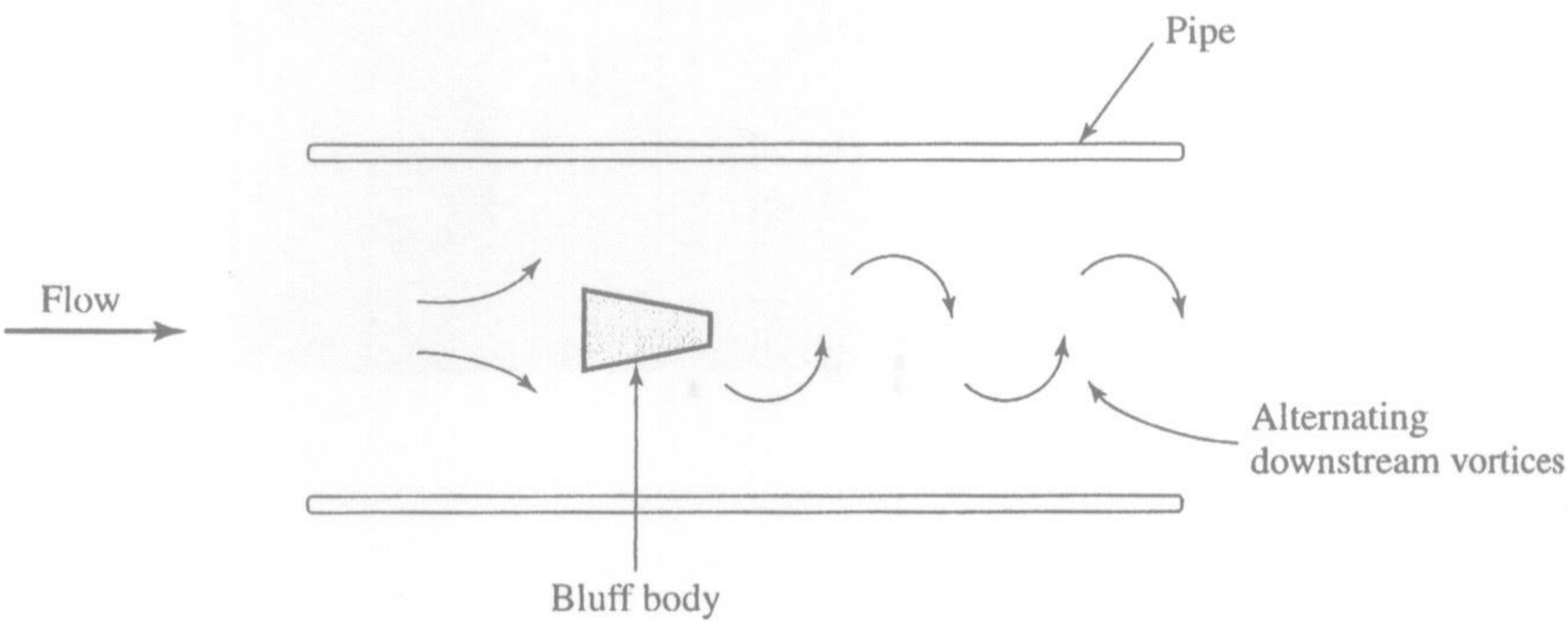
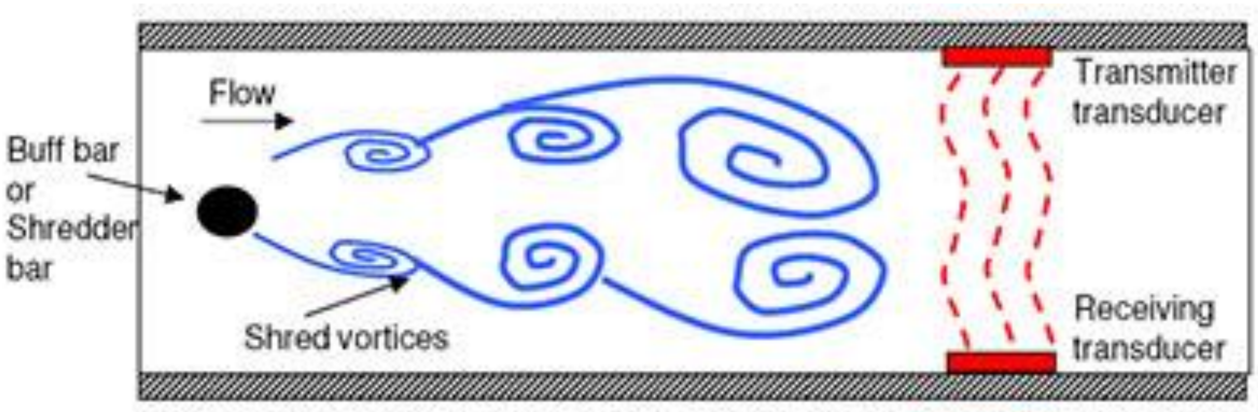
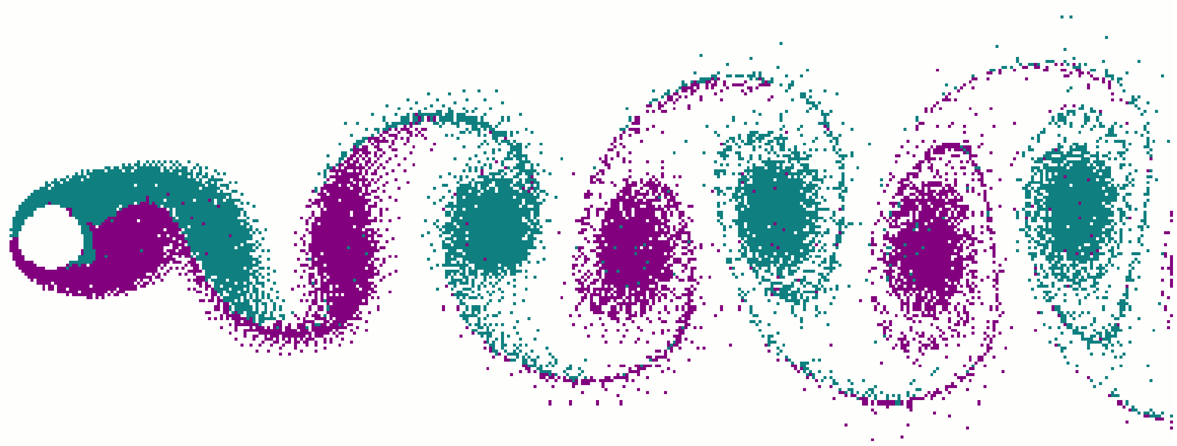


Figure 7.19 Vortex-shedding flowmeter.

Basic idea: Oscillating vortices on the back of the body

2-Vortex Shedding Meters





2-Vortex Shedding Meters

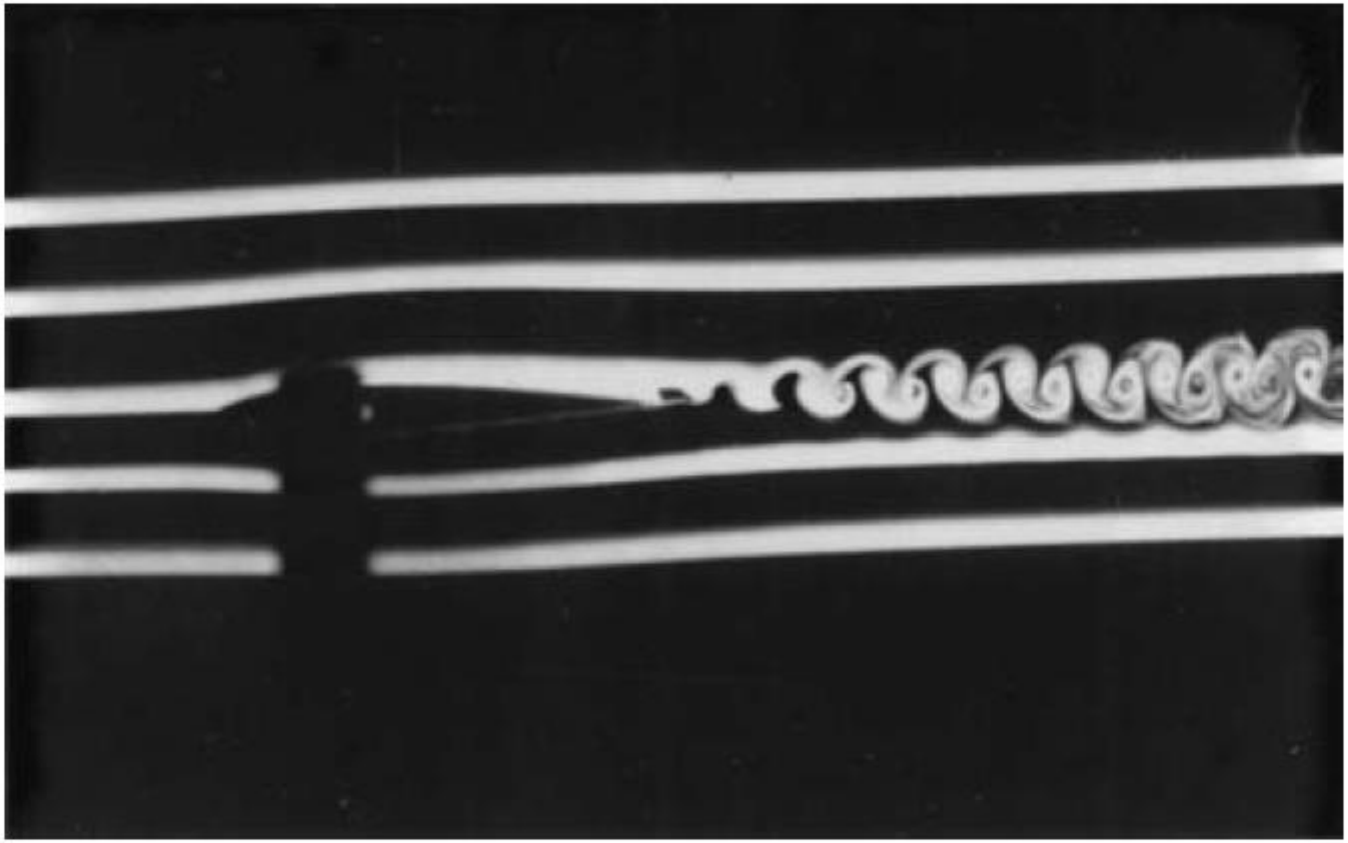


Figure 10.17 Smoke lines in this photograph reveal the vortex shedding behind a streamlined wing-shaped body in a moving flow. (Photograph by R. Figliola.)

2-Vortex Shedding Meters

Basic idea

The frequency f of the vortex depends on the flow velocity U over the body

For a given shape, U is related to the frequency f by Strouhal number St

$$St = fd / \overline{U}$$

f is the frequency of vortices

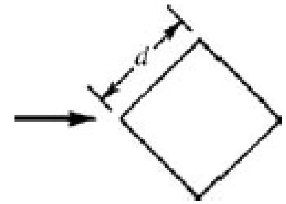
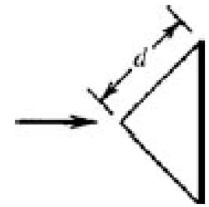
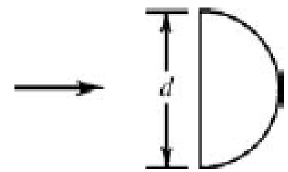
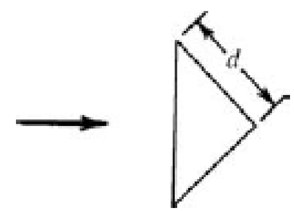
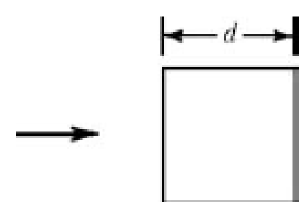
d is the characteristic length of the body

2-Vortex Shedding Meters

$$St = fd/\overline{U}$$

$$\overline{U} = \frac{fd}{St}$$

Table 10.1 Shedder Shape and Strouhal Number

Cross Section	Strouhal Number[a]
	0.16
	0.19
	0.16
	0.15
	0.12

[a]For Reynolds number $Re_d \geq 10^4$. Strouhal number $St = fd/\overline{U}$.

2-Vortex Shedding Meters

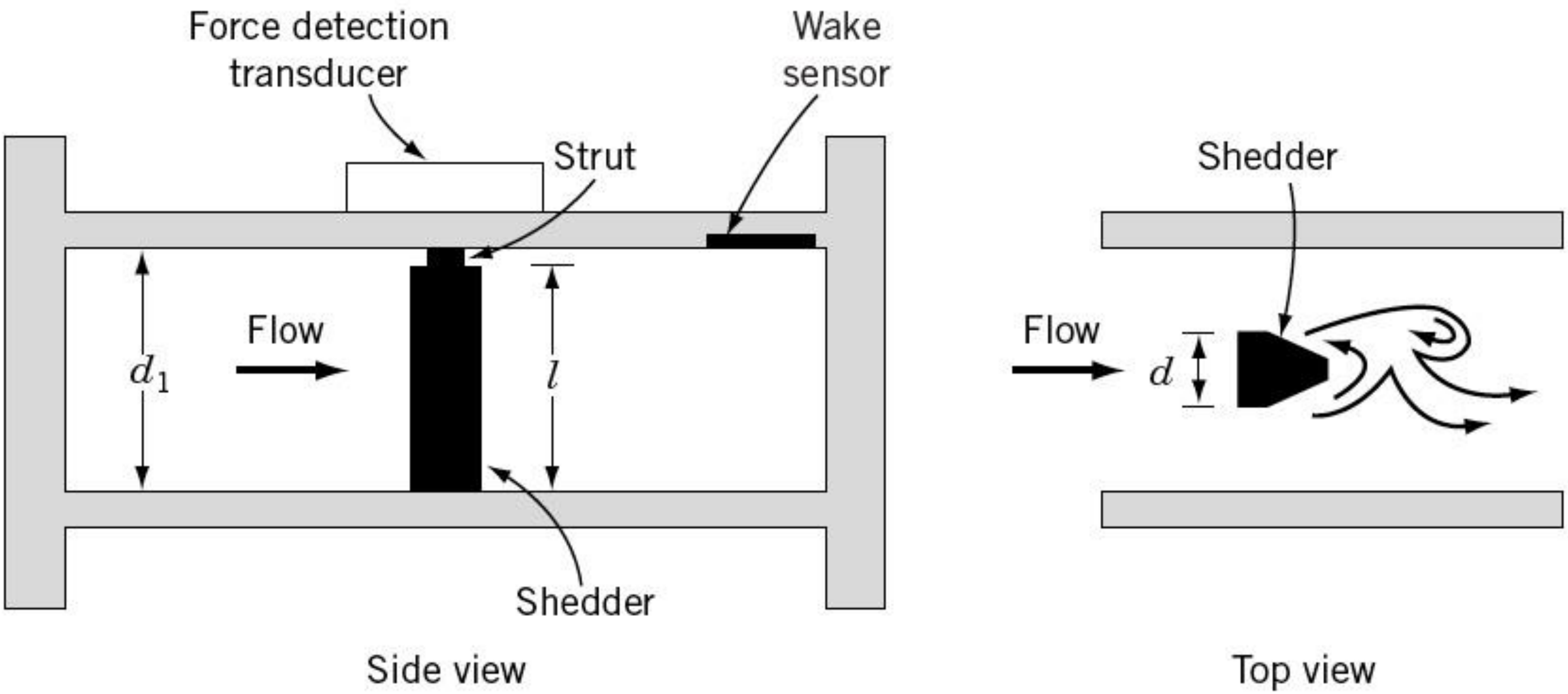


Figure 10.17 Vortex shedding flow meter. Different shedder shapes are available.

2-Vortex Shedding Meters

For a fixed Strouhal number

$$St = fd / \overline{U}$$

or $$\overline{U} = fd / St$$

$$Q = \overline{U}A = C\frac{fd}{St}\frac{\pi d_1^2}{4} = K_1 f$$

C is a constant to take into account the blockage effect that tends to increase the average velocity measured

K_1 is the K factor which is usually constant for $10^4 < Re_d < 10^7$

2-Vortex Shedding Meters

Some remarks about Vortex shedding meter

Frequency measurement is made using a strain gage or capacitance sensor to sense pressure oscillations

There is a low limit application for this meter for Re_d around 10000. At which St (Strouhal number) is not fixed.

The upper limit on using this meter is the onset of cavitations

Property variation (density and viscosity) affect the meter performance indirectly

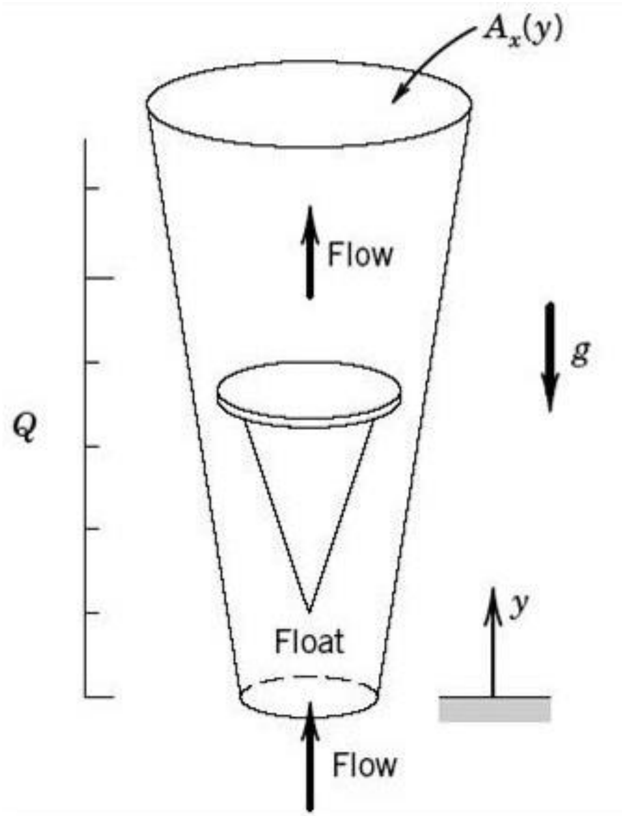
Density affect the strength of the shed vortex

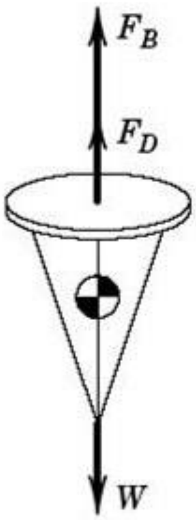
Viscosity affect the operational Reynolds number

No moving parts meter and turndown 20:1

3- Drag meter (Rotameter)
Variable area flow meter

3- Rotameter

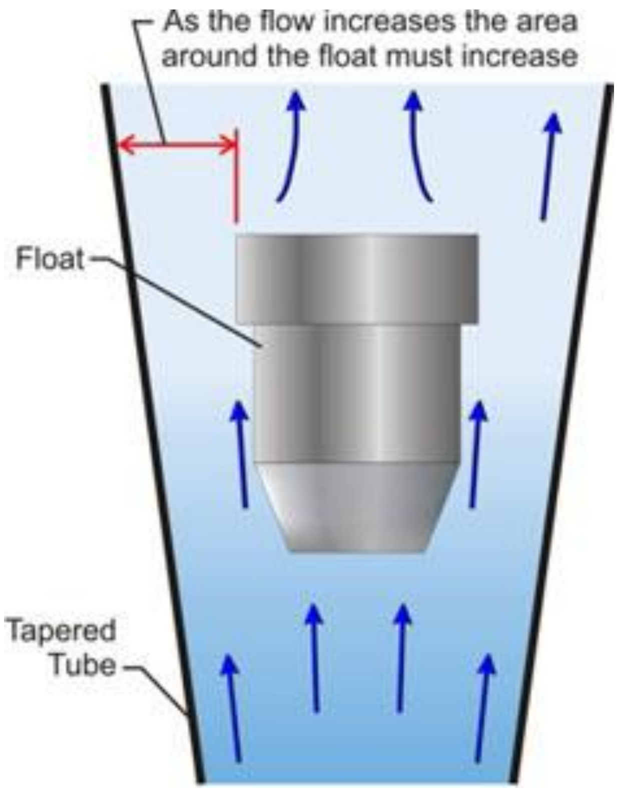


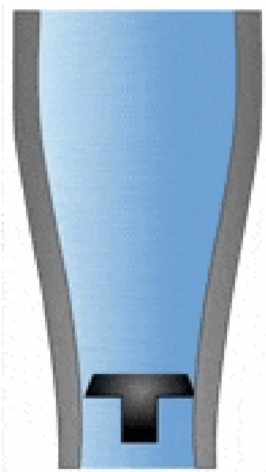


Taper tube

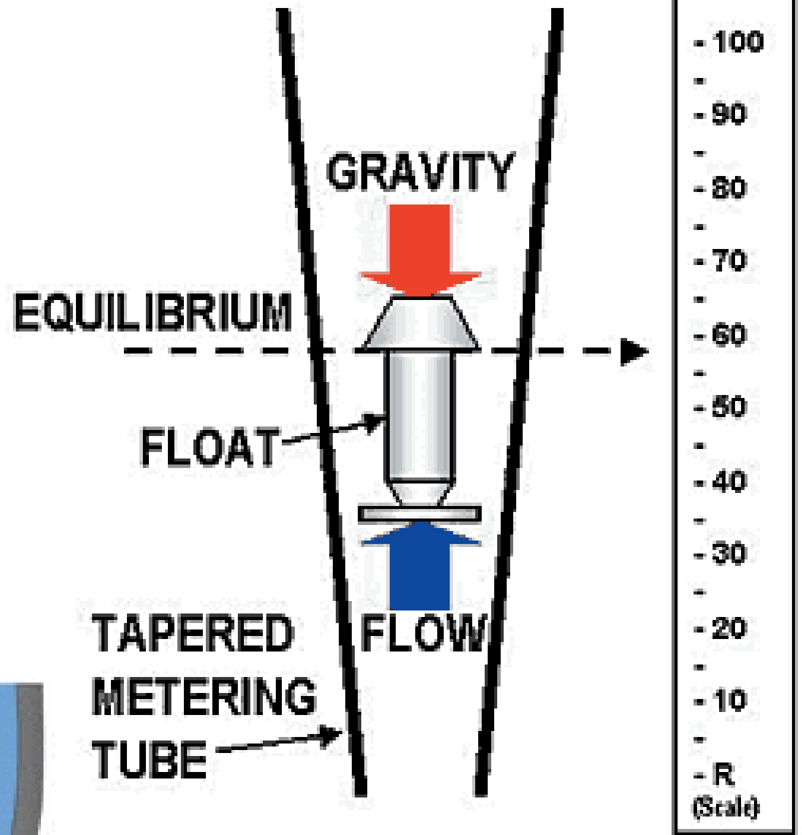
Float or Bob

3-Rotameter





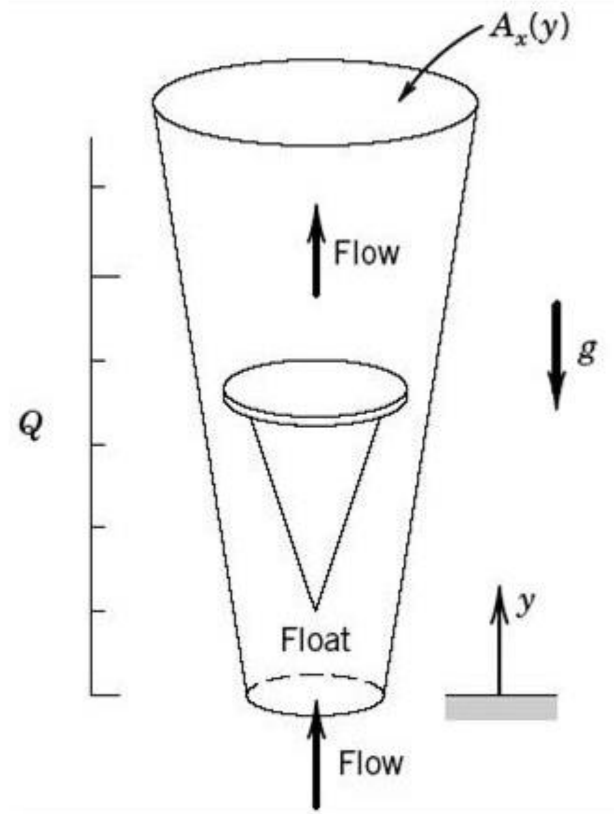
Flow rate
Scale



3-Rotameter

Basic idea

Balance of the float. As the velocity increases, the height of the float increases, and the flow area increases

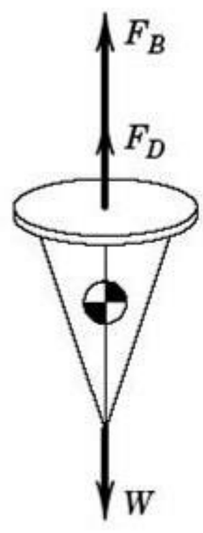


Three forces acting on the float

1-Wieght, W

2-Buoyancy, F_B

3-Drag, F_D

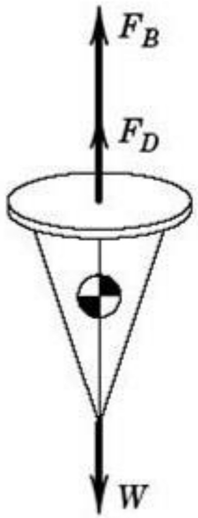


3-Rotameter

$$W = mg = \rho_b V_b g$$

$$F_B = \rho_f V_b g$$

$$F_D = C_D \rho_f \overline{U}^2 A_b / 2$$



At equilibrium $$W = F_D + F_B$$

$$\rho_b V_b g = \rho_f V_b g + C_D \rho_f \overline{U}^2 A_b / 2$$

$$\overline{U} = \sqrt{2(\rho_b - \rho_f) g V_b / C_D \rho_f A_b}$$

ρ_f=fluid density, ρ_b = float (body) density

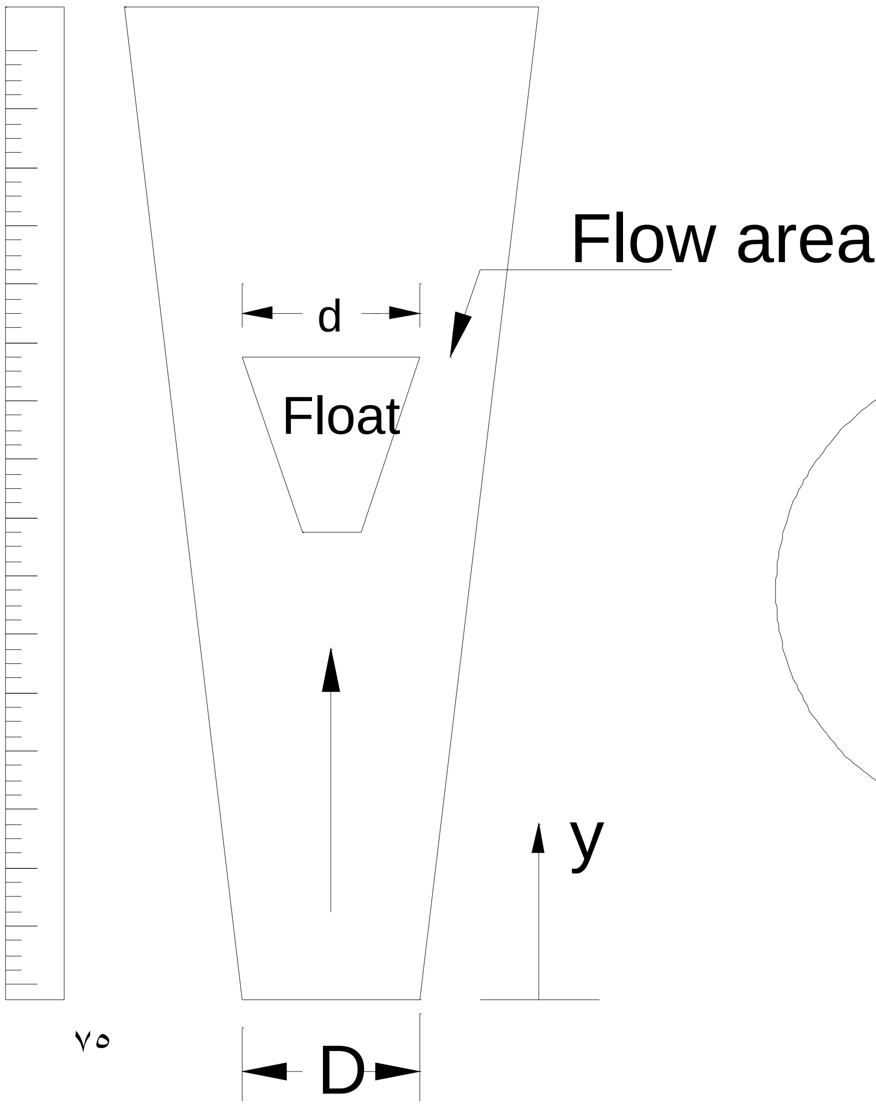
A_b is the frontal area of the float

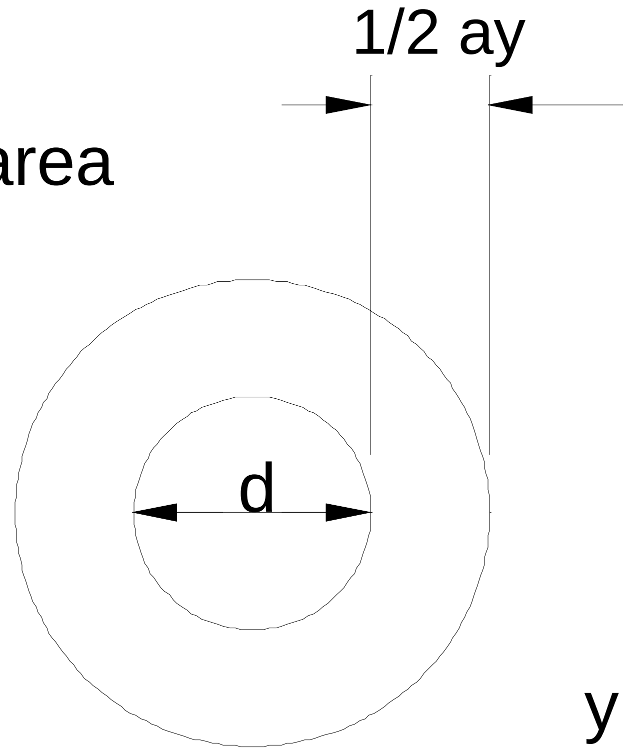
C_d Drag coefficient

3- Drag flow meter (Rotameter)

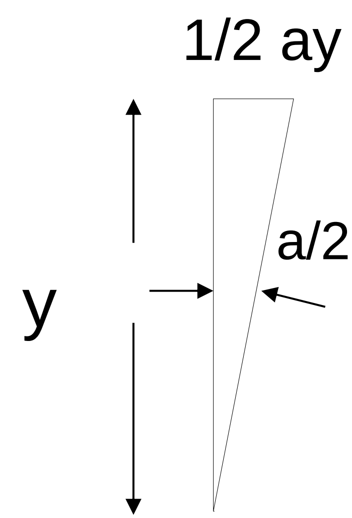
$$A=\frac{\pi}{4}\left[(D+ay)^2-d^2\right]=\frac{\pi}{4}\left[D^2+2ay+a^2y^2-d^2\right]\approx\frac{\pi}{4}\left[2ay\right]$$

$$D\approx d$$





Top view



a= tube taper

3-Rotameter

The volume flow rate through the rotameter is given by

$$Q = A\overline{U} = A\sqrt{2(\rho_b - \rho_f)gV_b / C_D \rho_f A_b}$$

Where

$$A = \frac{\pi}{4}\left[(D+ay)^2 - d^2\right]$$

A is the flow area between the bob or float and the wall of the rotameter

$$A \approx \frac{\pi}{4}[2ay]$$

D=diameter of the tube at inlet (D≈d)

d= bob (float) max. diameter

$$Q = A\overline{U} \approx C_1 y\sqrt{(\rho_b - \rho_f)/\rho_f}$$

C_1=Constant

For a rotameter that gives an indication that is independent of fluid density

$$\frac{\partial \dot{m}}{\partial \rho_f} = 0$$

Which gives

$$\rho_b = 2\rho_f$$

Therefore

$$\dot{m} \approx \frac{C_1 y \rho_b}{2}$$

3-Rotameter

$$Q = A\overline{U} = C_1 y\sqrt{(\rho_b - \rho_f)/\rho_f}$$

A is the annular flow area (between the float and the tube

C_1 is the meter constant

The flow rate depends on y

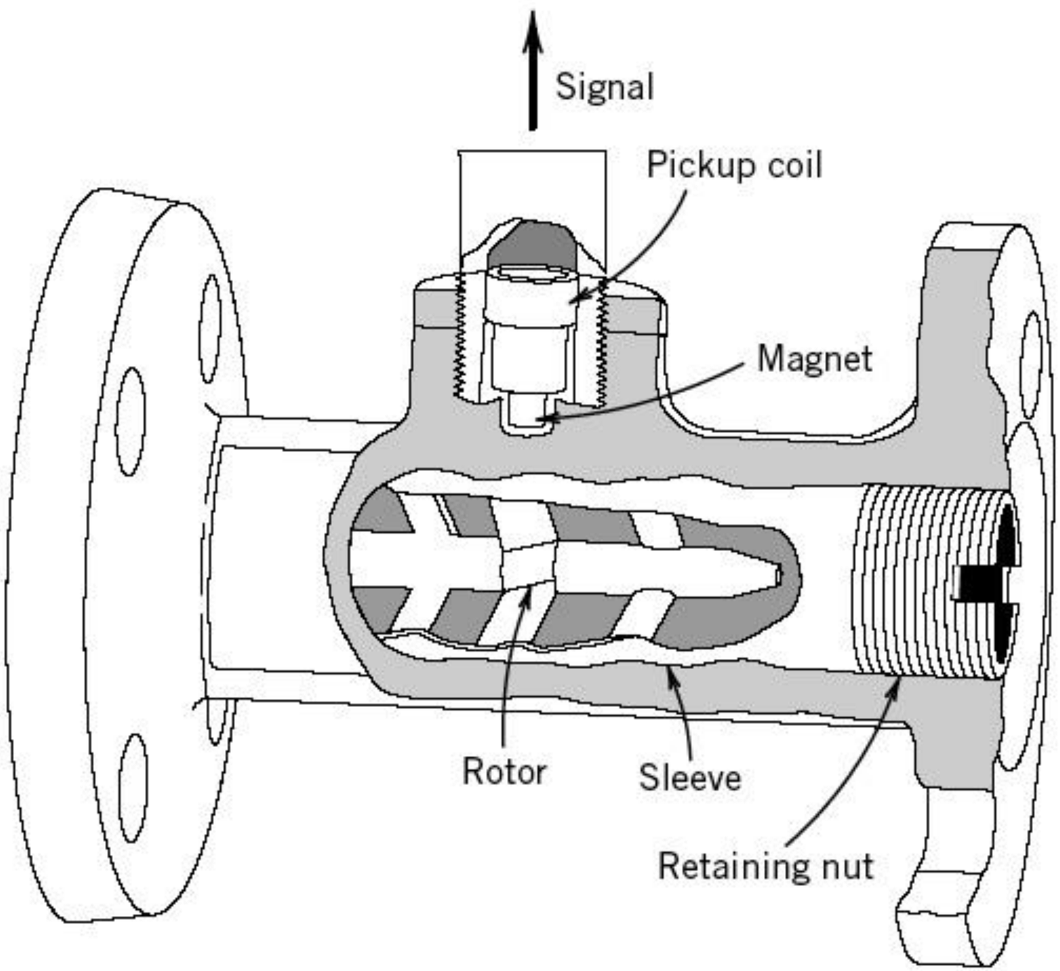
The float vertical position y is a direct measure of Q

Float with sharp edges are less sensitive to fluid viscosity changes with T.

Typical meter turndown is 10:1

Systematic uncertainty $\pm$2 % of the flow rate

4-Turbine flow meter



Basic idea

Flow due to angular momentum will rotate a rotor. The rotational speed is proportional to the flow rate of the fluid.

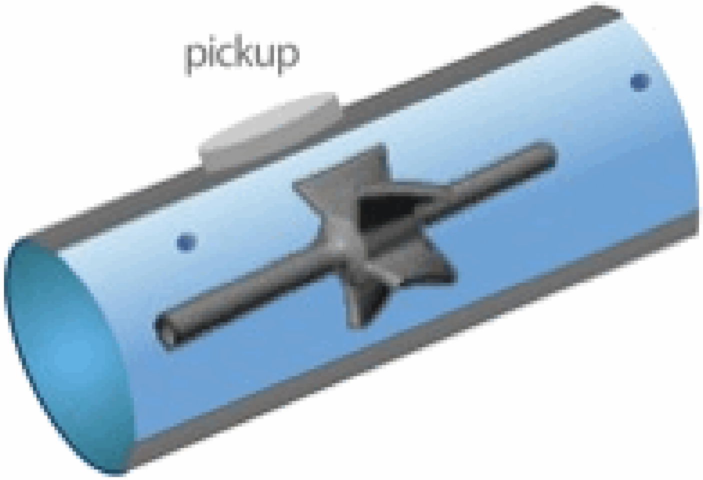


Figure 10.19 Cutaway view of a turbine flow meter. (Courtesy of Actaris Gas Division, Owenton, KY.)

4-Turbine flow meter

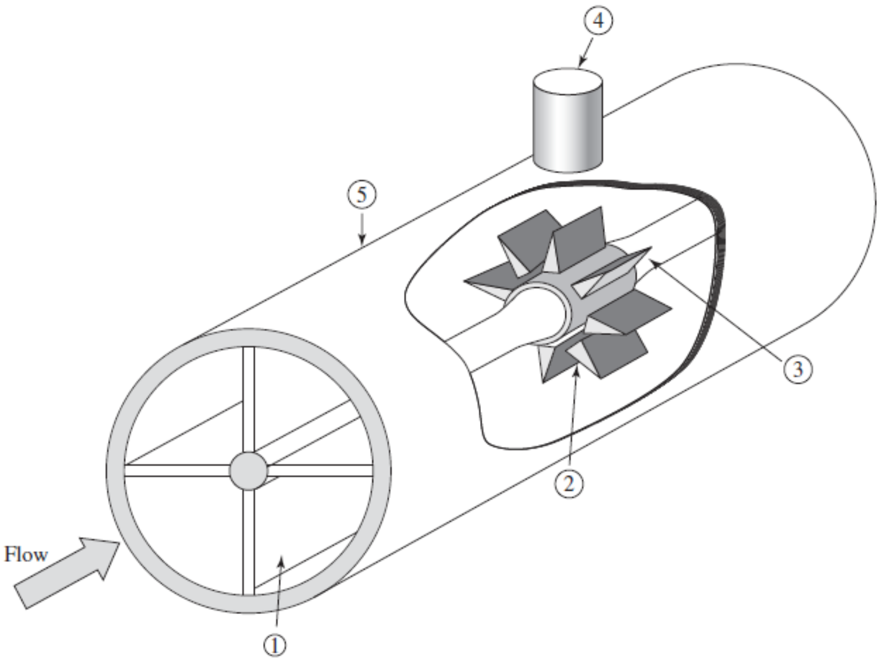
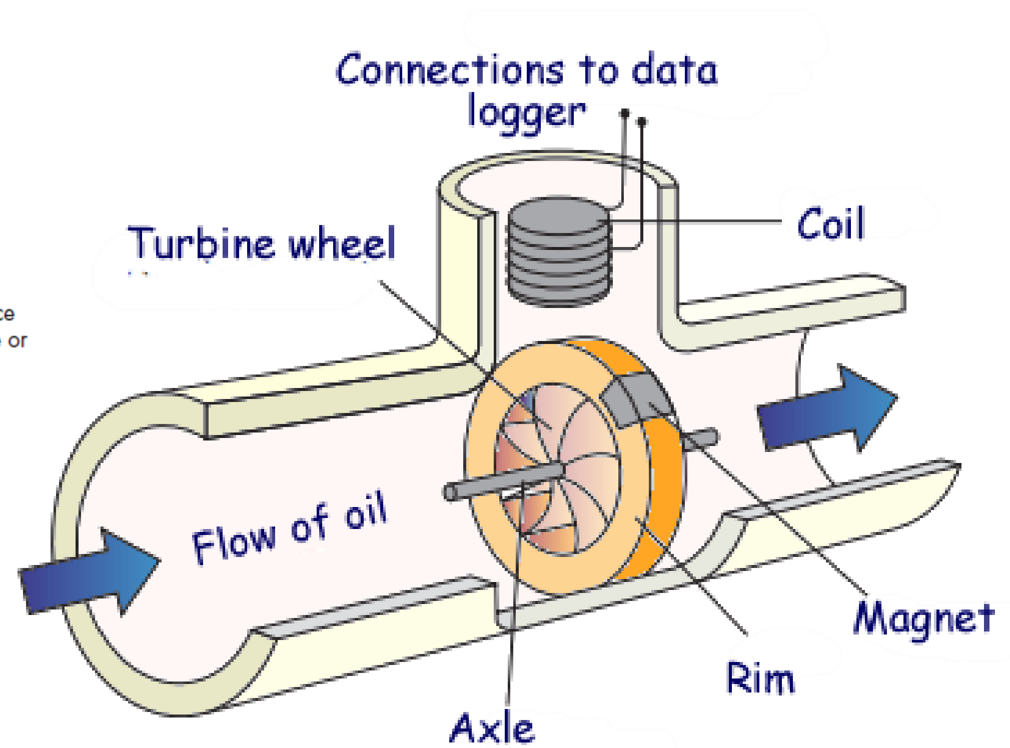
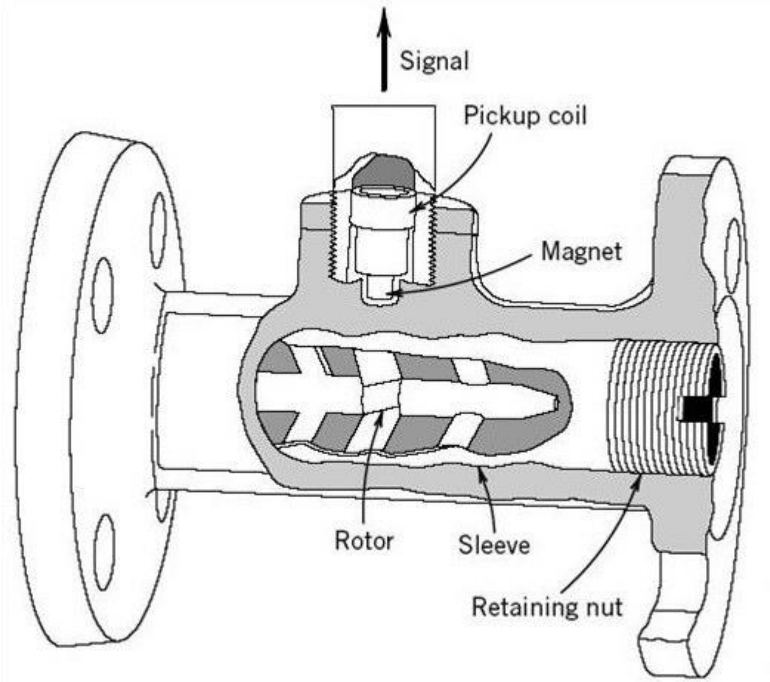


Figure 7.17 Schematic of turbine meter. (*1*) Inlet straightening vanes, (*2*) rotating turbine blades with embedded magnet, (*3*) smooth afterbody to reduce pressure drop, (*4*) reluctance pickup, (*5*) meter body for insert in pipe or flow channel.



4-Turbine flow meter



Make use of angular momentum principle

As the fluid velocity increases, the rotational speed ω increases

The coil can sense the passage of magnetic rotor blades producing pulse signal at frequency that proportional to the rotational speed of the rotor.

$$Q = K_1 \omega$$

K_1 is the K factor for the meter.

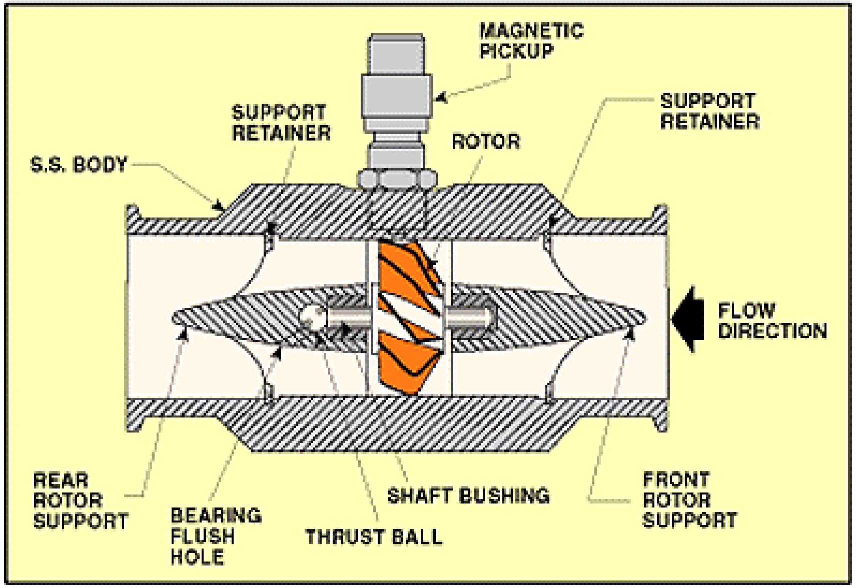


Figure 5: Turbine flowmeter consists of a multiple-bladed, free-spinning, permeable metal rotor housed in a non-magnetic stainless steel body. In operation, the rotating blades generate a frequency signal proportional to the liquid flow rate, which is sensed by the magnetic pickup and transferred to a read-out indicator

4-Turbine flow meter

Remarks about Turbine flow meters

Low pressure loss and very good accuracy

Typical instrument uncertainty is ±0.25% of flow rate

Turndown 20:1

Good candidate as local flow standard

Use for clean fluid

Rotational speed is sensitive to temperature changes

Careful installation is required to avoid errors caused by flow swirl

5-Transient Time and Doppler (ultrasonic) flow meters

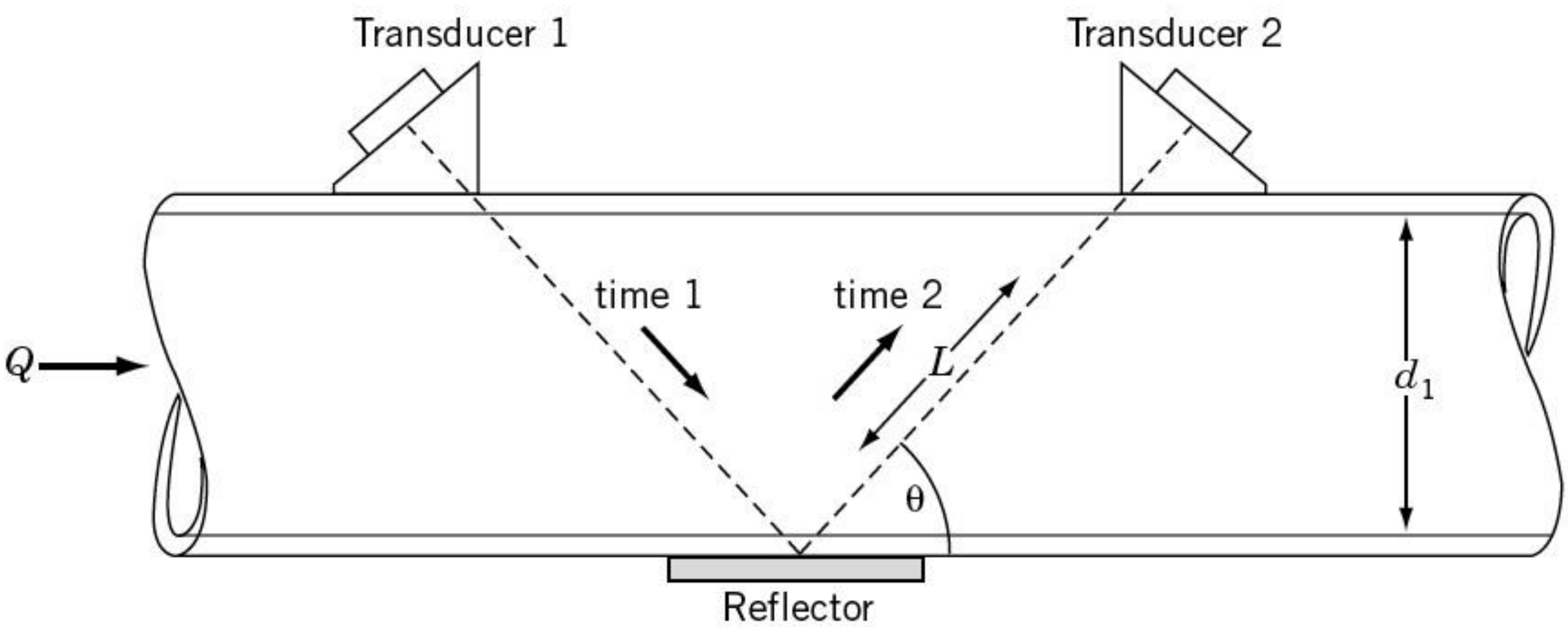


Figure 10.20 Principle of a transit time (ultrasonic) flow meter.

Ultrasonic meters use sound wave to determine flow rate.

5-Transient Time and Doppler (ultrasonic) flow meters

A pair of transducers separated by a distance are fixed outside the tube.

A reflector is installed at the opposite side of the tube

Each transducer act as a transducer and a receiver.

Ultrasonic wave is emitted by one transducer and receive by the other transducer

The difference in time is directly related to average velocity of the flow

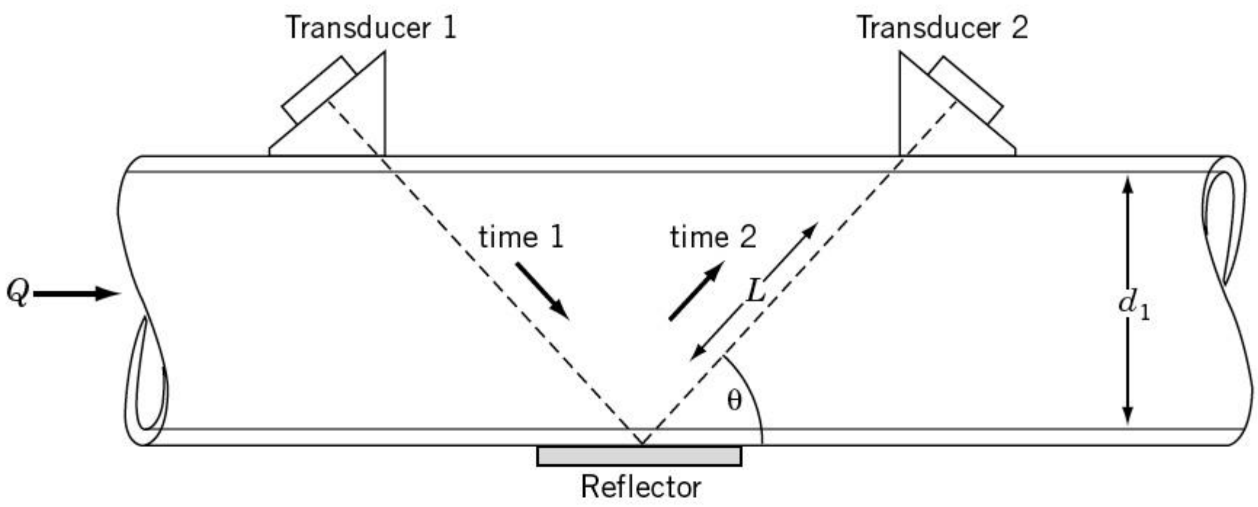


Figure 10.20 Principle of a transit time (ultrasonic) flow meter.

5-Transient Time and Doppler (ultrasonic) flow meters

Times measured by the transducers

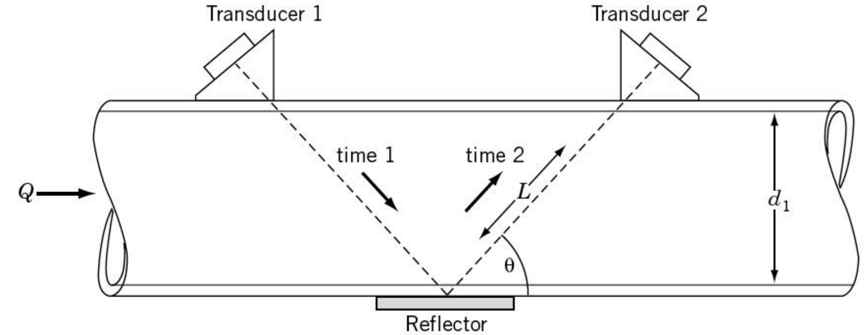


Figure 10.20 Principle of a transit time (ultrasonic) flow meter.

$$t_1 = \frac{2L}{a + \overline{U}\cos(\theta)} \qquad t_2 = \frac{2L}{a - \overline{U}\cos(\theta)}$$

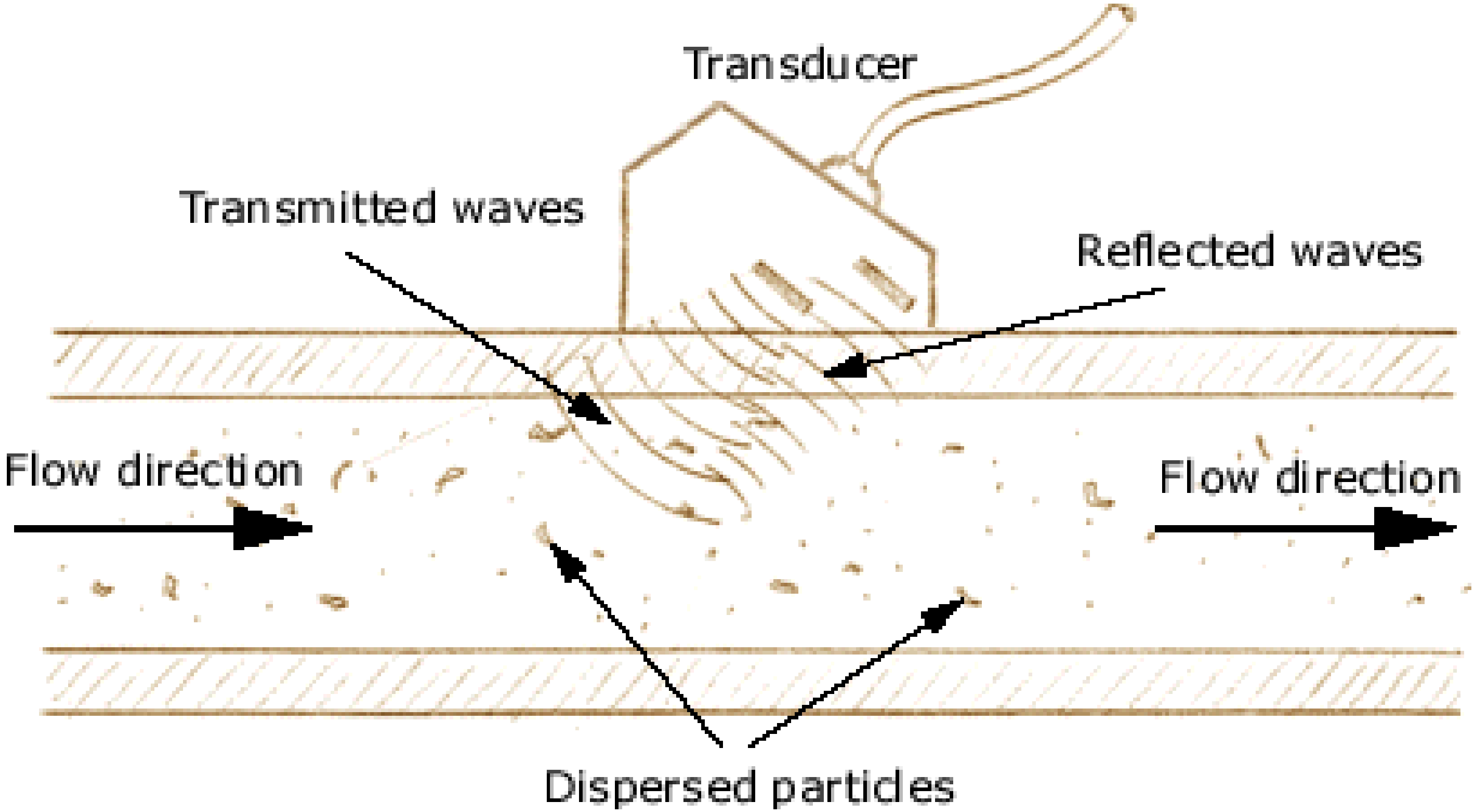
Knowing t_1 and t_2 one can get the velocity U

$$Q = \overline{U}A = \frac{K_1 \pi d_1 a^2 (t_2 - t_1)}{16\cot(\theta)}$$

a= speed of sound in fluid

5-Transient Time and Doppler (ultrasonic) flow meters

Doppler flow meters



5-Transient Time and Doppler (ultrasonic) flow meters

Doppler flow meters

Here the average velocity of the fluid particles (contaminants) suspended in the fluid is measured using sound waves.

A wave with frequency f is emitted to the flow

Due to the speed of the particles the received frequency is going to be different than the sent frequency. The difference between the two frequencies is called Doppler shift f_d

$$Q = \overline{U}A = \frac{\pi d_1^2 a f_d}{8 f \cos(\theta)}$$

Uncertainty ~2 %

a=speed of sound

6-Positive displacement meters

These meters contains mechanical elements with known volume

As the fluid move through these volumes, volume of the fluid displaced is measured through knowing the number of times these elements were it turned

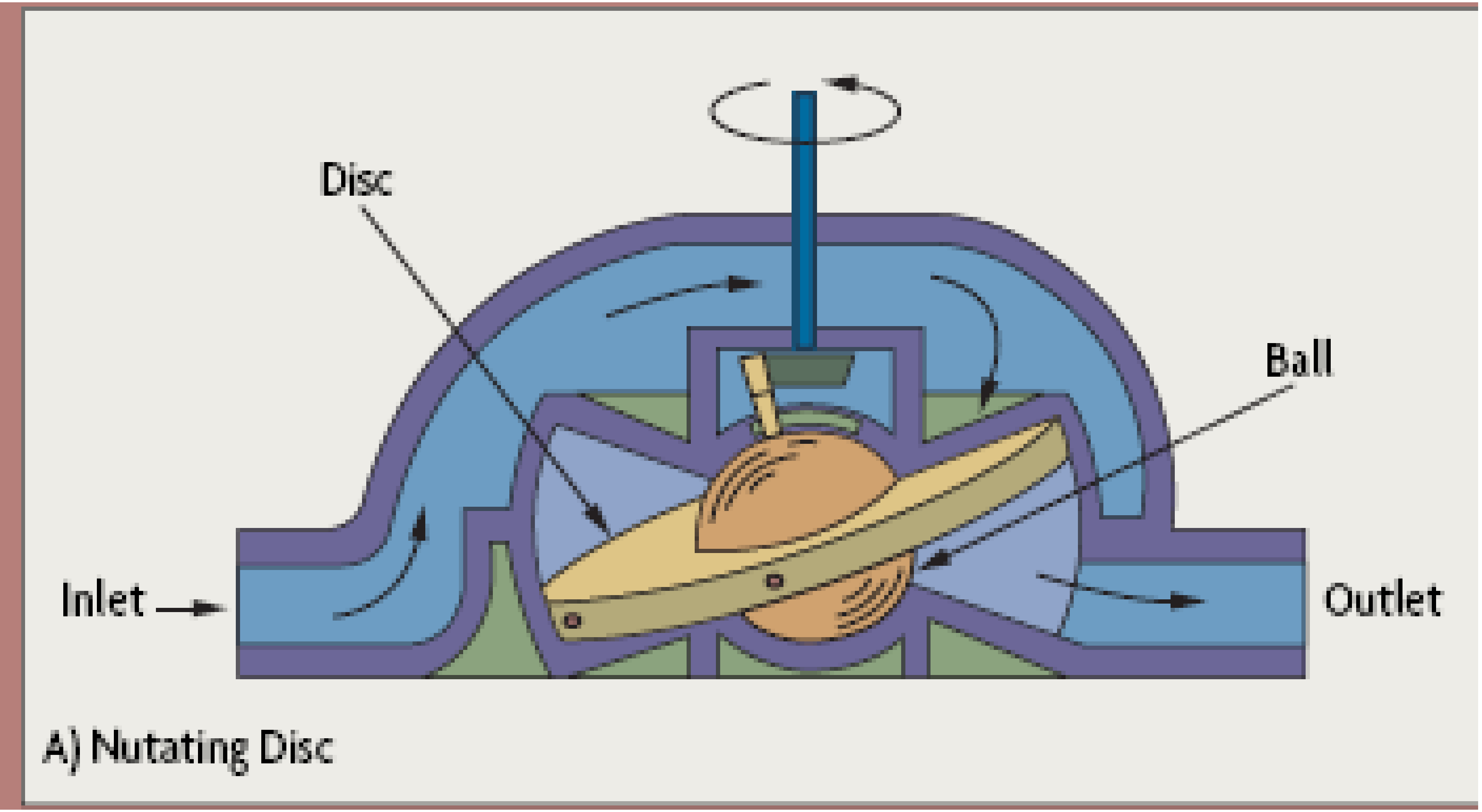
If the time is also recorded then the flow rate is given by

$$Q = V / t$$

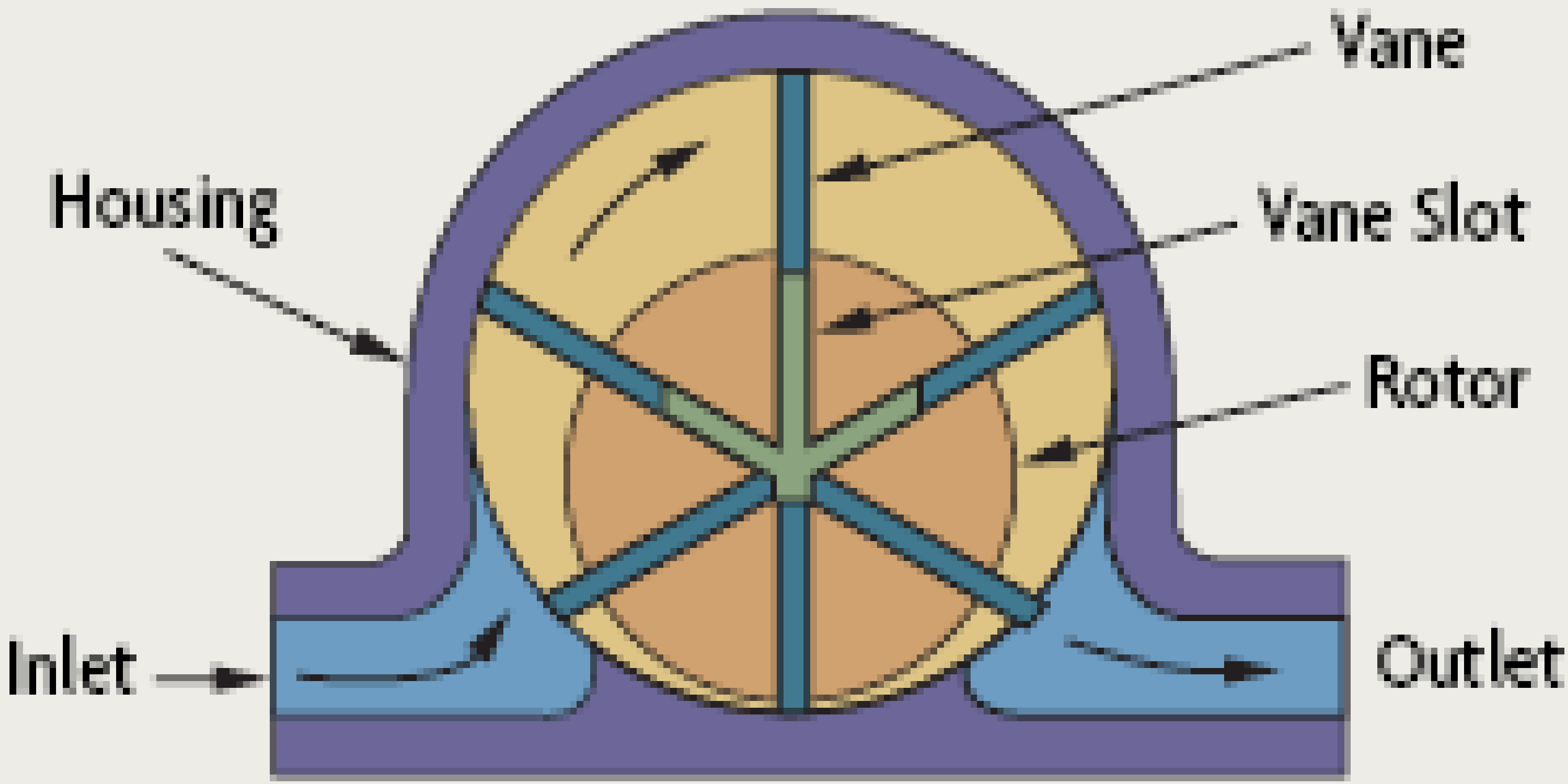
Uncertainty for these flow meter is low as $\pm$0.2 %

Common types: a) Nutating disk b) rotating vane

6-Positive displacement meters

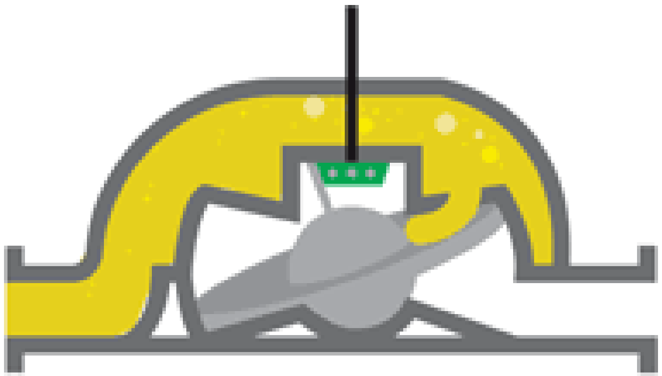


6-Positive displacement meters



B) Rotating Valve

6-Positive displacement meters



Nutating disk

Rotating vane

Thermal Mass flow meters

mass flow rate $\dot{m} = \rho U A$

In some application the density is not known exactly, so it is better to find the mass flow rate.

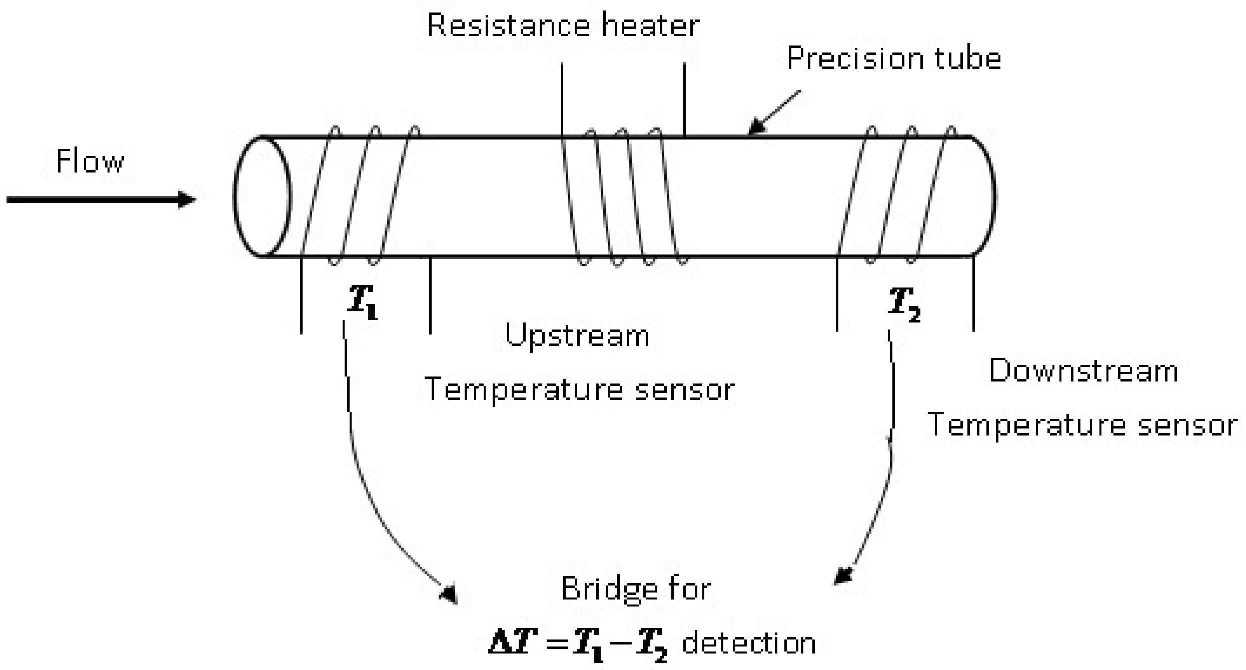
Basic two methods to measure the mass flow rate

1-Thermal flow meter

2-Coriolis flow meter (**Excluded**)

Thermal mass flow meter

Basic idea: heat is added to the flow, measure temperature rise, the heat input and then use



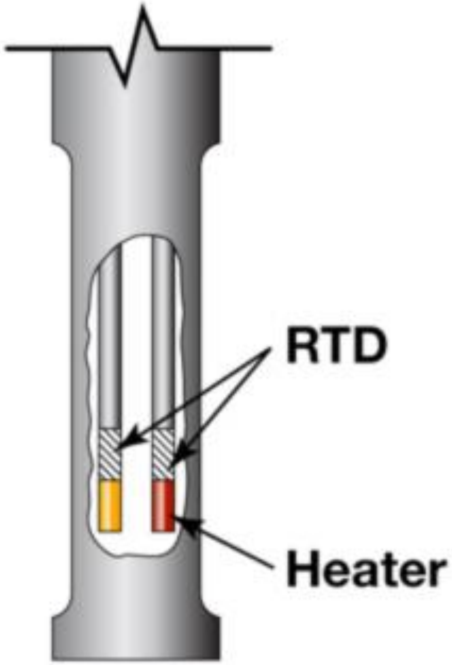
$$E = \dot{m} C_p \Delta T$$

Use above equation to find $\dot{m}$

Thermal mass flow meter

Another idea of a thermal mass flow meter using hot film anemometer

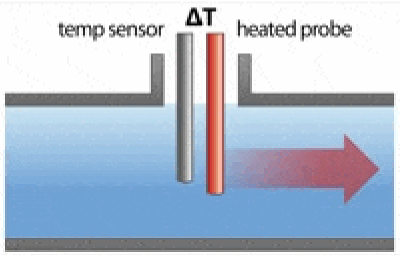
Use of a hot film anemometer with RTD upstream of the flow



if T_f is the film temperature, and T_s is the RTD sensor temperature, one may use

$$E = \left[C + B(\rho U)^{1/n} \right] (T_f - T_s)$$

Again the heat input equals the mass flow rate times the specific heat times the temperature difference



C, B and n are calibration constants

Flow meter calibration and standards

Establishing a steady flow in a loop, and then determine the volume or mass passing through the meter in a given time.

Flow meter calibration and standards

Liquid fluid

Uncertainty ±0.03%

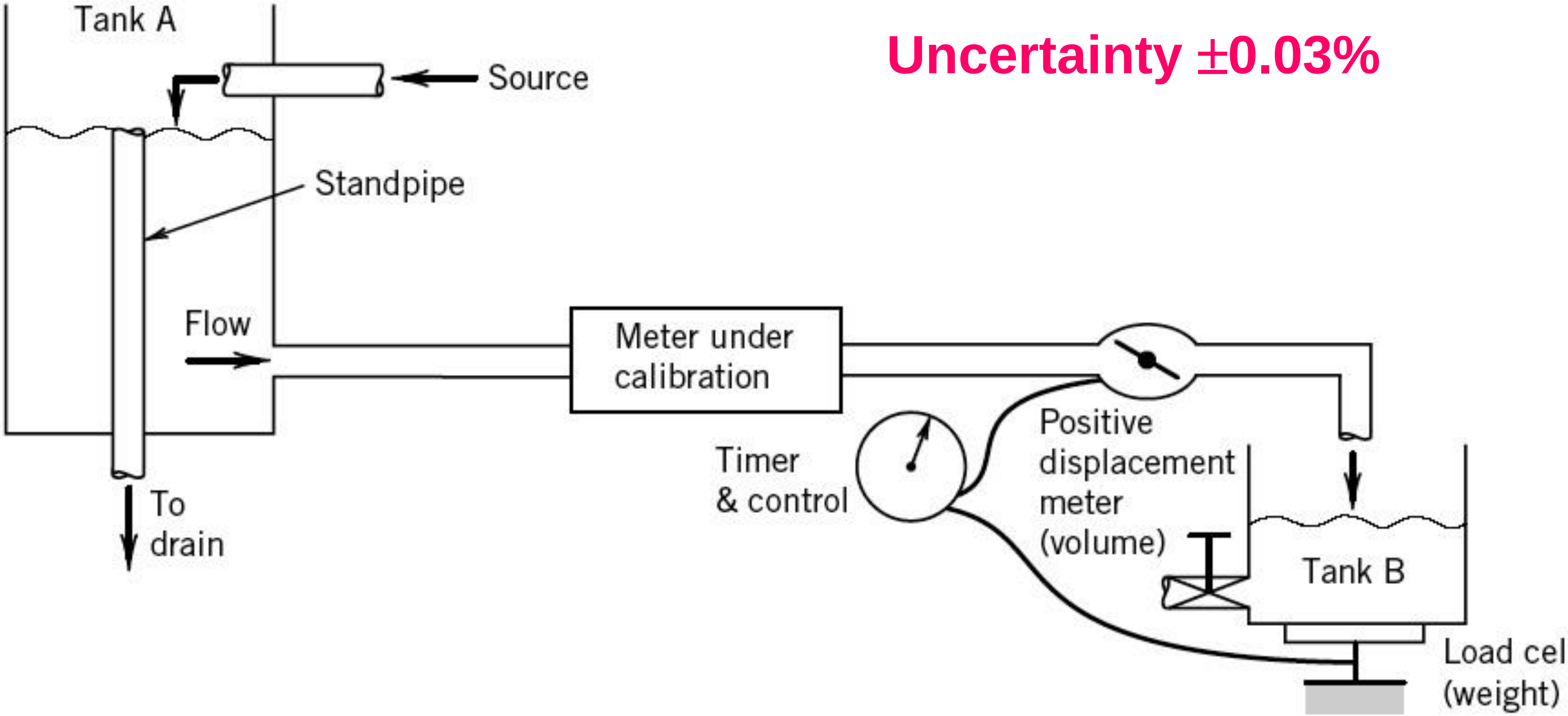


Figure 10.24 Flow diagram of a flow meter prover for liquids.

Estimating standard flow rate

Mainly for Gases (especially air)

The mass flow rate is constant whether it is standard or actual

$$Q_s \rho_s = Q_a \rho_a$$

Using ideal gas equation of state $\rho = \frac{P}{RT}$

$$Q_s = Q_a \frac{\rho_a}{\rho_s} = Q_a \frac{P_a}{T_a} \frac{T_s}{P_s}$$

standard condition T_s=20° C, P=760 mm Hg

Standard flow rate

ACMM and SCMM

Actual volume flow rate Q_a in m^3/min or ACMM or in ft^3/min [CFM]

Standard volume flow rate Q_s in m^3/min or SCMM or ft^3/min, [CFM]

SCMM=Standard Cubic Meter per Minute

SCFM=Standard Cubic Feet per Minute

ACMM=Actual Cubic Meter per Minute

ACFM=Actual Cubic Feet per Minute