Workshop Solutions to Sections 2.1 and 2.2 (1.1 & 1.2)

1) Find the domain of the function $f(x) = 9 - x^2$.
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$
Note: The domain of any polynomial is $\mathbb{R}$.

2) Find the range of the function $f(x) = 9 - x^2$.
Solution:
$$R_f = (-\infty, 9]$$

3) Find the domain of the function $f(x) = 6 - 2x$.
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

4) Find the range of the function $f(x) = 6 - 2x$.
Solution:
Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then
$$R_f = \mathbb{R} = (-\infty, \infty)$$

5) Find the domain of the function $f(x) = x^2 - 2x - 3$.
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$.
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

7) Find the domain of the function $f(x) = 5$.
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

8) Find the range of the function $f(x) = 5$.
Solution:
$$R_f = \{5\}$$

9) Find the domain of the function $f(x) = |x - 1|$.
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$
Note: The domain of an absolute value of any polynomial is $\mathbb{R}$.

10) Find the domain of the function $f(x) = |x + 5|$.
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

11) Find the domain of the function $f(x) = |x|$.
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

12) Find the range of the function $f(x) = |x|$.
Solution:
$$R_f = [0, \infty)$$
Note: The range of an absolute value of any polynomial is always $[0, \infty)$.

13) Find the domain of the function $f(x) = |3x - 6|$.
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

14) Find the domain of the function $f(x) = |9 - 3x|$.
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

15) Find the domain of the function
$$f(x) = \frac{x + 2}{x - 3}$$
Solution:
$f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So,
$$D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$$

16) Find the domain of the function
$$f(x) = \frac{x - 2}{x + 3}$$
Solution:
$f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So,
$$D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$$

17) Find the domain of the function $$f(x)=\frac{x+2}{x^2-9}$$ Solution: $f(x)$ is defined when $x^2-9\neq 0 \Longrightarrow x^2\neq 9 \Longrightarrow x\neq \pm 3$. So, $$D_f=\mathbb{R}\setminus\{-3,3\}=(-\infty,-3)\cup(-3,3)\cup(3,\infty)$$	18) Find the domain of the function $$f(x)=\frac{x+2}{x^2-5x+6}$$ Solution: $f(x)$ is defined when $x^2-5x+6\neq 0$ $\Longrightarrow (x-2)(x-3)\neq 0 \Longrightarrow x\neq 2$ or $x\neq 3$. So, $$D_f=\mathbb{R}\setminus\{2,3\}=(-\infty,2)\cup(2,3)\cup(3,\infty)$$
19) Find the domain of the function $$f(x)=\frac{x+2}{x^2-x-6}$$ Solution: $f(x)$ is defined when $x^2-x-6\neq 0$ $\Longrightarrow (x+2)(x-3)\neq 0 \Longrightarrow x\neq -2$ or $x\neq 3$. So, $$D_f=\mathbb{R}\setminus\{-2,3\}=(-\infty,-2)\cup(-2,3)\cup(3,\infty)$$	20) Find the domain of the function $$f(x)=\frac{x+2}{x^2+9}$$ Solution: $f(x)$ is defined when $x^2+9\neq 0$ but for any value x the denominator x^2+9 cannot be 0. So, $$D_f=\mathbb{R}=(-\infty,\infty)$$
21) Find the domain of the function $$f(x)=\sqrt[3]{x-3}$$ Solution: $$D_f=\mathbb{R}=(-\infty,\infty)$$ **Note:** The domain of an odd root of any polynomial is $\mathbb{R}$.	22) Find the domain of the function $$f(x)=\sqrt{x-3}$$ Solution: $f(x)$ is defined when $x-3\geq 0 \Longrightarrow x\geq 3$ because $f(x)$ is an even root. So, $$D_f=[3,\infty)$$
23) Find the domain of the function $$f(x)=\sqrt{3-x}$$ Solution: $f(x)$ is defined when $3-x\geq 0 \Longrightarrow -x\geq -3 \Longrightarrow x\leq 3$ because $f(x)$ is an even root. So, $$D_f=(-\infty,3]$$	24) Find the domain of the function $$f(x)=\sqrt{x+3}$$ Solution: $f(x)$ is defined when $x+3\geq 0 \Longrightarrow x\geq -3$ because $f(x)$ is an even root. So, $$D_f=[-3,\infty)$$
25) Find the domain of the function $$f(x)=\sqrt{-x}$$ Solution: $f(x)$ is defined when $-x\geq 0 \Longrightarrow x\leq 0$ because $f(x)$ is an even root. So, $$D_f=(-\infty,0]$$	26) Find the range of the function $$f(x)=\sqrt{-x}$$ Solution: $$R_f=[0,\infty)$$ **Note:** The range of an even root is always ≥ 0.
27) Find the domain of the function $$f(x)=\sqrt{9-x^2}$$ Solution: $f(x)$ is defined when $9-x^2\geq 0 \Longrightarrow -x^2\geq -9 \Longrightarrow$ $x^2\leq 9 \Longrightarrow \sqrt{x^2}\leq\sqrt{9} \Longrightarrow \lvert x\rvert\leq 3 \Longrightarrow -3\leq x\leq 3$. So, $$D_f=[-3,3]$$	28) Find the domain of the function $$f(x)=\frac{x+2}{\sqrt{x-3}}$$ Solution: $f(x)$ is defined when $x-3>0 \Longrightarrow x>3$. So, $$D_f=(3,\infty)$$
29) Find the domain of the function $$f(x)=\frac{x+2}{\sqrt{9-x^2}}$$ Solution: $f(x)$ is defined when $9-x^2>0 \Longrightarrow -x^2>-9$ $\Longrightarrow x^2<9 \Longrightarrow \sqrt{x^2}<\sqrt{9} \Longrightarrow \lvert x\rvert<3 \Longrightarrow -3<x<3$. So, $$D_f=(-3,3)$$	30) Find the domain of the function $$f(x)=\sqrt{x^2-9}$$ Solution: $f(x)$ is defined when $x^2-9\geq 0 \Longrightarrow x^2\geq 9$ $\Longrightarrow \sqrt{x^2}\geq\sqrt{9} \Longrightarrow \lvert x\rvert\geq 3 \Longrightarrow x\geq 3$ or $x\leq -3$. So, $$D_f=(-\infty,-3]\cup[3,\infty)$$

31) Find the range of the function
$$f(x)=\sqrt{x^2-9}$$
Solution:
$$R_f=[0,\infty)$$

32) Find the domain of the function
$$f(x)=\frac{x+2}{\sqrt{x^2-9}}$$
Solution:
$f(x)$ is defined when $x^2-9>0 \implies x^2>9$
$\implies \sqrt{x^2}>\sqrt{9} \implies |x|>3 \implies x>3$ or $x<-3$.
So,
$$D_f=(-\infty,-3)\cup(3,\infty)$$

33) Find the domain of the function
$$f(x)=\sqrt{9+x^2}$$
Solution:
$f(x)$ is defined when $9+x^2\geq 0$ but it is always true for any value x . So,
$$D_f=\mathbb{R}$$

34) Find the domain of the function
$$f(x)=\sqrt[4]{x^2-25}$$
Solution:
$f(x)$ is defined when $x^2-25\geq 0 \implies x^2\geq 25$
$\implies \sqrt{x^2}\geq\sqrt{25} \implies |x|\geq 5 \implies x\geq 5$ or $x\leq -5$.
So,
$$D_f=(-\infty,-5]\cup[5,\infty)$$

35) Find the domain of the function
$$f(x)=\sqrt[6]{16-x^2}$$
Solution:
$f(x)$ is defined when $16-x^2\geq 0 \implies -x^2\geq -16 \implies$
$x^2\leq 16 \implies \sqrt{x^2}\leq\sqrt{16} \implies |x|\leq 4 \implies -4\leq x\leq 4$.
So,
$$D_f=[-4,4]$$

36) Find the range of the function
$$f(x)=\sqrt{16-x^2}$$
Solution:
We know that $f(x)$ is defined when $16-x^2\geq 0$
$\implies -x^2\geq -16 \implies x^2\leq 16 \implies \sqrt{x^2}\leq\sqrt{16}$
$\implies |x|\leq 4 \implies -4\leq x\leq 4$. So,
$$D_f=[-4,4]$$
Using D_f we find the outputs vary from 0 to 4 . Hence,
$$R_f=[0,4]$$

37) Find the domain of the function
$$f(x)=\frac{x+|x|}{x}$$
Solution:
$f(x)$ is defined when $x\neq 0$. So,
$$D_f=\mathbb{R}\setminus\{0\}=(-\infty,0)\cup(0,\infty)$$

38) Find the domain of the function
$$f(x)=\begin{cases}-\dfrac{1}{x}, & x<0\\ x, & x\geq 0\end{cases}$$
Solution:
It is clear from the definition of the function $f(x)$ that
$$D_f=\mathbb{R}=(-\infty,\infty)$$

39) Find the domain of the function
$$f(x)=\frac{2-\sqrt{x}}{\sqrt{x^2+1}}$$
Solution:
$f(x)$ is defined when
1- $x\geq 0 \quad\implies\quad D_{\sqrt{x}}=[0,\infty)$
2- $x^2+1>0$ but this is always true for all x
$\implies D_{\sqrt{x^2+1}}=\mathbb{R}$.
Hence,
$$D_f=D_{\sqrt{x}}\cap D_{\sqrt{x^2+1}}=[0,\infty)\cap\mathbb{R}=[0,\infty)$$

40) Find the domain of the function
$$f(x)=\sqrt{x-1}+\sqrt{x+3}$$
Solution:
$f(x)$ is defined when
1- $x-1\geq 0 \implies x\geq 1 \implies D_{\sqrt{x-1}}=[1,\infty)$
2- $x+3\geq 0 \implies x\geq -3 \implies D_{\sqrt{x+3}}=[-3,\infty)$
Hence,
$$D_f=D_{\sqrt{x-1}}\cap D_{\sqrt{x+3}}=[1,\infty)\cap[-3,\infty)=[1,\infty)$$

41) The function $f(x)=3x^4+x^2+1$ is a polynomial function.

42) The function $f(x)=5x^3+x^2+7$ is a cubic function.

43) The function $f(x)=-3x^2+7$ is a quadratic function.

44) The function $f(x)=2x+3$ is a linear function.

45) The function $f(x)=x^7$ is a power function.

46) The function $f(x)=\frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x)=\frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x)=\sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.

50) The function $f(x) = 3^x$ is a general exponential function.

51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.

52) The function $f(x) = -3$ is a constant function.

53) The function $f(x) = \log_3 x$ is a general logarithmic function.

54) The function $f(x) = \ln x$ is a natural logarithmic function.

55) The function $f(x) = 3x^4 + x^2 + 1$ is

Solution:

$$f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$$

Hence,
$f(x)$ is an even function.

56) The function $f(x) = 9 - x^2$ is

Solution:

$$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$$

Hence,
$f(x)$ is an even function.

57) The function $f(x) = x^5 - x$ is

Solution:

$$\begin{aligned} f(-x) &= (-x)^5 - (-x) = -x^5 + x \\ &= -(x^5 - x) = -f(x) \end{aligned}$$

Hence,
$f(x)$ is an odd function.

58) The function $f(x) = 2 - \sqrt[5]{x}$ is

Solution:

$$\begin{aligned} f(-x) &= 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x} \\ &= -\left(-2 - \sqrt[5]{x}\right) \end{aligned}$$

Hence,
$f(x)$ is neither even nor odd.

59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is

Solution:

$$\begin{aligned} f(-x) &= 3(-x) + \frac{2}{\sqrt{(-x)^2+9}} = -3x + \frac{2}{\sqrt{x^2+9}} \\ &= -\left(3x - \frac{2}{\sqrt{x^2+9}}\right) \end{aligned}$$

Hence,
$f(x)$ is neither even nor odd.

60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is

Solution:

$$f(-x) = \frac{3}{\sqrt{(-x)^2+9}} = \frac{3}{\sqrt{x^2+9}} = f(x)$$

Hence,
$f(x)$ is an even function.

61) The function $f(x) = \sqrt{4 + x^2}$ is

Solution:

$$f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$$

Hence,
$f(x)$ is an even function.

62) The function $f(x) = 3$ is

Solution:

Since the graph of the constant function 3 is symmetric about the $y - axis$, then
$f(x)$ is an even function.

63) The function $f(x) = \frac{9-x^2}{x-2}$ is

Solution:

$$\begin{aligned} f(-x) &= \frac{9-(-x)^2}{(-x)-2} = \frac{9-x^2}{-x-2} \\ &= -\left(\frac{9-x^2}{x+2}\right) \end{aligned}$$

Hence,
$f(x)$ is neither even nor odd.

64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is

Solution:

$$f(-x) = \frac{(-x)^2-4}{(-x)^2+1} = \frac{x^2-4}{x^2+1} = f(x)$$

Hence,
$f(x)$ is an even function.

65) The function $f(x) = 3|x|$ is

Solution:

$$f(-x) = 3|(-x)| = 3|x| = f(x)$$

Hence,
$f(x)$ is an even function.

66) The function $f(x) = x^{-2}$ is

Solution:

$$f(x) = x^{-2} = \frac{1}{x^2}$$

$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is <u>Solution:</u> $f(-x) = (-x)^3 - 2(-x) + 5 \; = -x^3 + 2x + 5$ $= -(x^3 - 2x - 5)$ Hence, $f(x)$ is neither even nor odd.	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is <u>Solution:</u> $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \; = -\sqrt[3]{x^5} + x^3 - x$ $= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$ Hence, $f(x)$ is an odd function.
69) The function $f(x) = 7$ is <u>Solution:</u> Since the graph of the constant function 7 is symmetric about the $y - axis$, then $f(x)$ is an even function.	70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^3 - 4}{(-x)^3 + 1} = \dfrac{-x^3 - 4}{-x^3 + 1} = -\dfrac{x^3 + 4}{-x^3 + 1}$ Hence, $f(x)$ is neither even nor odd.
71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^2 - 1}{(-x)^3 + 3} = \dfrac{x^2 - 1}{-x^3 + 3} = -\dfrac{x^2 - 1}{x^3 - 3}$ Hence, $f(x)$ is neither even nor odd.	72) The function $f(x) = x^6 - 4x^2 + 1$ is <u>Solution:</u> $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)\backslash\{0\}$