

Workshop Solutions to Sections 3.1 and 3.2 (2.3)

1) $\lim_{x \rightarrow -2} (x^3 - 2x + 1) = (-2)^3 - 2(-2) + 1$ $= -8 + 4 + 1 = -3$	2) $\lim_{x \rightarrow 2} (3x^2 + x - 4) = 3(2)^2 + (2) - 4$ $= 12 + 2 - 4 = 10$
3) $\lim_{x \rightarrow 1} (x^2 + 3x - 5)^3 = ((1)^2 + 3(1) - 5)^3$ $= (1 + 3 - 5)^3 = (-1)^3 = -1$	4) $\lim_{x \rightarrow -2} (2x^3 + 3x^2 + 5) = 2(-2)^3 + 3(-2)^2 + 5$ $= 2(-8) + 3(4) + 5$ $= -16 + 12 + 5 = 1$
5) $\lim_{x \rightarrow -2} \frac{x^2 - 2}{x - 2} = \frac{(-2)^2 - 2}{(-2) - 2} = \frac{4 - 2}{-2 - 2} = \frac{2}{-4} = -\frac{1}{2}$	6) $\lim_{x \rightarrow 2} \frac{x^3 + 5}{x^2 + 1} = \frac{(2)^3 + 5}{(2)^2 + 1} = \frac{8 + 5}{4 + 1} = \frac{13}{5}$
7) $\lim_{x \rightarrow 0} \frac{x^2 + 3x + 5}{x^2 - 3} = \frac{(0)^2 + 3(0) + 5}{(0)^2 - 3} = \frac{0 + 0 + 5}{0 - 3}$ $= \frac{5}{-3} = -\frac{5}{3}$	8) $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 + x - 5} = \frac{(1) - 1}{(1)^2 + (1) - 5} = \frac{1 - 1}{1 + 1 - 5} = \frac{0}{-3} = 0$
9) $\lim_{x \rightarrow -1} \sqrt{x^3 - 10x + 7} = \sqrt{(-1)^3 - 10(-1) + 7}$ $= \sqrt{-1 + 10 + 7} = \sqrt{16} = 4$	10) $\lim_{x \rightarrow -1} \frac{1 - (x + 4)^{-2}}{x - 2} = \frac{1 - ((-1) + 4)^{-2}}{(-1) - 2}$ $= \frac{1 - (-1 + 4)^{-2}}{-3} = \frac{1 - (3)^{-2}}{-3} = \frac{1 - \frac{1}{3^2}}{-3}$ $= \frac{1 - \frac{1}{9}}{-3} = \frac{\frac{8}{9}}{-3} = \frac{8}{9} \times \frac{1}{-3} = \frac{8}{-27} = -\frac{8}{27}$
11) $\lim_{x \rightarrow -1} \frac{x^3 + 2x}{8 - 2x} = \frac{(-1)^3 + 2(-1)}{8 - 2(-1)} = \frac{-1 - 2}{8 + 2} = \frac{-3}{10}$ $= -\frac{3}{10}$	12) $\lim_{x \rightarrow 4} \frac{x^2 - 3x}{5 + x} = \frac{(4)^2 - 3(4)}{5 + (4)} = \frac{16 - 12}{5 + 4} = \frac{4}{9}$
13) $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{5 + x} = \frac{(4)^2 - 4(4)}{5 + (4)} = \frac{16 - 16}{5 + 4} = \frac{0}{9} = 0$	15) $\lim_{x \rightarrow 0} \frac{x^3 - 5x^2}{x^2} = \lim_{x \rightarrow 0} \frac{x^2(x - 5)}{x^2}$ $= \lim_{x \rightarrow 0} (x - 5) = (0) - 5 = -5$
14) $\lim_{x \rightarrow 4} \frac{3^{-1} - (2x - 5)^{-1}}{4 - x} = \lim_{x \rightarrow 4} \frac{\frac{1}{3} - \frac{1}{2x - 5}}{4 - x}$ $= \lim_{x \rightarrow 4} \frac{\frac{2x - 5 - 3}{3(2x - 5)}}{4 - x}$ $= \lim_{x \rightarrow 4} \frac{2x - 8}{3(2x - 5)(4 - x)}$ $= \lim_{x \rightarrow 4} \frac{2(x - 4)}{3(2x - 5)(4 - x)}$ $= \lim_{x \rightarrow 4} \frac{-2(4 - x)}{3(2x - 5)(4 - x)} = \lim_{x \rightarrow 4} \frac{-2}{3(2x - 5)}$ $= \frac{-2}{3(2(4) - 5)} = \frac{-2}{3(8 - 5)} = \frac{-2}{9} = -\frac{2}{9}$	16) $\lim_{x \rightarrow 6} \frac{x - 6}{x^2 - 36} = \lim_{x \rightarrow 6} \frac{x - 6}{(x - 6)(x + 6)} = \lim_{x \rightarrow 6} \frac{1}{x + 6}$ $= \frac{1}{(6) + 6} = \frac{1}{12}$
17) $\lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6} = \lim_{x \rightarrow 6} \frac{(x - 6)(x + 6)}{x - 6} = \lim_{x \rightarrow 6} (x + 6)$ $= (6) + 6 = 12$	18) $\lim_{x \rightarrow -6} \frac{x + 6}{x^2 - 36} = \lim_{x \rightarrow -6} \frac{x + 6}{(x - 6)(x + 6)} = \lim_{x \rightarrow -6} \frac{1}{x - 6}$ $= \frac{1}{(-6) - 6} = \frac{1}{-12} = -\frac{1}{12}$
19) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x^2 + 3x + 9)}{x - 3}$ $= \lim_{x \rightarrow 3} (x^2 + 3x + 9) = (3)^2 + 3(3) + 9$ $= 9 + 9 + 9 = 27$	20) $\lim_{x \rightarrow 3} \frac{x - 3}{x^3 - 27} = \lim_{x \rightarrow 3} \frac{x - 3}{(x - 3)(x^2 + 3x + 9)}$ $= \lim_{x \rightarrow 3} \frac{1}{x^2 + 3x + 9} = \frac{1}{(3)^2 + 3(3) + 9}$ $= \frac{1}{9 + 9 + 9} = \frac{1}{27}$

$21) \lim_{x \rightarrow -2} \frac{x+2}{x^3+8} = \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x^2-2x+4)}$ $= \lim_{x \rightarrow -2} \frac{1}{x^2-2x+4}$ $= \frac{1}{(-2)^2-2(-2)+4} = \frac{1}{4+4+4} = \frac{1}{12}$	$22) \lim_{x \rightarrow -2} \frac{x^3+8}{x+2} = \lim_{x \rightarrow -2} \frac{(x+2)(x^2-2x+4)}{x+2}$ $= \lim_{x \rightarrow -2} (x^2-2x+4) = (-2)^2-2(-2)+4$ $= 4+4+4 = 12$
$23) \lim_{x \rightarrow 4} \frac{x^2-3x-4}{x-4} = \lim_{x \rightarrow 4} \frac{(x-4)(x+1)}{x-4} = \lim_{x \rightarrow 4} (x+1)$ $= (4)+1 = 5$	$24) \lim_{x \rightarrow 3} \frac{x^2+4x-21}{x^2-8x+15} = \lim_{x \rightarrow 3} \frac{(x+7)(x-3)}{(x-5)(x-3)} = \lim_{x \rightarrow 3} \frac{x+7}{x-5}$ $= \frac{(3)+7}{(3)-5} = \frac{10}{-2} = -5$
$25) \lim_{x \rightarrow 0} \frac{x}{1-(1-x)^2} = \lim_{x \rightarrow 0} \frac{x}{1-(1-2x+x^2)}$ $= \lim_{x \rightarrow 0} \frac{x}{1-1+2x-x^2}$ $= \lim_{x \rightarrow 0} \frac{x}{2x-x^2} = \lim_{x \rightarrow 0} \frac{x}{x(2-x)}$ $= \lim_{x \rightarrow 0} \frac{1}{2-x} = \frac{1}{2-(0)} = \frac{1}{2}$	$26) \lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6}-2}{x-2} = \lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6}-2}{(x+6)-8} = \lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6}-2}{(\sqrt[3]{x+6})^3-8}$ $= \lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6}-2}{(\sqrt[3]{x+6}-2)((\sqrt[3]{x+6})^2+2\sqrt[3]{x+6}+4)}$ $= \lim_{x \rightarrow 2} \frac{1}{(\sqrt[3]{x+6})^2+2\sqrt[3]{x+6}+4}$ $= \frac{1}{(\sqrt[3]{(2)+6})^2+2\sqrt[3]{(2)+6}+4} = \frac{1}{4+4+4} = \frac{1}{12} \text{ deleted}$
$27) \lim_{x \rightarrow 0} \frac{\sqrt{x+25}-5}{x}$ $= \lim_{x \rightarrow 0} \left[\frac{\sqrt{x+25}-5}{x} \times \frac{\sqrt{x+25}+5}{\sqrt{x+25}+5} \right]$ $= \lim_{x \rightarrow 0} \frac{(x+25)-25}{x(\sqrt{x+25}+5)}$ $= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+25}+5)}$ $= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+25}+5} = \frac{1}{\sqrt{(0)+25}+5}$ $= \frac{1}{5+5} = \frac{1}{10}$	$28) \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+25}-5} = \lim_{x \rightarrow 0} \left[\frac{x}{\sqrt{x+25}-5} \times \frac{\sqrt{x+25}+5}{\sqrt{x+25}+5} \right]$ $= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+25}+5)}{(x+25)-25}$ $= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+25}+5)}{x}$ $= \lim_{x \rightarrow 0} (\sqrt{x+25}+5) = \sqrt{(0)+25}+5$ $= 5+5 = 10$
$29) \lim_{x \rightarrow 2} \frac{x-2}{2-\sqrt{6-x}} = \lim_{x \rightarrow 2} \left[\frac{x-2}{2-\sqrt{6-x}} \times \frac{2+\sqrt{6-x}}{2+\sqrt{6-x}} \right]$ $= \lim_{x \rightarrow 2} \frac{(x-2)(2+\sqrt{6-x})}{4-(6-x)}$ $= \lim_{x \rightarrow 2} \frac{(x-2)(2+\sqrt{6-x})}{4-6+x}$ $= \lim_{x \rightarrow 2} \frac{(x-2)(2+\sqrt{6-x})}{x-2}$ $= \lim_{x \rightarrow 2} (2+\sqrt{6-x}) = 2+\sqrt{6-(2)}$ $= 2+2 = 4$	$30) \lim_{x \rightarrow 2} \frac{2-\sqrt{6-x}}{x+2} = \frac{2-\sqrt{6-(2)}}{(2)+2} = \frac{2-2}{4} = 0$ $31) \lim_{x \rightarrow 3} \frac{1-\sqrt{x-2}}{2-\sqrt{x+1}}$ $= \lim_{x \rightarrow 3} \left[\frac{1-\sqrt{x-2}}{2-\sqrt{x+1}} \times \frac{1+\sqrt{x-2}}{1+\sqrt{x-2}} \right]$ $\times \frac{2+\sqrt{x+1}}{2+\sqrt{x+1}}$ $= \lim_{x \rightarrow 3} \left[\frac{1-(x-2)}{4-(x+1)} \times \frac{2+\sqrt{x+1}}{1+\sqrt{x-2}} \right]$ $= \lim_{x \rightarrow 3} \left[\frac{3-x}{3-x} \times \frac{2+\sqrt{x+1}}{1+\sqrt{x-2}} \right]$ $= \lim_{x \rightarrow 3} \frac{2+\sqrt{x+1}}{1+\sqrt{x-2}} = \frac{2+\sqrt{(3)+1}}{1+\sqrt{(3)-2}} = \frac{2+2}{1+1}$ $= \frac{4}{2} = 2$

32) If $2x \leq f(x) \leq 3x^2 - 8$, then

$$\lim_{x \rightarrow 2} f(x) =$$

Solution:

$$\lim_{x \rightarrow 2} 2x = 2(2) = 4$$

and

$$\lim_{x \rightarrow 2} (3x^2 - 8) = 3(2)^2 - 8 = 12 - 8 = 4$$

It follows from the Sandwich Theorem that

$$\lim_{x \rightarrow 2} f(x) = 4$$

$$33) \lim_{x \rightarrow 0} \left[x \cos \left(x + \frac{1}{x} \right) \right] =$$

We know that the cosine of any angle is between -1 and 1 . So,

$$-1 \leq \cos \left(x + \frac{1}{x} \right) \leq 1$$

Now, multiply throughout by x , we get

$$-x \leq x \cos \left(x + \frac{1}{x} \right) \leq x$$

But $\lim_{x \rightarrow 0} x = 0$ and $\lim_{x \rightarrow 0} (-x) = 0$.

It follows from the Sandwich Theorem that

$$\lim_{x \rightarrow 0} \left[x \cos \left(x + \frac{1}{x} \right) \right] = 0$$

$$34) \lim_{x \rightarrow 0} \left[x \sin \left(\frac{1}{x} \right) \right] =$$

We know that the sine of any angle is between -1 and 1 . So,

$$-1 \leq \sin \left(\frac{1}{x} \right) \leq 1$$

Now, multiply throughout by x , we get

$$-x \leq x \sin \left(\frac{1}{x} \right) \leq x$$

But $\lim_{x \rightarrow 0} x = 0$ and $\lim_{x \rightarrow 0} (-x) = 0$.

It follows from the Sandwich Theorem that

$$\lim_{x \rightarrow 0} \left[x \sin \left(\frac{1}{x} \right) \right] = 0$$

35) If $\frac{x^2+1}{x-1} \leq f(x) \leq x-1$, then

$$\lim_{x \rightarrow 0} f(x) =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{x^2 + 1}{x - 1} = \frac{(0)^2 + 1}{(0) - 1} = \frac{1}{-1} = -1$$

and

$$\lim_{x \rightarrow 0} (x - 1) = (0) - 1 = -1$$

It follows from the Sandwich Theorem that

$$\lim_{x \rightarrow 0} f(x) = -1$$

36) If $4(x-1) \leq f(x) \leq x^3 + x - 2$, then

$$\lim_{x \rightarrow 1} f(x) =$$

Solution:

$$\lim_{x \rightarrow 1} (4(x-1)) = 4((1)-1) = 4 \times 0 = 0$$

and

$$\lim_{x \rightarrow 1} (x^3 + x - 2) = (1)^3 + (1) - 2 = 1 + 1 - 2 = 0$$

It follows from the Sandwich Theorem that

$$\lim_{x \rightarrow 1} f(x) = 0$$

37) If

$$\lim_{x \rightarrow 3} \frac{f(x) + 4}{x - 1} = 3,$$

then

$$\lim_{x \rightarrow 3} f(x) =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{f(x) + 4}{x - 1} &= \frac{\lim_{x \rightarrow 3} (f(x) + 4)}{\lim_{x \rightarrow 3} (x - 1)} = \frac{\lim_{x \rightarrow 3} f(x) + \lim_{x \rightarrow 3} (4)}{\lim_{x \rightarrow 3} (x) - \lim_{x \rightarrow 3} (1)} \\ &= \frac{\lim_{x \rightarrow 3} f(x) + 4}{3 - 1} = \frac{\lim_{x \rightarrow 3} f(x) + 4}{2} \end{aligned}$$

Now

$$\frac{\lim_{x \rightarrow 3} f(x) + 4}{2} = 3$$

$$\lim_{x \rightarrow 3} f(x) + 4 = 6 \Leftrightarrow \lim_{x \rightarrow 3} f(x) = 2$$

$$\begin{aligned}
 38) \lim_{x \rightarrow 2} \frac{2^{-1} - (3x - 4)^{-1}}{2 - x} &= \lim_{x \rightarrow 2} \frac{\frac{1}{2} - \frac{1}{3x - 4}}{2 - x} \\
 &= \lim_{x \rightarrow 2} \frac{\frac{3x - 4 - 2}{2(3x - 4)}}{2 - x} \\
 &= \lim_{x \rightarrow 2} \frac{\frac{3x - 6}{2(3x - 4)}}{2 - x} \\
 &= \lim_{x \rightarrow 2} \frac{\frac{3(x - 2)}{2(3x - 4)}}{2 - x} \\
 &= \lim_{x \rightarrow 2} \frac{3(x - 2)}{2(3x - 4)(2 - x)} \\
 &= \lim_{x \rightarrow 2} \frac{-3(2 - x)}{2(3x - 4)(2 - x)} = \lim_{x \rightarrow 2} \frac{-3}{2(3x - 4)} \\
 &= \frac{-3}{2(3(2) - 4)} = \frac{-3}{2 \times 2} = -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 39) \lim_{x \rightarrow 0} \frac{(x + 1)^3 - 1}{x} &= \lim_{x \rightarrow 0} \frac{(x^3 + 3x^2 + 3x + 1) - 1}{x} \\
 &= \lim_{x \rightarrow 0} \frac{x^3 + 3x^2 + 3x}{x} \\
 &= \lim_{x \rightarrow 0} \frac{x(x^2 + 3x + 3)}{x} = \lim_{x \rightarrow 0} (x^2 + 3x + 3) \\
 &= (0)^2 + 3(0) + 3 = 3
 \end{aligned}$$

40) If

$$\lim_{x \rightarrow 1} \frac{f(x) + 3x}{x^2 - 5f(x)} = 1,$$

then

$$\lim_{x \rightarrow 1} f(x) =$$

Solution:

$$\begin{aligned}
 \lim_{x \rightarrow 1} \frac{f(x) + 3x}{x^2 - 5f(x)} &= \frac{\lim_{x \rightarrow 1} (f(x) + 3x)}{\lim_{x \rightarrow 1} (x^2 - 5f(x))} \\
 &= \frac{\lim_{x \rightarrow 1} f(x) + \lim_{x \rightarrow 1} (3x)}{\lim_{x \rightarrow 1} (x^2) - \lim_{x \rightarrow 1} (5f(x))} \\
 &= \frac{\lim_{x \rightarrow 1} f(x) + 3(1)}{(1)^2 - 5 \lim_{x \rightarrow 1} f(x)} = \frac{\lim_{x \rightarrow 1} f(x) + 3}{1 - 5 \lim_{x \rightarrow 1} f(x)}
 \end{aligned}$$

Now

$$\frac{\lim_{x \rightarrow 1} f(x) + 3}{1 - 5 \lim_{x \rightarrow 1} f(x)} = 1$$

$$\begin{aligned}
 \lim_{x \rightarrow 1} f(x) + 3 &= (1) \left(1 - 5 \lim_{x \rightarrow 1} f(x) \right) \\
 \Leftrightarrow \lim_{x \rightarrow 1} f(x) + 3 &= 1 - 5 \lim_{x \rightarrow 1} f(x) \\
 \Leftrightarrow \lim_{x \rightarrow 1} f(x) + 5 \lim_{x \rightarrow 1} f(x) &= 1 - 3 \\
 \Leftrightarrow 6 \lim_{x \rightarrow 1} f(x) &= -2 \\
 \Leftrightarrow \lim_{x \rightarrow 1} f(x) &= \frac{-2}{6} = -\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 41) \lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x^2 + x - 20} &= \lim_{x \rightarrow 4} \frac{(x - 2)(x - 4)}{(x - 4)(x + 5)} \\
 &= \lim_{x \rightarrow 4} \frac{x - 2}{x + 5} = \frac{(4) - 2}{(4) + 5} = \frac{2}{9}
 \end{aligned}$$

$$\begin{aligned}
 42) \lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - x - 6} &= \lim_{x \rightarrow -2} \frac{(x + 2)(x^2 - 2x + 4)}{(x - 3)(x + 2)} \\
 &= \lim_{x \rightarrow -2} \frac{x^2 - 2x + 4}{x - 3} = \frac{(-2)^2 - 2(-2) + 4}{(-2) - 3} \\
 &= \frac{4 + 4 + 4}{-5} = \frac{12}{-5} = -\frac{12}{5}
 \end{aligned}$$

$$\begin{aligned}
 43) \lim_{x \rightarrow 1} \left[\frac{x^2 - 2}{x + 4} + x^2 - 2x \right] &= \frac{(1)^2 - 2}{(1) + 4} + (1)^2 - 2(1) \\
 &= \frac{1 - 2}{1 + 4} + 1 - 2 = \frac{-1}{5} - 1 = \frac{-1 - 5}{5} = -\frac{6}{5}
 \end{aligned}$$

$$\begin{aligned}
 44) \lim_{x \rightarrow -2} \frac{4x^2 + 6x - 4}{2x^2 - 8} &= \lim_{x \rightarrow -2} \frac{2(2x^2 + 3x - 2)}{2(x^2 - 4)} \\
 &= \lim_{x \rightarrow -2} \frac{2x^2 + 3x - 2}{x^2 - 4} \\
 &= \lim_{x \rightarrow -2} \frac{(2x - 1)(x + 2)}{(x - 2)(x + 2)} \\
 &= \lim_{x \rightarrow -2} \frac{2x - 1}{x - 2} = \frac{2(-2) - 1}{(-2) - 2} = \frac{-4 - 1}{-2 - 2} \\
 &= \frac{-5}{-4} = \frac{5}{4}
 \end{aligned}$$

$$\begin{aligned}
 45) \lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x^5 - x^3} &= \lim_{x \rightarrow -1} \frac{(x - 3)(x + 1)}{x^3(x^2 - 1)} \\
 &= \lim_{x \rightarrow -1} \frac{(x - 3)(x + 1)}{x^3(x - 1)(x + 1)} \\
 &= \lim_{x \rightarrow -1} \frac{x - 3}{x^3(x - 1)} = \frac{(-1) - 3}{(-1)^3((-1) - 1)} \\
 &= \frac{-1 - 3}{(-1)(-2)} = \frac{-4}{2} = -2
 \end{aligned}$$

$$\begin{aligned}
 46) \lim_{x \rightarrow 3} \frac{\sqrt{2x + 1}(x^2 - 9)}{(2x + 3)(x - 3)} &= \lim_{x \rightarrow 3} \frac{\sqrt{2x + 1}(x - 3)(x + 3)}{(2x + 3)(x - 3)} \\
 &= \lim_{x \rightarrow 3} \frac{\sqrt{2x + 1}(x + 3)}{2x + 3} = \frac{\sqrt{2(3) + 1}((3) + 3)}{2(3) + 3} \\
 &= \frac{6\sqrt{7}}{9} = \frac{2\sqrt{7}}{3}
 \end{aligned}$$

$$\begin{aligned}
 47) \lim_{x \rightarrow 1} \frac{\sqrt{3 - 2x} - 1}{x - 1} &= \lim_{x \rightarrow 1} \left[\frac{\sqrt{3 - 2x} - 1}{x - 1} \times \frac{\sqrt{3 - 2x} + 1}{\sqrt{3 - 2x} + 1} \right] \\
 &= \lim_{x \rightarrow 1} \frac{(3 - 2x) - 1}{(x - 1)(\sqrt{3 - 2x} + 1)} \\
 &= \lim_{x \rightarrow 1} \frac{2 - 2x}{(x - 1)(\sqrt{3 - 2x} + 1)} \\
 &= \lim_{x \rightarrow 1} \frac{2(1 - x)}{(x - 1)(\sqrt{3 - 2x} + 1)} = \\
 &= \lim_{x \rightarrow 1} \frac{-2(x - 1)}{(x - 1)(\sqrt{3 - 2x} + 1)} = \\
 &= \lim_{x \rightarrow 1} \frac{-2}{\sqrt{3 - 2x} + 1} = \frac{-2}{\sqrt{3 - 2(1)} + 1} \\
 &= \frac{-2}{\sqrt{3 - 2} + 1} = \frac{-2}{2} = -1
 \end{aligned}$$

$$\begin{aligned}
 48) \lim_{x \rightarrow 0} \frac{(x + 1)^2 - 1}{x} &= \lim_{x \rightarrow 0} \frac{(x^2 + 2x + 1) - 1}{x} \\
 &= \lim_{x \rightarrow 0} \frac{x^2 + 2x}{x} = \lim_{x \rightarrow 0} \frac{x(x + 2)}{x} \\
 &= \lim_{x \rightarrow 0} (x + 2) = (0) + 2 = 2
 \end{aligned}$$

$$\begin{aligned}
 49) \lim_{x \rightarrow 1} \frac{\sqrt{2x + 2} - 2}{\sqrt{3x - 2} - 1} &= \lim_{x \rightarrow 1} \left[\frac{\sqrt{2x + 2} - 2}{\sqrt{3x - 2} - 1} \times \frac{\sqrt{2x + 2} + 2}{\sqrt{2x + 2} + 2} \times \frac{\sqrt{3x - 2} + 1}{\sqrt{3x - 2} + 1} \right] \\
 &= \lim_{x \rightarrow 1} \left[\frac{(2x + 2) - 4}{(3x - 2) - 1} \times \frac{\sqrt{3x - 2} + 1}{\sqrt{2x + 2} + 2} \right] \\
 &= \lim_{x \rightarrow 1} \left[\frac{2x - 2}{3x - 3} \times \frac{\sqrt{3x - 2} + 1}{\sqrt{2x + 2} + 2} \right] \\
 &= \lim_{x \rightarrow 1} \left[\frac{2(x - 1)}{3(x - 1)} \times \frac{\sqrt{3x - 2} + 1}{\sqrt{2x + 2} + 2} \right] \\
 &= \lim_{x \rightarrow 1} \left[\frac{2}{3} \times \frac{\sqrt{3x - 2} + 1}{\sqrt{2x + 2} + 2} \right] = \frac{2}{3} \times \frac{\sqrt{3(1) - 2} + 1}{\sqrt{2(1) + 2} + 2} \\
 &= \frac{2}{3} \times \frac{\sqrt{1} + 1}{\sqrt{4} + 2} = \frac{2}{3} \times \frac{2}{4} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 50) \lim_{x \rightarrow 2} \frac{3 - \sqrt{2x + 5}}{x - 2} &= \lim_{x \rightarrow 2} \left[\frac{3 - \sqrt{2x + 5}}{x - 2} \times \frac{3 + \sqrt{2x + 5}}{3 + \sqrt{2x + 5}} \right] \\
 &= \lim_{x \rightarrow 2} \frac{9 - (2x + 5)}{(x - 2)(3 + \sqrt{2x + 5})} \\
 &= \lim_{x \rightarrow 2} \frac{4 - 2x}{2(2 - x)} \\
 &= \lim_{x \rightarrow 2} \frac{2(2 - x)}{2(2 - x)} \\
 &= \lim_{x \rightarrow 2} \frac{-2}{-2} \\
 &= \lim_{x \rightarrow 2} \frac{-2}{3 + \sqrt{2x + 5}} = \frac{-2}{3 + \sqrt{2(2) + 5}} \\
 &= \frac{-2}{3 + \sqrt{9}} = \frac{-2}{6} = -\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 53) \lim_{x \rightarrow 0} \frac{\sqrt{x + 4} - 2}{x} &= \lim_{x \rightarrow 0} \left[\frac{\sqrt{x + 4} - 2}{x} \times \frac{\sqrt{x + 4} + 2}{\sqrt{x + 4} + 2} \right] \\
 &= \lim_{x \rightarrow 0} \frac{(x + 4) - 4}{x(\sqrt{x + 4} + 2)} \\
 &= \lim_{x \rightarrow 0} \frac{1}{x(\sqrt{x + 4} + 2)} \\
 &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x + 4} + 2} = \frac{1}{\sqrt{(0) + 4} + 2} \\
 &= \frac{1}{\sqrt{4} + 2} = \frac{1}{4}
 \end{aligned}$$

56) If

$$\lim_{x \rightarrow 1} f(x) = 3$$

and

$$\lim_{x \rightarrow 1} g(x) = -4$$

then

$$\lim_{x \rightarrow 1} h(x) = -1$$

then

$$\begin{aligned}
 \lim_{x \rightarrow 1} \left[\frac{5f(x)}{2g(x)} + h(x) \right] &= \frac{\lim_{x \rightarrow 1} 5f(x)}{\lim_{x \rightarrow 1} 2g(x)} + \lim_{x \rightarrow 1} h(x) \\
 &= \frac{5 \lim_{x \rightarrow 1} f(x)}{2 \lim_{x \rightarrow 1} g(x)} + \lim_{x \rightarrow 1} h(x) \\
 &= \frac{5(3)}{2(-4)} + (-1) = \frac{15}{-8} - 1 = -\frac{15}{8} - 1 \\
 &= \frac{-15 - 8}{8} = -\frac{23}{8}
 \end{aligned}$$

$$\begin{aligned}
 51) \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 + 1} &= \frac{(-1)^2 + 3(-1) + 2}{(-1)^2 + 1} = \frac{1 - 3 + 2}{1 + 1} \\
 &= \frac{0}{2} = 0
 \end{aligned}$$

52) If

$$\lim_{x \rightarrow k} f(x) = -\frac{1}{2}$$

and

$$\lim_{x \rightarrow k} g(x) = \frac{2}{3}$$

Then

$$\lim_{x \rightarrow k} \frac{f(x)}{g(x)} = \frac{-\frac{1}{2}}{\frac{2}{3}} = -\frac{1}{2} \times \frac{3}{2} = -\frac{3}{4}$$

$$\begin{aligned}
 54) \lim_{x \rightarrow -1} \frac{x^2 - 5x - 6}{x + 1} &= \lim_{x \rightarrow -1} \frac{(x - 6)(x + 1)}{x + 1} = \lim_{x \rightarrow -1} (x - 6) \\
 &= (-1) - 6 = -7
 \end{aligned}$$

$$\begin{aligned}
 55) \lim_{x \rightarrow 0} \frac{(x + 3)^{-1} - 3^{-1}}{x} &= \lim_{x \rightarrow 0} \frac{\frac{1}{x + 3} - \frac{1}{3}}{x} = \lim_{x \rightarrow 0} \frac{3 - (x + 3)}{3(x + 3)x} \\
 &= \lim_{x \rightarrow 0} \frac{-x}{3x(x + 3)} = \lim_{x \rightarrow 0} \frac{-1}{3(x + 3)} \\
 &= \frac{-1}{3((0) + 3)} = \frac{-1}{9} = -\frac{1}{9}
 \end{aligned}$$

57) If

$$\lim_{x \rightarrow 1} g(x) = -4$$

and

$$\lim_{x \rightarrow 1} h(x) = -1$$

then

$$\begin{aligned}
 \lim_{x \rightarrow 1} \sqrt{g(x)h(x)} &= \sqrt{\left[\lim_{x \rightarrow 1} g(x) \right] \left[\lim_{x \rightarrow 1} h(x) \right]} = \sqrt{(-4)(-1)} \\
 &= \sqrt{4} = 2
 \end{aligned}$$

58) If

$$\lim_{x \rightarrow 1} f(x) = 3$$

$$\lim_{x \rightarrow 1} g(x) = -4$$

and

$$\lim_{x \rightarrow 1} h(x) = -1$$

then

$$\begin{aligned}
 \lim_{x \rightarrow 1} [2f(x)g(x)h(x)] &= 2 \left[\lim_{x \rightarrow 1} f(x) \right] \left[\lim_{x \rightarrow 1} g(x) \right] \left[\lim_{x \rightarrow 1} h(x) \right] \\
 &= 2(3)(-4)(-1) = 24
 \end{aligned}$$