Workshop Solutions to Sections 5.1 and 5.2 (chapter 4)

1) The absolute maximum value of $f(x) = x^3 - 2x^2$ in $[-1,2]$ is at $x =$

Solution:

Since $f(x)$ is a continuous on $[-1,2]$, we can use the Closed Interval Method,

$$f(x) = x^3 - 2x^2$$
$$f'(x) = 3x^2 - 4x$$

Now, we find the critical numbers of $f(x)$ when

$f'(x) = 0 \implies 3x^2 - 4x = 0 \implies x(3x-4) = 0$

$\implies x = 0$ or $x = \frac{4}{3}$

Thus,

$f(-1) = (-1)^3 - 2(-1)^2 = -1 - 2 = -3$

$f(2) = (2)^3 - 2(2)^2 = 8 - 8 = 0$

$f(0) = (0)^3 - 2(0)^2 = 0 - 0 = 0$

$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 = \frac{64}{27} - \frac{32}{9} = -\frac{32}{27}$

Hence, we see that the absolute maximum value is 0 at $x = 0$ and $x = 2$

2) The absolute minimum value of $f(x) = x^3 - 3x^2 + 1$ in $\left[-\frac{1}{2}, 4\right]$ is

Solution:

Since $f(x)$ is a continuous on $\left[-\frac{1}{2}, 4\right]$, we can use the Closed Interval Method,

$$f(x) = x^3 - 3x^2 + 1$$
$$f'(x) = 3x^2 - 6x$$

Now, we find the critical numbers of $f(x)$ when

$f'(x) = 0 \implies 3x^2 - 6x = 0 \implies 3x(x-2) = 0$

$\implies x = 0$ or $x = 2$

Thus,

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$

$f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$

$f(0) = (0)^3 - 3(0)^2 + 1 = 0 - 0 + 1 = 1$

$f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$

Hence, we see that the absolute minimum value is -3 at $x = 2$

3) The absolute maximum point of $f(x) = 3x^2 - 12x + 1$ in $[0,3]$ is

Solution:

Since $f(x)$ is a continuous on $[0,3]$, we can use the Closed Interval Method,

$$f(x) = 3x^2 - 12x + 1$$
$$f'(x) = 6x - 12$$

Now, we find the critical numbers of $f(x)$ when

$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$

$\implies x = 2$

Thus,

$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$

$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$

$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$

Hence, we see that the absolute maximum point is $(0,1)$.

4) The absolute minimum point of $f(x) = 3x^2 - 12x + 1$ in $[0,3]$ is

Solution:

Since $f(x)$ is a continuous on $[0,3]$, we can use the Closed Interval Method,

$$f(x) = 3x^2 - 12x + 1$$
$$f'(x) = 6x - 12$$

Now, we find the critical numbers of $f(x)$ when

$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$

$\implies x = 2$

Thus,

$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$

$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$

$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$

Hence, we see that the absolute minimum point is $(2,-11)$.

5) The absolute minimum point of $f(x) = 3x^2 - 12x + 2$ in $[0,3]$ is

Solution:

Since $f(x)$ is a continuous on $[0,3]$, we can use the Closed Interval Method,

$$f(x) = 3x^2 - 12x + 2$$
$$f'(x) = 6x - 12$$

Now, we find the critical numbers of $f(x)$ when

$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$

$\implies x = 2$

Thus,

$f(0) = 3(0)^2 - 12(0) + 2 = 0 - 0 + 2 = 2$

$f(3) = 3(3)^2 - 12(3) + 2 = 27 - 36 + 2 = -7$

$f(2) = 3(2)^2 - 12(2) + 2 = 12 - 24 + 2 = -10$

Hence, we see that the absolute minimum point is $(2,-10)$.

6) The values in $(-3,3)$ which make $f(x) = x^3 - 9x$ satisfy Rolle's Theorem on $[-3,3]$ are deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[-3,3]$.

2- $f(x)$ is differentiable on $(-3,3)$,

$f'(x) = 3x^2 - 9$

3- $f(-3) = (-3)^3 - 9(-3) = -27 + 27 = 0 = f(3)$

Then there is a number $c \in (-3,3)$ such that

$f'(c) = 0 \implies 3c^2 - 9 = 0 \implies 3c^2 = 9$

$\implies c^2 = 3 \implies c = \pm\sqrt{3}$

Hence, the values are $\pm\sqrt{3} \in (-3,3)$.

7) The values in $(0,2)$ which make $f(x) = x^3 - 3x^2 + 2x + 5$ satisfy Rolle's Theorem on $[0,2]$ are deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,2]$.

2- $f(x)$ is differentiable on $(0,2)$,

$f'(x) = 3x^2 - 6x + 2$

3- $f(0) = (0)^3 - 3(0)^2 + 2(0) + 5 = 5 = f(2)$

Then there is a number $c \in (0,2)$ such that

$$f'(c) = 0 \quad \Rightarrow \quad 3c^2 - 6c + 2 = 0$$

$$\Rightarrow \quad c = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} = \frac{6 \pm \sqrt{36-24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm \sqrt{3 \times 4}}{6} = \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} = \frac{3}{3} \pm \frac{\sqrt{3}}{3}$$

$$= 1 \pm \frac{\sqrt{3}}{3}$$

Hence, the values are $1 \pm \frac{\sqrt{3}}{3} \in (0,2)$.

8) The value c in $(0,5)$ which makes $f(x) = x^2 - x - 6$ satisfy the Mean Value Theorem on $[0,5]$ is deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,5]$.

2- $f(x)$ is differentiable on $(0,5)$,

$f'(x) = 2x - 1$

Then there is a number $c \in (0,5)$ such that

$$f'(c) = \frac{f(5) - f(0)}{5 - 0}$$

$$\Rightarrow \quad 2c - 1 = \frac{[(5)^2 - (5) - 6] - [(0)^2 - (0) - 6]}{5}$$

$$\Rightarrow \quad 2c - 1 = \frac{(14) - (-6)}{5}$$

$$\Rightarrow \quad 2c - 1 = \frac{14 + 6}{5}$$

$$\Rightarrow \quad 2c - 1 = 4$$

$$\Rightarrow \quad 2c = 4 + 1$$

$$\Rightarrow \quad c = \frac{5}{2}$$

Hence, the value c is $\frac{5}{2} \in (0,5)$.

9) The value c in $(0,2)$ makes $f(x) = x^3 - x$ satisfied the Mean Value Theorem on $[0,2]$ are deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,2]$.

2- $f(x)$ is differentiable on $(0,2)$,

$f'(x) = 3x^2 - 1$

Then there is a number $c \in (0,3)$ such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$\Rightarrow \quad 3c^2 - 1 = \frac{[(2)^3 - (2)] - [(0)^3 - (0)]}{2}$$

$$\Rightarrow \quad 3c^2 - 1 = \frac{(6) - (0)}{2}$$

$$\Rightarrow \quad 3c^2 - 1 = \frac{6}{2}$$

$$\Rightarrow \quad 3c^2 - 1 = 3$$

$$\Rightarrow \quad 3c^2 = 3 + 1$$

$$\Rightarrow \quad c^2 = \frac{4}{3}$$

$$\Rightarrow \quad c = \pm\sqrt{\frac{4}{3}}$$

$$\Rightarrow \quad c = \pm\frac{2}{\sqrt{3}}$$

Hence, the value c is $\frac{2}{\sqrt{3}} \in (0,2)$ but $-\frac{2}{\sqrt{3}} \notin (0,2)$.

10) The value in $(0,1)$ which makes $f(x) = 3x^2 + 2x + 5$ satisfy the Mean Value Theorem on $[0,1]$ is deleted

Solution:

$\because$ $f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,1]$.

2- $f(x)$ is differentiable on $(0,1)$,

$f'(x) = 6x + 2$

Then there is a number $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 6c + 2 = \frac{[3(1)^2 + 2(1) + 5] - [3(0)^2 + 2(0) + 5]}{1}$$

$$\Rightarrow \quad 6c + 2 = (3 + 2 + 5) - (0 + 0 + 5)$$

$$\Rightarrow \quad 6c + 2 = 10 - 5$$

$$\Rightarrow \quad 6c + 2 = 5$$

$$\Rightarrow \quad 6c = 5 - 2$$

$$\Rightarrow \quad 6c = 3$$

$$\Rightarrow \quad c = \frac{3}{6}$$

$$\Rightarrow \quad c = \frac{1}{2}$$

Hence, the values are $\frac{1}{2} \in (0,1)$.

11) The critical numbers of the function $f(x) = x^3 + 3x^2 - 9x + 1$ are

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 + 6x - 9 = 0$$

$$\Rightarrow \quad 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow \quad x^2 + 2x - 3 = 0$$

$$\Rightarrow \quad (x + 3)(x - 1) = 0$$

$$\Rightarrow \quad x = -3 \quad \text{or} \quad x = 1$$

12) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Rightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow \quad x^2 + 2x - 3 = 0 \\ &\Rightarrow \quad (x+3)(x-1) = 0 \\ &\Rightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

$-3 \qquad\qquad 1$

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-3,1)$

13) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Rightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow \quad x^2 + 2x - 3 = 0 \\ &\Rightarrow \quad (x+3)(x-1) = 0 \\ &\Rightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

$-3 \qquad\qquad 1$

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-3) \cup (1,\infty)$

14) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Rightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow \quad x^2 + 2x - 3 = 0 \\ &\Rightarrow \quad (x+3)(x-1) = 0 \\ &\Rightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

$-3 \qquad\qquad 1$

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-3,28)$.

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - 9(-3) + 1 \\ &= -27 + 27 + 27 + 1 = 28 \end{aligned}$$

15) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 + 6x - 9 = 0 \\ &\Rightarrow \quad 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow \quad x^2 + 2x - 3 = 0 \\ &\Rightarrow \quad (x+3)(x-1) = 0 \\ &\Rightarrow \quad x = -3 \quad \text{or} \quad x = 1 \end{aligned}$$

$-3 \qquad\qquad 1$

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(1,-4)$.

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - 9(1) + 1 \\ &= 1 + 3 - 9 + 1 = -4 \end{aligned}$$

16) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave upward on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Rightarrow \quad 6x + 6 = 0 \\ &\Rightarrow \quad 6x = -6 \\ &\Rightarrow \quad x = -\frac{6}{6} \\ &\Rightarrow \quad x = -1 \end{aligned}$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-1,\infty)$

17) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave downward on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Rightarrow \quad 6x + 6 = 0 \\ &\Rightarrow \quad 6x = -6 \\ &\Rightarrow \quad x = -\frac{6}{6} \\ &\Rightarrow \quad x = -1 \end{aligned}$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty,-1)$

18) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 + 6x - 9$$
$$f''(x) = 6x + 6$$
$$f''(x) = 0 \implies 6x + 6 = 0$$
$$\implies 6x = -6$$
$$\implies x = -\frac{6}{6}$$
$$\implies x = -1$$

-1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-1,12)$.

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 1$$
$$= -1 + 3 + 9 + 1 = 12$$

19) The critical numbers of the function $f(x) = x^3 - 3x^2 - 9x + 1$ are

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$$
$$\implies 3(x^2 - 2x - 3) = 0$$
$$\implies x^2 - 2x - 3 = 0$$
$$\implies (x+1)(x-3) = 0$$
$$\implies x = -1 \quad \text{or} \quad x = 3$$

20) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$$
$$\implies 3(x^2 - 2x - 3) = 0$$
$$\implies x^2 - 2x - 3 = 0$$
$$\implies (x+1)(x-3) = 0$$
$$\implies x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1,3)$

21) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$$
$$\implies 3(x^2 - 2x - 3) = 0$$
$$\implies x^2 - 2x - 3 = 0$$
$$\implies (x+1)(x-3) = 0$$
$$\implies x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-1) \cup (3,\infty)$

22) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$$
$$\implies 3(x^2 - 2x - 3) = 0$$
$$\implies x^2 - 2x - 3 = 0$$
$$\implies (x+1)(x-3) = 0$$
$$\implies x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1,6)$.

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1$$
$$= -1 - 3 + 9 + 1 = 6.$$

23) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$$
$$\implies 3(x^2 - 2x - 3) = 0$$
$$\implies x^2 - 2x - 3 = 0$$
$$\implies (x+1)(x-3) = 0$$
$$\implies x = -1 \quad \text{or} \quad x = 3$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(3,-26)$.

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 1$$
$$= 27 - 27 - 27 + 1 = -26.$$

24) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave upward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$
$$f''(x) = 0 \implies 6x - 6 = 0$$
$$\implies 6x = 6$$
$$\implies x = \frac{6}{6}$$
$$\implies x = 1$$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$

25) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave downward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$
$$f''(x) = 0 \implies 6x - 6 = 0$$
$$\implies 6x = 6$$
$$\implies x = \frac{6}{6}$$
$$\implies x = 1$$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$

26) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has an inflection point at

<u>Solution:</u>

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$
$$f''(x) = 0 \implies 6x - 6 = 0$$
$$\implies 6x = 6$$
$$\implies x = \frac{6}{6}$$
$$\implies x = 1$$

1

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -10)$.

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 1$$
$$= 1 - 3 - 9 + 1 = -10$$

27) The critical numbers of the function $f(x) = x^3 + 3x^2 - 9x + 5$ are

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$$
$$\implies 3(x^2 + 2x - 3) = 0$$
$$\implies x^2 + 2x - 3 = 0$$
$$\implies (x + 3)(x - 1) = 0$$
$$\implies x = -3 \quad \text{or} \quad x = 1$$

28) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$$
$$\implies 3(x^2 + 2x - 3) = 0$$
$$\implies x^2 + 2x - 3 = 0$$
$$\implies (x + 3)(x - 1) = 0$$
$$\implies x = -3 \quad \text{or} \quad x = 1$$

-3 1

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-3,1)$.

29) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is increasing on

<u>Solution:</u>

$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$$
$$\implies 3(x^2 + 2x - 3) = 0$$
$$\implies x^2 + 2x - 3 = 0$$
$$\implies (x + 3)(x - 1) = 0$$
$$\implies x = -3 \quad \text{or} \quad x = 1$$

-3 1

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -3) \cup (1, \infty)$.

30) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$

$\implies 3(x^2 + 2x - 3) = 0$

$\implies x^2 + 2x - 3 = 0$

$\implies (x + 3)(x - 1) = 0$

$\implies x = -3 \quad \text{or} \quad x = 1$

$-3 \qquad\qquad 1$

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(1,0)$.

$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$

$= 1 + 3 - 9 + 5 = 0$

31) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$

$\implies 3(x^2 + 2x - 3) = 0$

$\implies x^2 + 2x - 3 = 0$

$\implies (x + 3)(x - 1) = 0$

$\implies x = -3 \quad \text{or} \quad x = 1$

$-3 \qquad\qquad 1$

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-3,32)$.

$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$

$= -27 + 27 + 27 + 5 = 32$

32) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has an inflection point at

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$f''(x) = 0 \implies 6x + 6 = 0$

$\implies 6x = -6$

$\implies x = -\frac{6}{6}$

$\implies x = -1$

-1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-1,16)$.

$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 5$

$= -1 + 3 + 9 + 5 = 16$

33) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave downward on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$f''(x) = 0 \implies 6x + 6 = 0$

$\implies 6x = -6$

$\implies x = -\frac{6}{6}$

$\implies x = -1$

-1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty,-1)$.

34) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave upward on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$f''(x) = 0 \implies 6x + 6 = 0$

$\implies 6x = -6$

$\implies x = -\frac{6}{6}$

$\implies x = -1$

-1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-1,\infty)$.

35) The critical numbers of the function $f(x) = x^3 - 3x^2 - 9x + 5$ are

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$

$\implies 3(x^2 - 2x - 3) = 0$

$\implies x^2 - 2x - 3 = 0$

$\implies (x + 1)(x - 3) = 0$

$\implies x = -1 \quad \text{or} \quad x = 3$

36) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$
\begin{aligned}
f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 6x - 9 = 0 \\
&\Rightarrow \quad 3(x^2 - 2x - 3) = 0 \\
&\Rightarrow \quad x^2 - 2x - 3 = 0 \\
&\Rightarrow \quad (x+1)(x-3) = 0 \\
&\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3
\end{aligned}
$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (3, \infty)$.

37) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$
\begin{aligned}
f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 6x - 9 = 0 \\
&\Rightarrow \quad 3(x^2 - 2x - 3) = 0 \\
&\Rightarrow \quad x^2 - 2x - 3 = 0 \\
&\Rightarrow \quad (x+1)(x-3) = 0 \\
&\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3
\end{aligned}
$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1, 3)$.

38) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$
\begin{aligned}
f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 6x - 9 = 0 \\
&\Rightarrow \quad 3(x^2 - 2x - 3) = 0 \\
&\Rightarrow \quad x^2 - 2x - 3 = 0 \\
&\Rightarrow \quad (x+1)(x-3) = 0 \\
&\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3
\end{aligned}
$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1, 10)$.

$$
\begin{aligned}
f(-1) &= (-1)^3 - 3(-1)^2 - 9(-1) + 5 \\
&= -1 - 3 + 9 + 5 = 10.
\end{aligned}
$$

39) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$
\begin{aligned}
f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 6x - 9 = 0 \\
&\Rightarrow \quad 3(x^2 - 2x - 3) = 0 \\
&\Rightarrow \quad x^2 - 2x - 3 = 0 \\
&\Rightarrow \quad (x+1)(x-3) = 0 \\
&\Rightarrow \quad x = -1 \quad \text{or} \quad x = 3
\end{aligned}
$$

-1 $\qquad$ 3

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(3, -22)$.

$$
\begin{aligned}
f(3) &= (3)^3 - 3(3)^2 - 9(3) + 5 \\
&= 27 - 27 - 27 + 5 = -22.
\end{aligned}
$$

40) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$
\begin{aligned}
f''(x) = 0 \quad &\Rightarrow \quad 6x - 6 = 0 \\
&\Rightarrow \quad 6x = 6 \\
&\Rightarrow \quad x = \frac{6}{6} \\
&\Rightarrow \quad x = 1
\end{aligned}
$$

1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

41) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$
\begin{aligned}
f''(x) = 0 \quad &\Rightarrow \quad 6x - 6 = 0 \\
&\Rightarrow \quad 6x = 6 \\
&\Rightarrow \quad x = \frac{6}{6} \\
&\Rightarrow \quad x = 1
\end{aligned}
$$

1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

42) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \implies 6x - 6 = 0$
$\implies 6x = 6$
$\implies x = \frac{6}{6}$
$\implies x = 1$

1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -6)$.

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 5$$
$$= 1 - 3 - 9 + 5 = -6$$

43) The critical numbers of the function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ are

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

44) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is increasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (2, \infty)$.

45) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is decreasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1, 2)$.

46) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has a relative maximum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $\left(-1, \frac{13}{6}\right)$.

$$f(-1) = \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) + 1$$
$$= -\frac{1}{3} - \frac{1}{2} + 2 + 1 = \frac{13}{6}$$

47) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has a relative minimum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\implies (x+1)(x-2) = 0$
$\implies x = -1$ or $x = 2$

-1 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $\left(2, -\frac{7}{3}\right)$.

$$f(2) = \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - 2(2) + 1$$
$$= \frac{8}{3} - \frac{4}{2} - 4 + 1 = -\frac{7}{3}$$

48) The function $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$ concave upward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2-2\left(\frac{1}{2}\right)x-2=x^2-x-2$$

$$f''(x)=2x-1$$

$$f''(x)=0 \quad\Longrightarrow\quad 2x-1=0$$
$$\Longrightarrow\quad 2x=1$$
$$\Longrightarrow\quad x=\frac{1}{2}$$

$\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $\left(\frac{1}{2},\infty\right)$.

49) The function $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$ concave downward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2-2\left(\frac{1}{2}\right)x-2=x^2-x-2$$

$$f''(x)=2x-1$$

$$f''(x)=0 \quad\Longrightarrow\quad 2x-1=0$$
$$\Longrightarrow\quad 2x=1$$
$$\Longrightarrow\quad x=\frac{1}{2}$$

$\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $\left(-\infty,\frac{1}{2}\right)$.

50) The function $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$ has an inflection point at

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2-2\left(\frac{1}{2}\right)x-2=x^2-x-2$$

$$f''(x)=2x-1$$

$$f''(x)=0 \quad\Longrightarrow\quad 2x-1=0$$
$$\Longrightarrow\quad 2x=1$$
$$\Longrightarrow\quad x=\frac{1}{2}$$

$\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $\left(\frac{1}{2},-\frac{1}{12}\right)$.

$$f\left(\frac{1}{2}\right)=\frac{1}{3}\left(\frac{1}{2}\right)^3-\frac{1}{2}\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)+1$$
$$=\frac{1}{24}-\frac{1}{8}-1+1=-\frac{1}{12}$$

51) The critical numbers of the function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ are

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$$f'(x)=0 \quad\Longrightarrow\quad x^2+x-2=0$$
$$\Longrightarrow\quad (x+2)(x-1)=0$$
$$\Longrightarrow\quad x=-2 \quad\text{or}\quad x=1$$

52) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ is increasing on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$$f'(x)=0 \quad\Longrightarrow\quad x^2+x-2=0$$
$$\Longrightarrow\quad (x+2)(x-1)=0$$
$$\Longrightarrow\quad x=-2 \quad\text{or}\quad x=1$$

-2 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-2)\cup(1,\infty)$.

53) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ is decreasing on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$$f'(x)=0 \quad\Longrightarrow\quad x^2+x-2=0$$
$$\Longrightarrow\quad (x+2)(x-1)=0$$
$$\Longrightarrow\quad x=-2 \quad\text{or}\quad x=1$$

-2 $\qquad$ 1

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2,1)$.

54) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$
has a relative maximum point

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad\Rightarrow\quad x^2+x-2=0$
$\Rightarrow\quad (x+2)(x-1)=0$
$\Rightarrow\quad x=-2 \quad \text{or} \quad x=1$

$-2 \qquad 1$

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $\left(-2,\frac{13}{3}\right)$.

$$f(-2)=\frac{1}{3}(-2)^3+\frac{1}{2}(-2)^2-2(-2)+1$$
$$=-\frac{8}{3}+\frac{4}{2}+4+1=\frac{13}{3}$$

55) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$
has a relative minimum point

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$

$f'(x)=0 \quad\Rightarrow\quad x^2+x-2=0$
$\Rightarrow\quad (x+2)(x-1)=0$
$\Rightarrow\quad x=-2 \quad \text{or} \quad x=1$

$-2 \qquad 1$

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $\left(1,-\frac{1}{6}\right)$.

$$f(1)=\frac{1}{3}(1)^3+\frac{1}{2}(1)^2-2(1)+1$$
$$=\frac{1}{3}+\frac{1}{2}-2+1=-\frac{1}{6}$$

56) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ concave
upward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$
$$f''(x)=2x+1$$

$f''(x)=0 \quad\Rightarrow\quad 2x+1=0$
$\Rightarrow\quad 2x=-1$
$\Rightarrow\quad x=-\frac{1}{2}$

$-\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $\left(-\frac{1}{2},\infty\right)$.

57) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ concave
downward on

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$
$$f''(x)=2x+1$$

$f''(x)=0 \quad\Rightarrow\quad 2x+1=0$
$\Rightarrow\quad 2x=-1$
$\Rightarrow\quad x=-\frac{1}{2}$

$-\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $\left(-\infty,-\frac{1}{2}\right)$.

58) The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ has an
inflection point at

Solution:

$$f'(x)=3\left(\frac{1}{3}\right)x^2+2\left(\frac{1}{2}\right)x-2=x^2+x-2$$
$$f''(x)=2x+1$$

$f''(x)=0 \quad\Rightarrow\quad 2x+1=0$
$\Rightarrow\quad 2x=-1 \quad\Rightarrow\quad x=-\frac{1}{2}$

$-\frac{1}{2}$

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $\left(-\frac{1}{2},\frac{25}{12}\right)$.

$$f\left(-\frac{1}{2}\right)=\frac{1}{3}\left(-\frac{1}{2}\right)^3+\frac{1}{2}\left(-\frac{1}{2}\right)^2-2\left(-\frac{1}{2}\right)+1$$
$$=-\frac{1}{24}+\frac{1}{8}+1+1=\frac{25}{12}$$

59) The critical numbers of the function
$f(x)=x^3-12x+3$ are

Solution:

$$f'(x)=3x^2-12$$

$f'(x)=0 \quad\Rightarrow\quad 3x^2-12=0$
$\Rightarrow\quad 3(x^2-4)=0$
$\Rightarrow\quad x^2-4=0$
$\Rightarrow\quad x^2=4$
$\Rightarrow\quad x=\pm 2$

60) The function $f(x) = x^3 - 12x + 3$ is increasing on

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$

$\implies 3(x^2 - 4) = 0$

$\implies x^2 - 4 = 0$

$\implies x^2 = 4$

$\implies x = \pm 2$

Critical points: -2, 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -2) \cup (2, \infty)$.

61) The function $f(x) = x^3 - 12x + 3$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$

$\implies 3(x^2 - 4) = 0$

$\implies x^2 - 4 = 0$

$\implies x^2 = 4$

$\implies x = \pm 2$

Critical points: -2, 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2, 2)$.

62) The function $f(x) = x^3 - 12x + 3$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$

$\implies 3(x^2 - 4) = 0$

$\implies x^2 - 4 = 0$

$\implies x^2 = 4$

$\implies x = \pm 2$

Critical points: -2, 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(-2, 19)$.

$f(-2) = (-2)^3 - 12(-2) + 3$

$= -8 + 24 + 3 = 19.$

63) The function $f(x) = x^3 - 12x + 3$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 12$$

$f'(x) = 0 \implies 3x^2 - 12 = 0$

$\implies 3(x^2 - 4) = 0$

$\implies x^2 - 4 = 0$

$\implies x^2 = 4$

$\implies x = \pm 2$

Critical points: -2, 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2, -13)$.

$f(2) = (2)^3 - 12(2) + 3$

$= 8 - 24 + 3 = -13$

64) The function $f(x) = x^3 - 12x + 3$ concave upward on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$f''(x) = 0 \implies 6x = 0$

$\implies x = \frac{0}{6}$

$\implies x = 0$

Critical point: 0

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(0, \infty)$.

65) The function $f(x) = x^3 - 12x + 3$ concave downward on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$f''(x) = 0 \implies 6x = 0$

$\implies x = \frac{0}{6}$

$\implies x = 0$

Critical point: 0

$-$	$+$	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 0)$.

66) The function $f(x) = x^3 - 12x + 3$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 12$$
$$f''(x) = 6x$$
$$f''(x) = 0 \quad \Rightarrow \quad 6x = 0$$
$$\Rightarrow \quad x = \frac{0}{6}$$
$$\Rightarrow \quad x = 0$$

0

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(0,3)$.

$f(0) = (0)^3 - 12(0)^2 + 3$

$= 0 - 0 + 3 = 3$

67) The critical numbers of the function $f(x) = x^3 - 3x^2 + 1$ are

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

68) The function $f(x) = x^3 - 3x^2 + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.

69) The function $f(x) = x^3 - 3x^2 + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(0,2)$.

70) The function $f(x) = x^3 - 3x^2 + 1$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(0,1)$.

$f(0) = (0)^3 - 3(0)^2 + 1$

$= 0 - 0 + 1 = 1.$

71) The function $f(x) = x^3 - 3x^2 + 1$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3(x^2 - 2x) = 0$$
$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2$$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2,-3)$.

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 12 + 1 = -3.$

72) The function $f(x) = x^3 - 3x^2 + 1$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \Rightarrow 6x - 6 = 0$
$\Rightarrow 6x = 6$
$\Rightarrow x = \frac{6}{6}$
$\Rightarrow x = 1$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

73) The function $f(x) = x^3 - 3x^2 + 1$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \Rightarrow 6x - 6 = 0$
$\Rightarrow 6x = 6$
$\Rightarrow x = \frac{6}{6}$
$\Rightarrow x = 1$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

74) The function $f(x) = x^3 - 3x^2 + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

$f''(x) = 0 \Rightarrow 6x - 6 = 0$
$\Rightarrow 6x = 6$
$\Rightarrow x = \frac{6}{6}$
$\Rightarrow x = 1$

1

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -1)$.

$f(1) = (1)^3 - 3(1)^2 + 1$
$= 1 - 3 + 1 = -1$

75) The critical numbers of the function $f(x) = x^3 - 3x^2 + 2$ are

Solution:

$$f'(x) = 3x^2 - 6x$$

$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$
$\Rightarrow 3(x^2 - 2x) = 0$
$\Rightarrow x^2 - 2x = 0$
$\Rightarrow x(x - 2) = 0$
$\Rightarrow x = 0$ or $x = 2$

76) The function $f(x) = x^3 - 3x^2 + 2$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$
$\Rightarrow 3(x^2 - 2x) = 0$
$\Rightarrow x^2 - 2x = 0$
$\Rightarrow x(x - 2) = 0$
$\Rightarrow x = 0$ or $x = 2$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.

77) The function $f(x) = x^3 - 3x^2 + 2$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$
$\Rightarrow 3(x^2 - 2x) = 0$
$\Rightarrow x^2 - 2x = 0$
$\Rightarrow x(x - 2) = 0$
$\Rightarrow x = 0$ or $x = 2$

0 2

$+$	$-$	$+$	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(0,2)$.

78) The function $f(x) = x^3 - 3x^2 + 2$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 6x = 0 \\ &\Rightarrow \quad 3(x^2 - 2x) = 0 \\ &\Rightarrow \quad x^2 - 2x = 0 \\ &\Rightarrow \quad x(x-2) = 0 \\ &\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2 \end{aligned}$$

0 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2, -2)$.

$$\begin{aligned} f(2) &= (2)^3 - 3(2)^2 + 2 \\ &= 8 - 12 + 2 = -2. \end{aligned}$$

79) The function $f(x) = x^3 - 3x^2 + 2$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 6x = 0 \\ &\Rightarrow \quad 3(x^2 - 2x) = 0 \\ &\Rightarrow \quad x^2 - 2x = 0 \\ &\Rightarrow \quad x(x-2) = 0 \\ &\Rightarrow \quad x = 0 \quad \text{or} \quad x = 2 \end{aligned}$$

0 2

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(0,2)$.

$$\begin{aligned} f(0) &= (0)^3 - 3(0)^2 + 2 \\ &= 0 - 0 + 2 = 2. \end{aligned}$$

80) The function $f(x) = x^3 - 3x^2 + 2$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Rightarrow \quad 6x - 6 = 0 \\ &\Rightarrow \quad 6x = 6 \\ &\Rightarrow \quad x = \frac{6}{6} \\ &\Rightarrow \quad x = 1 \end{aligned}$$

1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

81) The function $f(x) = x^3 - 3x^2 + 2$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Rightarrow \quad 6x - 6 = 0 \\ &\Rightarrow \quad 6x = 6 \\ &\Rightarrow \quad x = \frac{6}{6} \\ &\Rightarrow \quad x = 1 \end{aligned}$$

1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

82) The function $f(x) = x^3 - 3x^2 + 2$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$\begin{aligned} f''(x) = 0 \quad &\Rightarrow \quad 6x - 6 = 0 \\ &\Rightarrow \quad 6x = 6 \\ &\Rightarrow \quad x = \frac{6}{6} \\ &\Rightarrow \quad x = 1 \end{aligned}$$

1

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1,0)$.

$$\begin{aligned} f(1) &= (1)^3 - 3(1)^2 + 2 \\ &= 1 - 3 + 2 = 0 \end{aligned}$$

83) The critical numbers of the function $f(x) = x^3 - 6x^2 - 36x$ are

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$\begin{aligned} f'(x) = 0 \quad &\Rightarrow \quad 3x^2 - 12x - 36 = 0 \\ &\Rightarrow \quad 3(x^2 - 4x - 12) = 0 \\ &\Rightarrow \quad x^2 - 4x - 12 = 0 \\ &\Rightarrow \quad (x+2)(x-6) = 0 \\ &\Rightarrow \quad x = -2 \quad \text{or} \quad x = 6 \end{aligned}$$

84) The function $f(x) = x^3 - 6x^2 - 36x$ is decreasing on

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$

$\implies 3(x^2 - 4x - 12) = 0$

$\implies x^2 - 4x - 12 = 0$

$\implies (x+2)(x-6) = 0$

$\implies x = -2$ or $x = 6$

-2 $\qquad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2,6)$.

85) The function $f(x) = x^3 - 6x^2 - 36x$ is increasing on

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$

$\implies 3(x^2 - 4x - 12) = 0$

$\implies x^2 - 4x - 12 = 0$

$\implies (x+2)(x-6) = 0$

$\implies x = -2$ or $x = 6$

-2 $\qquad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty,-2) \cup (6,\infty)$.

86) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative minimum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$

$\implies 3(x^2 - 4x - 12) = 0$

$\implies x^2 - 4x - 12 = 0$

$\implies (x+2)(x-6) = 0$

$\implies x = -2$ or $x = 6$

-2 $\qquad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(6,-216)$.

$f(6) = (6)^3 - 6(6)^2 - 36(6)$

$= 216 - 216 - 216 = -216$

87) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative maximum value at the point

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$

$\implies 3(x^2 - 4x - 12) = 0$

$\implies x^2 - 4x - 12 = 0$

$\implies (x+2)(x-6) = 0$

$\implies x = -2$ or $x = 6$

-2 $\qquad$ 6

+	−	+	Sign of $f'(x)$
↗	↘	↗	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-2,40)$.

$f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$

$= -8 - 24 + 72 = 40$

88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$f''(x) = 0 \implies 6x - 12 = 0$

$\implies 6x = 12$

$\implies x = \frac{12}{6}$

$\implies x = 2$

2

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(2,-88)$.

$f(2) = (2)^3 - 6(2)^2 - 36(2)$

$= 8 - 24 - 72 = -88$

89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on

<u>Solution:</u>

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$f''(x) = 0 \implies 6x - 12 = 0$

$\implies 6x = 12$

$\implies x = \frac{12}{6}$

$\implies x = 2$

2

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 2)$.

90) The function $f(x) = x^3 - 6x^2 - 36x$ concave upward on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$
$$f''(x) = 6x - 12$$

$f''(x) = 0 \quad \Rightarrow \quad 6x - 12 = 0$

$\Rightarrow \quad 6x = 12$

$\Rightarrow \quad x = \frac{12}{6}$

$\Rightarrow \quad x = 2$

2

−	+	Sign of $f''(x)$
∩	∪	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(2, \infty)$.

91) The critical numbers of the function $f(x) = -x^3 - 6x^2 - 9x + 1$ are

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \quad \Rightarrow \quad -3x^2 - 12x - 9 = 0$

$\Rightarrow \quad -3(x^2 + 4x + 3) = 0$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\Rightarrow \quad (x+3)(x+1) = 0$

$\Rightarrow \quad x = -3$ or $x = -1$

92) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \quad \Rightarrow \quad -3x^2 - 12x - 9 = 0$

$\Rightarrow \quad -3(x^2 + 4x + 3) = 0$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\Rightarrow \quad (x+3)(x+1) = 0$

$\Rightarrow \quad x = -3$ or $x = -1$

-3 $\quad -1$

−	+	−	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-\infty, -3) \cup (-1, \infty)$.

93) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \quad \Rightarrow \quad -3x^2 - 12x - 9 = 0$

$\Rightarrow \quad -3(x^2 + 4x + 3) = 0$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\Rightarrow \quad (x+3)(x+1) = 0$

$\Rightarrow \quad x = -3$ or $x = -1$

-3 $\quad -1$

−	+	−	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-3, -1)$.

94) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \quad \Rightarrow \quad -3x^2 - 12x - 9 = 0$

$\Rightarrow \quad -3(x^2 + 4x + 3) = 0$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\Rightarrow \quad (x+3)(x+1) = 0$

$\Rightarrow \quad x = -3$ or $x = -1$

-3 $\quad -1$

−	+	−	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(-3,1)$.

$f(-3) = -(-3)^3 - 6(-3)^2 - 9(-3) + 1$
$\quad = 27 - 54 + 27 + 1 = 1.$

95) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$f'(x) = 0 \quad \Rightarrow \quad -3x^2 - 12x - 9 = 0$

$\Rightarrow \quad -3(x^2 + 4x + 3) = 0$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\Rightarrow \quad (x+3)(x+1) = 0$

$\Rightarrow \quad x = -3$ or $x = -1$

-3 $\quad -1$

−	+	−	Sign of $f'(x)$
↘	↗	↘	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1,5)$.

$f(-1) = -(-1)^3 - 6(-1)^2 - 9(-1) + 1$
$\quad = 1 - 6 + 9 + 1 = 5.$

96) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = -3x^2 - 12x - 9$$
$$f''(x) = -6x - 12$$

$f''(x) = 0 \implies -6x - 12 = 0$
$\implies -6x = 12$
$\implies x = -\frac{12}{6}$
$\implies x = -2$

−2

+	−	Sign of $f''(x)$
∪	∩	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-2,3)$.

$f(-2) = -(-2)^3 - 6(-2)^2 - 9(-2) + 1$
$= 8 - 24 + 18 + 1 = 3$

97) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave downward on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$
$$f''(x) = -6x - 12$$

$f''(x) = 0 \implies -6x - 12 = 0$
$\implies -6x = 12$
$\implies x = -\frac{12}{6}$
$\implies x = -2$

−2

+	−	Sign of $f''(x)$
∪	∩	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-2, \infty)$.

98) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave upward on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$
$$f''(x) = -6x - 12$$

$f''(x) = 0 \implies -6x - 12 = 0$
$\implies -6x = 12$
$\implies x = -\frac{12}{6}$
$\implies x = -2$

−2

+	−	Sign of $f''(x)$
∪	∩	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-\infty, -2)$.