MEP 365

Thermal Measurements

Ch. 9 Pressure & Velocity Measurements

March 2017

Contents

1-Introduction

2-Reference instruments

- **3-Pressure transducers**
- 4-Pressure measurement in moving fluid

5-Fluid velocity measuring systems

Static & Dynamic pressure measurements

Prandtl & Pitot tube

Thermal anemometry

Ultrasonic velocity measurement

-Time transit

-Doppler effect

PIV (Particle Image Velocimetry)

1-Introduction

Pressure =Force/area=N/m²=Pa

$$P = \frac{F}{m^2} = Pa$$

As altitude increases, the pressure decreases



Standard atmospheric pressure at sea level

1atm= 101.32 kPa

14.696 psi

1.013 bar.

101.32*1000=ρ**gh**

h=

760 mm Hg 10350.8 mm H₂O 29.92 in Hg 407.523 in H₂O

1 Torr = 1mm Hg =13600*9.81*(1/1000)= =133.4 Pa

Gage and absolute pressure

$$P_{gage} = P_{abs} - P_{atm}$$



 $P_{gage} = P_{abs} - P_{atm}$

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Hydrostatic pressure variation with depth



2-Pressure Reference instruments

McLeod Gauge Barometer Manometer Deadweight tester

McLeod Gauge



(a) Sensing position

A gas of unknown pressure is trapped at known volume when the gauge is inverted

Used for sub





$$p_{1}V_{1} = p_{2}V_{2}$$

$$p_{2} = p_{1} + \gamma y$$

$$V_{2} = yA$$

$$p_{1} = p_{2} - \gamma y$$

$$p_{1} = p_{1}(V_{1}/V_{2}) - \gamma y$$

$$p_{1}(V_{1}/V_{2} - 1) = \gamma y$$

$$p_{1} = \frac{\gamma A y^{2}}{V_{1} - yA}$$
(b) Indicating position

Barometer



Barometer

Fortin Barometer





Manometers







 $p_1 - p_2 = (\gamma_m - \gamma)H$

K=Sensitivity of the manometer=Output/input

$$K = \frac{\Delta H}{\Delta p} = \frac{1}{\gamma_m - \gamma}$$

Figure 9.5 U-tube manometer.

Micro-manometers

Used for measuring a very small differential pressure down to 0.005 mm H_2O

Micro-manometers



Used to measure small differential pressure up to $0.005 \text{ mm H}_2\text{O}$

Micrometer adjusting screw

> The reference position: The reservoir is adjusted up or down until the level of the manometer within the reservoir is the same as the set mark. Bringing the reservoir level back to the set mark (i.e. H) is a measure due to pressure difference

> > Figure 9.6 Micromanometer.

Inclined manometer

Used to measure small pressure difference.

One of the legs of the manometer is inclined (10 -30°) with the horizontal



Figure 9.7 Inclined tube manometer.





 $L = H / \sin \theta$ $\Delta P \approx \gamma L \sin \theta$ Sensitivity = $K = \frac{\Delta L}{\Delta p} = \frac{1}{\gamma \sin \theta}$

 θ is the angle measured from horizontal.

Elementary error associated with manometers

Scale adjustment error Zero error Temperature error Gravity error Capillary Meniscus error

Correction for mercury specific weight as a function of temperature

$$\gamma_{Hg} = \frac{133.084}{1 + 0.00006T}$$
 N/m³ T in °C

$$\gamma_{Hg} = \frac{848.707}{1+0.000101(T-32)}$$
 lb/ft³ T in °F

Gravity correction

$$e_1 = -(2.637 * 10^{-3} \cos(2\phi) + 9.6 * 10^{-8} z + 5 * 10^{-5})$$
 z in feet

$$e_1 = -(2.637 * 10^{-3} \cos(2\phi) + 2.9 * 10^{-8} z + 5 * 10^{-5})$$
 z in meter

 ϕ is the altitude in degrees

Capillary error can be reduced if a bore of diameter of 6 mm or greater is used.

General manometer uncertainty can be as low as 0.02-0.2%

Example 9.3

Inclined manometer, θ =30° is used to measure air pressure, nominal pressure =100 N/m^{2.}, u_{θ}=1 deg., γ_m =9770±0.5% manometer resolution 1 mm. manometer zero error = interpolation error. Estimate uncertainty in pressure

$$\Delta p = p_1 - p_2 = L(\gamma_m - \gamma) \sin(\theta)$$

For $\Delta p = 100 \text{ N/m}^2$
$$L = \frac{\Delta p}{(\gamma_m - \gamma) \sin(\theta)} = 21mm$$
$$u_{\Delta p} = \left[\left(\frac{\partial \Delta p}{\partial \gamma} u_{\gamma} \right)^2 + \left(\frac{\partial \Delta p}{\partial L} u_L \right)^2 + \left(\frac{\partial \Delta p}{\partial \theta} u_{\theta} \right)^2 \right]^{1/2}$$

$$u_{\Delta p} = \left[\left(\frac{\partial \Delta p}{\partial \gamma_m} u_{\gamma_m} \right)^2 + \left(\frac{\partial \Delta p}{\partial L} u_L \right)^2 + \left(\frac{\partial \Delta p}{\partial \theta} u_{\theta} \right)^2 \right]^{1/2}$$

 $u_{\gamma_m} = 9770 * 0.5 / 100 = 49 N / m^3$ $u_L = \sqrt{u_o^2 + u_c^2} = \sqrt{0.5^2 + 0.5^2} = 0.7 \text{ mm}$

$$u_{\theta} = 1^{\circ} = 0.0175 \text{ rad}$$

$$T_1 = \frac{\partial \Delta P}{\partial \gamma_m} u_{\gamma_m} = L\sin(\theta) * u_{\gamma_m} = 0.51 \qquad T_2 = \frac{\partial \Delta P}{\partial L} u_L = (\gamma_m - \gamma)\sin(\theta) * u_L = 3.42$$

$$T_{3} = \frac{\partial \Delta P}{\partial \theta} u_{\theta} = (\gamma_{m} - \gamma) L \cos(\theta) * u_{\theta} = (21/1000) * \cos(30) * 0.0175 = 3.1$$

$$u_{\Delta p} = \sqrt{0.51^2 + 3.42^2 + 3.1^2} = 4.6 \,\mathrm{N/m^2}$$

Dead weight tester

Use the fact that P=F/A

Used for calibration of pressure sensors 70- $70*10^7 \text{ N/m}^2$

Uncertainty 0.05 to 0.01 % reading

$$P = \frac{F}{A} + \sum errors$$

Dead weight tester



Dead weight tester





Errors associated with deadweight tester

- 1-Air buoyancy
- 2-Variation in local gravity
- 3-Unceratinty in mass of pistons and weights
- 4-Shear effect (piston with cylinder movement)
- 5-Thermal expansion of piston area
- 6-Elastic deformation of piston

Simple correction for errors

$$p = p_i(1 + e_1 + e_2)$$

 P_i indicated pressureFor Jeddah
Location e_1 error due to gravity variationz=17 m
 $\phi=21.7 \text{ N}$

 $e_1 = -(2.637 * 10^{-3} \cos(2\phi) + 2.9 * 10^{-8} z + 5 * 10^{-5})$ z in meter

 $e_1 = -(2.637 * 10^{-3} \cos(2\phi) + 9.6 * 10^{-8} z + 5 * 10^{-5})$ z in feet

Example 9.4

 p_i =100.00 psi, ϕ =34°, z=841 feet, γ_{air} =0.076 lb/ft³, γ_{mass} =496 lb/ft³.Correct the indicated pressure

$$e_2 = -\gamma_{air} / \gamma_{mass} = -0.076 / 496 = -0.000154$$

$$e_1 = -(2.637 * 10^{-3} \cos(2\phi) + 9.6 * 10^{-8} z + 5 * 10^{-5})$$
 z in feet

e₁=-0.001119

p=100.00*(1-0.000154-0.001119)=99.87 lb/in²

3- Pressure transducers

Bourdon tube Bellows Diaphragms Capsule

3-Pressure Transducers



- Bourden tube
- Diaphragm
- Capsule
- Bellows

• Potentiometer

• LVDT

LVDT-Linear Variable Differential Transducer

3-Pressure Transducers



Capsule

Bourdon tube





Bourdon tube

Bourdon tube



37
Bourdon tube



Bellows



As p_1-p_2 increases the displacement increases

Use secondary element to translate the motion into a an rA electrical signal

Converting the linear movement into electrical signal



Potentiometer pressure Transducer

Voltage divider circuit-Excluding meter loading error



Figure 6.9 Voltage divider circuit.

$$E_{AB} = E_i \, \frac{R_x}{R_T}$$

Potentiometer circuit with loading error

$$I = \frac{E_i}{\left(R_2 + R_{eq}\right)}$$

$$I = \frac{E_i}{\left[R_2 + \frac{R_1 R_m}{R_1 + R_m}\right]}$$

$$E_o = I * R_{eq}$$

$$\frac{E_o}{E_i} = \frac{\frac{R_1 R_m}{R_1 + R_m}}{\left[R_2 + \frac{R_1 R_m}{R_1 + R_m}\right]}$$



Potentiometer circuit with loading error



loading effect make R_m as large as possible

Diaphragm pressure transducer

Diaphragm pressure transducer



Diaphragm deforms due to pressure difference acting at both sides

Diaphragm can be used for either static or dynamic pressure measurements Use stain gauge to covert diaphragm movement into electrical signal

Strain Gage

Strain gages are the most popular electrical elements used in force or pressure measurements. The strain gage measures pressure indirectly by measuring the deflection it produces in a calibrated primary sensor. The resistance strain gage is a resistive element, which changes in length, hence resistance, as the force applied to the base on which it is mounted causes stretching or compression



Strain gage



Bonded strain gage

Use stain gauge to covert diaphragm movement into electrical signal



R₁ N V strain gauge



Quarter-bridge strain gauge circuit

Use of strain gauges for pressure measurements



Another means to convert the mechanical movement of a primary pressure transducer into electrical signal





AC Voltage is supplied to the primary coil. The amplitude output from the secondary coils is proportional to the core movement.



AC Voltage is supplied to the primary coil. The amplitude output from the secondary coils is proportional to the core movement.

For short core displacement the voltage output from the secondary coil is proportional to the displacement x



For additional information about LVDT see Ch. 12 in your book or search the internet



Capacitance pressure transducer



Figure 9.13 Capacitance pressure transducer. In this schematic, the diaphragm is conductive and its deflection exaggerated.

Capacitance pressure transducer



Figure 9.14 Capacitance pressure transducer.

Capacitance pressure transducer

The capacitance changes with t and area A

$$C = c \epsilon A / t$$

C=capacitance in Farad

 ϵ =permittivity (=8.85E-12 F/m)

A is the overlapping area of the two plates

c=dielectric constant (c=1 for air,c=80 for water)



Figure 9.14 Capacitance pressure transducer.

t is the distance between the two plates

Sensitivity

$$K = \frac{\partial C}{\partial t} = -\frac{c\varepsilon A}{t^2}$$

The measured voltage will be proportional with the separation distance t $E_o = E_i \frac{C_1}{C}$

Piezoelectric crystal elements



A piezoelectric disk generates a voltage when deformed

Piezoelectric crystal elements

Generally used for transient (dynamic pressure measurement)

Under compression, tension or shear the crystal will deform and develop a surface charge q which is proportional to force acting

The relation between the charge develop and the pressure is given by

 $q = K_q p A$ K_q is the sensitivity coefficient

voltage develop across the electrode $E_{_{o}}=q\,/\,C$

Using $C = c \epsilon A / t$ Gives $E_o = K_q p t / c \epsilon = K_E p t$

For Quartz, the most common material used K_q =2.2E-9 Coulombs/N

Piezoelectric crystal elements

The charged produced due to deformation is a measure of pressure



Figure 9.15 Piezoelectric pressure transducer.

Cross section in a piezoelectric crystal pressure sensor

pressure measurement in moving fluid



Figure 9.17 Streamline flow over a bluff body.

Streamlines A and B

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flow can not be perpendicular to stream line

pressure measurement in moving fluid



Figure 9.17 Streamline flow over a bluff body.

U₂=0.0 Stagnation point

$$p_1 + \rho U_1^2 / 2 = p_2 = p_t$$

The term $ho U_1^2/2$ is called dynamic pressure

pressure measurement in moving fluid



Figure 9.17 Streamline flow over a bluff body.

pressure at location 1, 3, and 4 is called static pressure. It is the pressure sensed by fluid particle as it moves with same velocity as local flow

since $U_4 > U_3$ due to mass conservation. $p_3 > p_4$

Total pressure measurement



$$p_t = p_s + \rho U^2 / 2$$

Static pressure measurement



Static pressure measurement

Improved Prandtl static tube Flow • flow must be 8 Taps smooth around the probe Support stem frontal area of the probe should not exceed 5% of pipe flow area

transducer



Figure 9.20 Improved Prandtl tube for static pressure. (a) Design. (b) Relative static error along tube length.









Pitot tube

correction for viscous flow i.e. $Re_r < 500$

$$\operatorname{Re}_{r} = \frac{Ur}{v} \quad \text{r the probe radius}$$
$$p_{v} = C_{v} p_{i}$$

 p_i is the indicated pressure and $C_{\rm v}$ is the correction factor given by

$$C_v = 1 + (4/\text{Re}_r)$$

High velocity flow

Stagnation temperature

assuming isentropic process

Define Mach number M as

$$M = \frac{U}{a}$$

$$\frac{U^2}{2} = C_p (T_t - T_x)$$
$$\frac{T_x}{T_t} = \left(\frac{p_x}{p_t}\right)^{(k-1)/k}$$

 $a = \sqrt{kRT_x}$

$$p_{v} = p_{t} - p_{x} = \frac{1}{2} \rho U_{x}^{2} \left[1 + M^{2} / 4 + (2 - k)M^{4} / 24 + ... \right]$$


Anemo=wind

Anemometer=measurement of wind force and velocity

The heat transfer from an object at T_s subjected to a fluid at T_∞ is given by

$$Q = hA(T_s - T_\infty)$$

as the fluid velocity increases, the heat transfer increases. Therefore the heat can be related to the velocity

$$Q = I^2 R = A + BU^n$$

King's Law

A, B and n are constants

The basic idea of thermal anemometry is to subject a metallic temperature sensor to the flow, and relates the resistance to the temperature, and relates the heat transfer to the fluid velocity.

Relation between resistance $R_s = R_o \left[1 + \alpha (T_s - T_o)\right]$

If the resistance is kept constant. The current must be changed with velocity. The relation between heat and velocity is

$$Q = I^2 R = A + BU^n$$



Figure 9.26 Schematic of a hot-wire probe.

Hotwire anemometer shapes

Handheld hotwire anemometer





hot-wire

tungsten or platinum wire, L=1 to 4mm, d=1.5 to 15 μ m. It can be used in non-conducting media

hot-film

a thin $2\mu m$ platinum or gold film deposited on glass and covered with high thermal conductivity coating. The coating is to electrically insulated the film and for mechanical protection. It can be used in either conducting or non conducting media

Modes of operation



imeanzer

constant current

The resistance as well as the sensor temperature change while the current is fixed. Bridge voltage is a measure of velocity

constant resistance

Most common for velocity measurements. Resistance and temperature is kept constant. The circuit has a closed loop to adjust the voltage and therefore the current to bring the resistance to set point value.

applied voltage
$$E^2 = C + DU^n$$
 using an $E_1 = KU$

Constant current mode



Thermal anemometry showing constant current mode



The resistance is kept constant i.e. the temperature is kept constant. The applied voltage changes

Thermal anemometry showing constant resistance mode. The voltage E is proportional to velocity

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 $E^2 = C + DU^n$

Doppler effect

Sound wave



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Ultrasonic & Doppler effect velocity measurements

Frequency of sound for a source moving toward and a way from a receiver

Doppler effect



Sound or Light wave

Ultrasonic velocity measurements



The difference between the sent and received frequency is related to object velocity

$$C = \lambda f$$
 C=speed of sound, λ =wavelength, f=frequency
Period T=1/f

Doppler effect

Source is moving towards the receiver



Doppler effect

Source is moving away from the receiver



$$f_R = \frac{f}{1 + \frac{V_s}{c}}$$

Doppler effect General case

$$f_o = \frac{c \pm V_o}{c \mp V_s} f_s$$

 $+V_o$ when the observer is moving towards the source $-V_o$ when the observer is moving away from the source $-V_s$ when the source is moving towards the object $+V_s$ when the source is moving away from the object C is the speed of sound

 $f_{\rm o}$ is the observer frequency $f_{\rm s}$ is the source frequency

Ultrasonic & Doppler effect velocity measurements

Two ideas

Time transient (To be seen later in Ch. 10)

Doppler effect (Explained in Ch. 9)

Doppler Anemometry

Basic idea: The frequency of light or sound emitted from a source that is traveling toward or a way from the observer is shifted from its original value by an amount proportional to its speed (Doppler 1853)

Small particles are suspended in the fluid are used to generate Doppler effect. Either acoustic waves or light waves are used. If laser beam is used then it is called Laser Doppler Anemometer (LDA)

For an observer watching frequency from particles in the flow

$$f_s = f_i \pm f_D$$

where

f_s is the scattered light as seen by the observer

- f_i is the frequency of the incident beam (in the order of 10^{14} Hz)
- $_{\Lambda^{q}}$ f_D is the Doppler shift (in the order 10³-10⁷ Hz)

Doppler Anemometry



Laser Doppler Anemometry

Dual laser beam mode to overcome the difficulties in detecting the Doppler shift



Figure 9.27 Laser Doppler anemometer. Shown in the dual-beam mode of operation.

Doppler Anemometry

Relation between velocity, Doppler shift and wavelength

$$U = \frac{\lambda}{2\sin(\theta/2)} f_D$$

where

 $\boldsymbol{\lambda}$ is the wavelength of the beam

f_D is the Doppler shift

 $\boldsymbol{\theta}$ lines scattering angle, see the figure

speed of light c and relation with wavelength λ and frequency ν

$$c = \lambda f = 3*10^8 m/s$$

Doppler Anemometry

Example 9.11

Doppler laser anemometer λ =632.8 nm. θ =11° , f_D=1.41 MHz. Estimate the velocity U

$$U = \frac{\lambda}{2\sin(\theta/2)} f_D$$

$$U = \frac{632.8 \times 10^{-9}}{2\sin(11/2)} 1.41 \times 10^{6} = 4.655 \, m/s$$

Particle Image Velocimetry (PIV)

In this method the full field 2D velocity is measured.

The technique is based on tracking the displacement of particles

The image of particles that are suspended in the flow is captured by a camera at predefined frequencies

The images are recorded and the traveled distances by the particles are calculated. the velocity is found using

$$U = \frac{\Delta x_i}{\Delta t_i}$$

PIV Particle Image Velocimetry

2D laser sheet flashes on the particles and at the same time a camera takes images.

The travel distance can be found from the comparison of the two images.

Particle Image Velocimetry

