Workshop Solutions to Sections 3.4 and 3.5 (2.2 & 2.5)

1) $\lim\limits_{x\to3^+}\dfrac{2}{x-3}=$

Solution:

If $x\to3^+$, then $x>3 \implies x-3>0$

$$\therefore \lim_{x\to3^+}\frac{2}{x-3}=\infty$$

2) $\lim\limits_{x\to3^-}\dfrac{2}{x-3}=$

Solution:

If $x\to3^-$, then $x<3 \implies x-3<0$

$$\therefore \lim_{x\to3^-}\frac{2}{x-3}=-\infty$$

3) $\lim\limits_{x\to3^+}\dfrac{-2}{x-3}=$

Solution:

If $x\to3^+$, then $x>3 \implies x-3>0$

$$\therefore \lim_{x\to3^+}\frac{-2}{x-3}=-\infty$$

4) $\lim\limits_{x\to3^-}\dfrac{-2}{x-3}=$

Solution:

If $x\to3^-$, then $x<3 \implies x-3<0$

$$\therefore \lim_{x\to3^-}\frac{2}{x-3}=\infty$$

5) $\lim\limits_{x\to-3^+}\dfrac{2}{x+3}=$

Solution:

If $x\to-3^+$, then $x>-3 \implies x+3>0$

$$\therefore \lim_{x\to-3^+}\frac{2}{x+3}=\infty$$

6) $\lim\limits_{x\to-3^-}\dfrac{2}{x+3}=$

Solution:

If $x\to-3^-$, then $x<-3 \implies x+3<0$

$$\therefore \lim_{x\to-3^-}\frac{2}{x+3}=-\infty$$

7) $\lim\limits_{x\to2^+}\dfrac{3x-1}{x-2}=$

Solution:

If $x\to2^+$, then $x>2 \implies x-2>0$ and $3x-1>0$

$$\therefore \lim_{x\to2^+}\frac{3x-1}{x-2}=\infty$$

8) $\lim\limits_{x\to2^-}\dfrac{3x-1}{x-2}=$

Solution:

If $x\to2^-$, then $x<2 \implies x-2<0$ and $3x-1>0$

$$\therefore \lim_{x\to2^-}\frac{3x-1}{x-2}=-\infty$$

9) $\lim\limits_{x\to-2^+}\dfrac{1-x}{(x+2)^2}=$

Solution:

If $x\to-2^+$, then $x>-2$

$\implies 1-x>0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to-2^+}\frac{1-x}{(x+2)^2}=\infty$$

10) $\lim\limits_{x\to-2^-}\dfrac{1-x}{(x+2)^2}=$

Solution:

If $x\to-2^-$, then $x<-2$

$\implies 1-x>0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to-2^+}\frac{1-x}{(x+2)^2}=\infty$$

11) $\lim\limits_{x\to-2^+}\dfrac{x-1}{(x+2)^2}=$

Solution:

If $x\to-2^+$, then $x>-2$

$\implies x-1<0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to-2^+}\frac{x-1}{(x+2)^2}=-\infty$$

12) $\lim\limits_{x\to-2^-}\dfrac{x-1}{(x+2)^2}=$

Solution:

If $x\to-2^-$, then $x<-2$

$\implies x-1<0$ and $(x+2)^2>0$

$$\therefore \lim_{x\to-2^-}\frac{x-1}{(x+2)^2}=-\infty$$

13) $\lim\limits_{x\to2^+}\dfrac{6x-1}{x^2-4}=$

Solution:

If $x\to2^+$, then $x^2>4$

$\implies x^2-4>0$ and $6x-1>0$

$$\therefore \lim_{x\to2^+}\frac{6x-1}{x^2-4}=\infty$$

14) $\lim\limits_{x\to2^-}\dfrac{6x-1}{x^2-4}=$

Solution:

If $x\to2^-$, then $x^2<4$

$\implies x^2-4<0$ and $6x-1>0$

$$\therefore \lim_{x\to2^+}\frac{6x-1}{x^2-4}=-\infty$$

15) $\lim\limits_{x \to -2^+} \dfrac{6x-1}{x^2-4} =$

Solution:

If $x \to -2^+$, then $x^2 < 4$

$$\Longrightarrow \; x^2 - 4 < 0 \text{ and } 6x - 1 < 0$$

$$\therefore \quad \lim_{x \to 2^+} \frac{6x-1}{x^2-4} = \infty$$

16) $\lim\limits_{x \to -2^-} \dfrac{6x-1}{x^2-4} =$

Solution:

If $x \to -2^-$, then $x^2 > 4$

$$\Longrightarrow \; x^2 - 4 > 0 \text{ and } 6x - 1 < 0$$

$$\therefore \quad \lim_{x \to 2^+} \frac{6x-1}{x^2-4} = -\infty$$

17) $\lim\limits_{x \to -2^-} \dfrac{6x-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{6x-1}{x^2-x-6} = \frac{6x-1}{(x-3)(x+2)}$$

If $x \to -2^-$, then $x < -2$

$$\Longrightarrow \; x - 3 < 0, \; x + 2 < 0 \text{ and } 6x - 1 < 0$$

$$\therefore \quad \lim_{x \to -2^-} \frac{6x-1}{x^2-x-6} = -\infty$$

18) $\lim\limits_{x \to -2^+} \dfrac{6x-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{6x-1}{x^2-x-6} = \frac{6x-1}{(x-3)(x+2)}$$

If $x \to -2^+$, then $x > -2$

$$\Longrightarrow \; x - 3 < 0, \; x + 2 > 0 \text{ and } 6x - 1 < 0$$

$$\therefore \quad \lim_{x \to -2^+} \frac{6x-1}{x^2-x-6} = \infty$$

19) $\lim\limits_{x \to 3^+} \dfrac{-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{-1}{x^2-x-6} = \frac{-1}{(x-3)(x+2)}$$

If $x \to 3^+$, then $x > 3$

$$\Longrightarrow \; x - 3 > 0, \; x + 2 > 0 \text{ and } -1 < 0$$

$$\therefore \quad \lim_{x \to 3^+} \frac{-1}{x^2-x-6} = -\infty$$

20) $\lim\limits_{x \to 3^-} \dfrac{-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{-1}{x^2-x-6} = \frac{-1}{(x-3)(x+2)}$$

If $x \to 3^-$, then $x < 3$

$$\Longrightarrow \; x - 3 < 0, \; x + 2 > 0 \text{ and } -1 < 0$$

$$\therefore \quad \lim_{x \to 3^-} \frac{-1}{x^2-x-6} = \infty$$

21) $\lim\limits_{x \to (\pi/2)^+} \tan x =$

Solution:

$$\lim_{x \to (\pi/2)^+} \tan x = -\infty$$

22) $\lim\limits_{x \to (\pi/2)^-} \tan x =$

Solution:

$$\lim_{x \to (\pi/2)^-} \tan x = \infty$$

23) The vertical asymptote of $f(x) = \frac{1-x}{2x+1}$ is

Solution:

We see that the function $f(x)$ is not defined when $2x + 1 = 0 \;\Longrightarrow\; x = -\frac{1}{2}$. Since

$$\lim_{x \to \left(-\frac{1}{2}\right)^+} \frac{1-x}{2x+1} = \infty$$

and

$$\lim_{x \to \left(-\frac{1}{2}\right)^-} \frac{1-x}{2x+1} = -\infty$$

then, $x = -\frac{1}{2}$ is a vertical asymptote.

24) The vertical asymptote of $f(x) = \frac{3-x}{x^2-4}$ is

Solution:

We see that the function $f(x)$ is not defined when $x^2 - 4 = 0 \;\Longrightarrow\; x = \pm 2$. Since

$$\lim_{x \to 2^+} \frac{3-x}{x^2-4} = \infty, \qquad \lim_{x \to 2^-} \frac{3-x}{x^2-4} = -\infty$$

and

$$\lim_{x \to -2^+} \frac{3-x}{x^2-4} = -\infty, \qquad \lim_{x \to -2^-} \frac{3-x}{x^2-4} = \infty$$

then, $x = \pm 2$ are vertical asymptotes.

25) The vertical asymptote of $f(x) = \frac{3-x}{x^2-x-6}$ is

Solution:

$$f(x) = \frac{3-x}{x^2-x-6} = \frac{3-x}{(x-3)(x+2)} = \frac{-(x-3)}{(x-3)(x+2)}$$
$$= -\frac{1}{x+2}$$

We see that the function $f(x)$ is not defined when $x^2-x-6=0 \implies (x-3)(x+2)=0 \implies x=3$ or $x=-2$. Since

$$\lim_{x\to3}\frac{3-x}{x^2-x-6} = \lim_{x\to3}\frac{3-x}{(x-3)(x+2)}$$
$$= \lim_{x\to3}\frac{-(x-3)}{(x-3)(x+2)} = \lim_{x\to3}\frac{-1}{x+2} = -\frac{1}{5}$$

then, $x=3$ is a removable discontinuity.

$$\lim_{x\to-2^+}\frac{3-x}{x^2-x-6} = \lim_{x\to-2^+}\frac{3-x}{(x-3)(x+2)} = \overset{-}{}\infty$$

and

$$\lim_{x\to-2^-}\frac{3-x}{x^2-x-6} = \lim_{x\to-2^-}\frac{3-x}{(x-3)(x+2)} = -\underset{+}{}\infty$$

then, $x=-2$ is a vertical asymptote only.

26) The vertical asymptote of $f(x) = \frac{7-x}{x^2-5x+6}$ is

Solution:

$$f(x) = \frac{7-x}{x^2-5x+6} = \frac{7-x}{(x-3)(x-2)}$$

We see that the function $f(x)$ is not defined when $x-3=0$ or $x-2=0 \implies x=3$ or $x=2$. Since

$$\lim_{x\to3^+}\frac{7-x}{x^2-5x+6} = \lim_{x\to3^+}\frac{7-x}{(x-3)(x-2)} = \infty$$
$$\lim_{x\to3^-}\frac{7-x}{x^2-5x+6} = \lim_{x\to3^-}\frac{7-x}{(x-3)(x-2)} = -\infty$$

and

$$\lim_{x\to2^+}\frac{7-x}{x^2-5x+6} = \lim_{x\to2^+}\frac{7-x}{(x-3)(x-2)} = -\infty$$
$$\lim_{x\to2^-}\frac{7-x}{x^2-5x+6} = \lim_{x\to2^-}\frac{7-x}{(x-3)(x-2)} = \infty$$

then, $x=3$ and $x=2$ are vertical asymptotes.

27) The vertical asymptote of $f(x) = \frac{x-7}{x^2+5x+6}$ is

Solution:

$$f(x) = \frac{x-7}{x^2+5x+6} = \frac{x-7}{(x+3)(x+2)}$$

We see that the function $f(x)$ is not defined when $x+3=0$ or $x+2=0 \implies x=-3$ or $x=-2$. Since

$$\lim_{x\to-3^+}\frac{x-7}{x^2+5x+6} = \lim_{x\to-3^+}\frac{x-7}{(x+3)(x+2)} = \infty$$
$$\lim_{x\to-3^-}\frac{x-7}{x^2+5x+6} = \lim_{x\to-3^-}\frac{x-7}{(x+3)(x+2)} = -\infty$$

and

$$\lim_{x\to-2^+}\frac{x-7}{x^2+5x+6} = \lim_{x\to-2^+}\frac{x-7}{(x+3)(x+2)} = -\infty$$
$$\lim_{x\to-2^-}\frac{x-7}{x^2+5x+6} = \lim_{x\to-2^-}\frac{x-7}{(x+3)(x+2)} = \infty$$

then, $x=-3$ and $x=-2$ are vertical asymptotes.

28) The vertical asymptote of $f(x) = \frac{x-7}{x^2+3x}$ is

Solution:

$$f(x) = \frac{x-7}{x^2+3x} = \frac{x-7}{x(x+3)}$$

We see that the function $f(x)$ is not defined when $x=0$ or $x+3=0 \implies x=0$ or $x=-3$. Since

$$\lim_{x\to-3^+}\frac{x-7}{x^2+3x} = \lim_{x\to-3^+}\frac{x-7}{x(x+3)} = \infty$$
$$\lim_{x\to-3^-}\frac{x-7}{x^2+3x} = \lim_{x\to-3^-}\frac{x-7}{x(x+3)} = -\infty$$

and

$$\lim_{x\to0^+}\frac{x-7}{x^2+3x} = \lim_{x\to0^+}\frac{x-7}{x(x+3)} = -\infty$$
$$\lim_{x\to0^-}\frac{x-7}{x^2+3x} = \lim_{x\to0^-}\frac{x-7}{x(x+3)} = \infty$$

then, $x=-3$ and $x=0$ are vertical asymptotes.

29) The vertical asymptote of $f(x) = \frac{x-7}{x^2-3x}$ is

Solution:

$$f(x) = \frac{x-7}{x^2-3x} = \frac{x-7}{x(x-3)}$$

We see that the function $f(x)$ is not defined when $x=0$ or $x-3=0 \implies x=0$ or $x=3$. Since

$$\lim_{x\to3^+}\frac{x-7}{x^2-3x} = \lim_{x\to3^+}\frac{x-7}{x(x-3)} = -\infty$$
$$\lim_{x\to3^-}\frac{x-7}{x^2-3x} = \lim_{x\to3^-}\frac{x-7}{x(x-3)} = \infty$$

and

$$\lim_{x\to0^+}\frac{x-7}{x^2-3x} = \lim_{x\to0^+}\frac{x-7}{x(x-3)} = \infty$$
$$\lim_{x\to0^-}\frac{x-7}{x^2-3x} = \lim_{x\to0^-}\frac{x-7}{x(x-3)} = -\infty$$

then, $x=3$ and $x=0$ are vertical asymptotes.

30) The vertical asymptotes of $f(x) = \frac{2x^2+1}{x^2-9}$ are

Solution:

$$f(x) = \frac{2x^2+1}{x^2-9} = \frac{2x^2+1}{(x+3)(x-3)}$$

We see that the function $f(x)$ is not defined when $x^2-9=0 \implies x=\pm3$. Since

$$\lim_{x\to3^+}\frac{2x^2+1}{x^2-9} = \lim_{x\to3^+}\frac{2x^2+1}{(x+3)(x-3)} = \infty$$
$$\lim_{x\to3^-}\frac{2x^2+1}{x^2-9} = \lim_{x\to3^-}\frac{2x^2+1}{(x+3)(x-3)} = -\infty$$

and

$$\lim_{x\to-3^+}\frac{2x^2+1}{x^2-9} = \lim_{x\to-3^+}\frac{2x^2+1}{(x+3)(x-3)} = -\infty$$
$$\lim_{x\to-3^-}\frac{2x^2+1}{x^2-9} = \lim_{x\to-3^-}\frac{2x^2+1}{(x+3)(x-3)} = \infty$$

then, $x=\pm3$ are vertical asymptotes.

31) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous at $a=2$ because

1 − $f(2)=\dfrac{(2)+1}{(2)^2-9}=\dfrac{3}{-5}=-\dfrac{3}{5}$

2 − $\lim\limits_{x\to 3^-}\dfrac{x+1}{x^2-9}=\lim\limits_{x\to 2}\dfrac{(2)+1}{(2)^2-9}=\dfrac{3}{-5}=-\dfrac{3}{5}$

3 − $\lim\limits_{x\to 2}\dfrac{x+1}{x^2-9}=f(2)$

OR

We know that $D_f=\mathbb{R}\setminus\{\pm 3\}$, so $\{2\}\in D_f$.

Note: Any function is continuous on its domain.

32) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at $a=\pm 3$ because we know that $D_f=\mathbb{R}\setminus\{\pm 3\}$, so $\{\pm 3\}\notin D_f$.

33) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at ± 3 because $\{\pm 3\}\notin D_f$.

34) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous on its domain which is $D_f=\mathbb{R}\setminus\{\pm 3\}$.

35) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 3, & x=0\end{cases}$ is continuous at $a=0$ because

1- $f(0)=3$

2- $\lim\limits_{x\to 0}\frac{\sin 3x}{x}=3\lim\limits_{x\to 0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim\limits_{x\to 0}f(x)=f(0)$

36) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 5, & x=0\end{cases}$ is discontinuous at $a=0$ because

1- $f(0)=5$

2- $\lim\limits_{x\to 0}\frac{\sin 3x}{x}=3\lim\limits_{x\to 0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim\limits_{x\to 0}f(x)\neq f(0)$

37) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 7, & x=1\end{cases}$ is discontinuous at $a=1$ because

1- $f(1)=7$

2- $\lim\limits_{x\to 1}\frac{2x^2-3x+1}{x-1}=\lim\limits_{x\to 1}\frac{(2x-1)(x-1)}{x-1}=\lim\limits_{x\to 1}(2x-1)=1$

3- $\lim\limits_{x\to 1}f(x)\neq f(1)$

38) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 1, & x=1\end{cases}$ is continuous at $a=1$ because

1- $f(1)=1$

2- $\lim\limits_{x\to 1}\frac{2x^2-3x+1}{x-1}=\lim\limits_{x\to 1}\frac{(2x-1)(x-1)}{x-1}=\lim\limits_{x\to 1}(2x-1)=1$

3- $\lim\limits_{x\to 1}f(x)=f(1)$

39) The function $f(x)=\frac{x^2-x-2}{x-2}$ is discontinuous at $a=2$ because $\{2\}\notin D_f$.

40) The function $f(x)=\begin{cases}2x+3, & x>2\\ 3x+1, & x\leq 2\end{cases}$ is continuous at $a=2$ because

1- $f(2)=3(2)+1=7$

2- $\lim\limits_{x\to 2^+}(2x+3)=2(2)+3=7$

$\lim\limits_{x\to 2^-}(3x+1)=3(2)+1=7$

$\therefore\ \lim\limits_{x\to 2}f(x)=7$

3- $\lim\limits_{x\to 2}f(x)=f(2)$

41) The function $f(x)=\frac{x+3}{\sqrt{x^2-4}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4>0 \implies x^2>4 \implies \sqrt{x^2}>\sqrt{4}$$
$$\implies |x|>2 \iff x>2 \text{ or } x<-2$$

Hence,

$D_f=(-\infty,-2)\cup(2,\infty)$.

42) The function $f(x)=\sqrt{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4\geq 0 \implies x^2\geq 4 \implies \sqrt{x^2}\geq\sqrt{4}$$
$$\implies |x|\geq 2 \iff x\geq 2 \text{ or } x\leq -2$$

Hence,

$D_f=(-\infty,-2]\cup[2,\infty)$.

43) The function $f(x)=\sqrt{4-x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4-x^2\geq 0 \implies -x^2\geq -4 \implies x^2\leq 4$$
$$\implies \sqrt{x^2}\leq\sqrt{4} \implies |x|\leq 2 \iff -2\leq x\leq 2$$

Hence,

$D_f=[-2,2]$.

44) The function $f(x)=\frac{x+3}{\sqrt{4-x^2}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4-x^2>0 \implies -x^2>-4 \implies x^2<4$$
$$\implies \sqrt{x^2}<\sqrt{4} \implies |x|<2 \iff -2<x<2$$

Hence,

$D_f=(-2,2)$.

45) The function $f(x)=\frac{x+1}{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4\neq 0 \implies x^2\neq 4 \implies x\neq\pm 2$$

Hence,

$D_f=\mathbb{R}\setminus\{\pm 2\}$

$=(-\infty,-2)\cup(-2,2)\cup(2,\infty)=\{x\in\mathbb{R}: x\neq\pm 2\}$.

46) The function $f(x) = \log_2(x+2)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x + 2 > 0 \implies x > -2$$

Hence,

$D_f = (-2, \infty)$.

47) The function $f(x) = \sqrt{x-1} + \sqrt{x+4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x - 1 \geq 0 \text{ and } x + 4 \geq 0 \implies x \geq 1 \ \cap \ x \geq -4$$

Hence,

$D_f = [1, \infty)$.

48) The function $f(x) = 5^x$ is continuous on its domain .

Hence,

$D_f = \mathbb{R} = (-\infty, \infty)$.

49) The function $f(x) = e^x$ is continuous on its domain .

Hence,

$D_f = \mathbb{R} = (-\infty, \infty)$.

50) The function $f(x) = \sin^{-1}(3x-5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1 \leq 3x - 5 \leq 1 \iff 4 \leq 3x \leq 6 \iff \frac{4}{3} \leq x \leq 2 .$$

Hence,

$D_f = \left[\frac{4}{3}, 2\right]$.

51) The function $f(x) = \cos^{-1}(3x+5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1 \leq 3x + 5 \leq 1 \iff -6 \leq 3x \leq -4 \iff -2 \leq x \leq -\frac{4}{3} .$$

Hence,

$D_f = \left[-2, -\frac{4}{3}\right]$.

52) The number c that makes $f(x) = \begin{cases} c + x , & x > 2 \\ 2x - c, & x \leq 2 \end{cases}$ is continuous at $x = 2$ is

Solution:

$\lim\limits_{x \to 2} f(x)$ exists if

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$$
$$\lim_{x \to 2^+} (c + x) = \lim_{x \to 2^-} (2x - c)$$
$$c + 2 = 4 - c$$
$$c + c = 4 - 2$$
$$2c = 2$$
$$c = 1$$

53) The number c that makes $f(x) = \begin{cases} cx^2 - 2x + 1, & x \leq -1 \\ 3x + 2 \quad , & x > -1 \end{cases}$ is continuous at -1 is

Solution:

$\lim\limits_{x \to -1} f(x)$ exists if

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$$
$$\lim_{x \to -1^+} (3x + 2) = \lim_{x \to -1^-} (cx^2 - 2x + 1)$$
$$3(-1) + 2 = c(-1)^2 - 2(-1) + 1$$
$$-1 = c + 3$$
$$c = -1 - 3$$
$$c = -4$$

54) The number c that makes

$f(x) = \begin{cases} \frac{\sin cx}{x} + 2x - 1, & x < 0 \\ 3x + 4 \quad , & x \geq 0 \end{cases}$ is continuous at 0 is

Solution:

$\lim\limits_{x \to 0} f(x)$ exists if

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$
$$\lim_{x \to 0^+} (3x + 4) = \lim_{x \to 0^-} \left(\frac{\sin cx}{x} + 2x - 1\right)$$
$$3(0) + 4 = c(1) + 2(0) - 1$$
$$4 = c - 1$$
$$c = 4 + 1$$
$$c = 5$$

55) The value c that makes $f(x) = \begin{cases} cx^2 + 2x, & x \leq 2 \\ x^3 - cx \ , & x > 2 \end{cases}$ is continuous at 2 is

Solution:

$\lim\limits_{x \to 2} f(x)$ exists if

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$$
$$\lim_{x \to 2^+} (x^3 - cx) = \lim_{x \to 2^-} (cx^2 + 2x)$$
$$(2)^3 - c(2) = c(2)^2 + 2(2)$$
$$8 - 2c = 4c + 4$$
$$-2c - 4c = 4 - 8$$
$$-6c = -4$$
$$c = \frac{-4}{-6}$$
$$c = \frac{2}{3}$$

56) The number c that makes $f(x) = \begin{cases} c^2x^2 - 1, & x \leq 3 \\ x + 5 \quad , & x > 3 \end{cases}$ is continuous at 3 is

Solution:

$\lim\limits_{x \to 3} f(x)$ exists if

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$$
$$\lim_{x \to 3^+} (x + 5) = \lim_{x \to 3^-} (c^2x^2 - 1)$$
$$(3) + 5 = c^2(3)^2 - 1$$
$$8 = 9c^2 - 1$$
$$9c^2 = 8 + 1$$
$$c^2 = 1$$
$$c = \pm 1$$

57) The number c that makes $f(x) = \begin{cases} x - 2 , & x > 5 \\ cx - 3, & x \leq 5 \end{cases}$ is continuous at 5 is

Solution:

$\lim\limits_{x \to 5} f(x)$ exists if

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^-} f(x)$$
$$\lim_{x \to 5^+} (x - 2) = \lim_{x \to 5^-} (cx - 3)$$
$$(5) - 2 = c(5) - 3$$
$$3 = 5c - 3$$
$$5c = 3 + 3$$
$$5c = 6$$
$$c = \frac{6}{5}$$

58) The number c that makes $f(x) = \begin{cases} x+3\,, & x > -1 \\ 2x - c, & x \le -1 \end{cases}$

is continuous at -1 is

<u>Solution:</u>

$\lim\limits_{x \to -1} f(x)$ exists if

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$$
$$\lim_{x \to -1^+} (x+3) = \lim_{x \to -1^-} (2x - c)$$
$$(-1) + 3 = 2(-1) - c$$
$$2 = -2 - c$$
$$c = -2 - 2$$
$$c = -4$$