Workshop Solutions to Section 2.5 (1.5)

How to find the domain and range of the exponential function $f(x) = a^x$?

1- If $f(x) = c.a^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$
2- If $f(x) = -c.a^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$
3- If $f(x) = c.e^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$
4- If $f(x) = -c.e^{\pm x} \pm k$ where c and k are positive constants, then
$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

1) Find the domain of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	2) Find the range of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $R_f = (0, \infty)$
3) Find the domain of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	4) Find the range of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $R_f = (-3, \infty)$
5) Find the domain of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $D_f = \mathbb{R}$	6) Find the range of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $R_f = (-\infty, 5)$
7) Find the domain of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	8) Find the range of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $R_f = (1, \infty)$
9) Find the domain of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	10) Find the range of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $R_f = (0, \infty)$
11) Find the domain of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	12) Find the range of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $R_f = (-3, \infty)$
13) Find the domain of the function $f(x) = e^x + 1$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	14) Find the domain of the function $f(x) = \frac{1}{1-e^x}$. Solution: $f(x)$ is defined when $1 - e^x \neq 0$ $\Leftrightarrow \quad e^x \neq 1 \quad \Leftrightarrow \quad \ln e^x \neq \ln 1$ $\Leftrightarrow \quad x \neq 0$ $\therefore \quad D_f = \mathbb{R} \setminus \{0\}$

15) Find the domain of the function $f(x) = \frac{1}{1+e^x}$. Solution: $f(x)$ is defined when $1 + e^x \neq 0$. But there is no value of x makes $1 + e^x = 0$. Therefore, $D_f = \mathbb{R}$	16) Find the domain of the function $f(x) = \sqrt{1 + 3^x}$. Solution: $f(x)$ is defined when $1 + 3^x \geq 0$. But $1 + 3^x > 0$ always. Therefore, $D_f = \mathbb{R}$
17) If $4^{(x+1)} = 8$, then $x =$ Solution: $4^{(x+1)} = 8$ $(2^2)^{(x+1)} = 2^3$ $2^{2(x+1)} = 2^3$ $2(x+1) = 3$ $2x + 2 = 3$ $2x = 3 - 2 = 1$ $\therefore \ x = \frac{1}{2}$	18) If $4^{(x-1)} = 8$, then $x =$ Solution: $4^{(x-1)} = 8$ $(2^2)^{(x-1)} = 2^3$ $2^{2(x-1)} = 2^3$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 3 + 2 = 5$ $\therefore \ x = \frac{5}{2}$
19) If $9^{(x+1)} = 27$, then $x =$ Solution: $9^{(x+1)} = 27$ $(3^2)^{(x+1)} = 3^3$ $3^{2(x+1)} = 3^3$ $2(x+1) = 3$ $2x + 2 = 3$ $2x = 3 - 2 = 1$ $\therefore \ x = \frac{1}{2}$	20) If $9^{(x-1)} = 27$, then $x =$ Solution: $9^{(x-1)} = 27$ $(3^2)^{(x-1)} = 3^3$ $3^{2(x-1)} = 3^3$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 3 + 2 = 5$ $\therefore \ x = \frac{5}{2}$
21) If $5^{2(x-1)} = 125$, then $x =$ Solution: $5^{2(x-1)} = 125$ $5^{2(x-1)} = 5^3$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 3 + 2 = 5$ $\therefore \ x = \frac{5}{2}$	22) If $5^{2(x+1)} = 125$, then $x =$ Solution: $5^{2(x+1)} = 125$ $5^{2(x+1)} = 5^3$ $2(x+1) = 3$ $2x + 2 = 3$ $2x = 3 - 2 = 1$ $\therefore \ x = \frac{1}{2}$