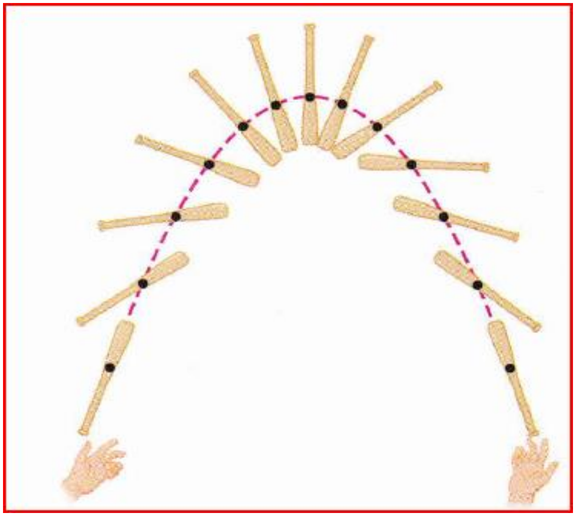

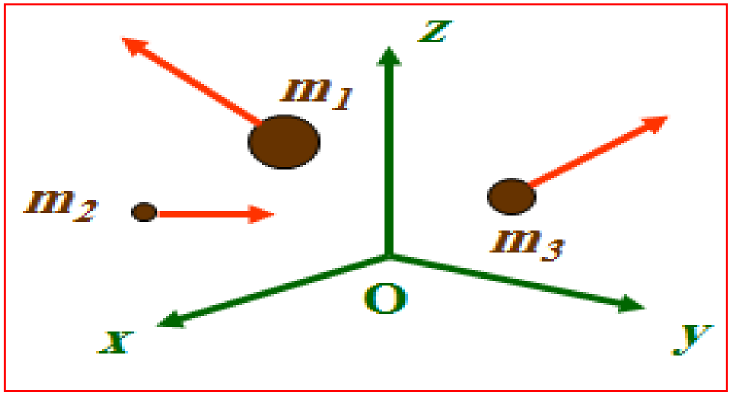

Center of Mass and Linear Momentum

9

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قسم الفيزياء
PHYSICS
جامعة الملك عبد العزيز

Learning objectives

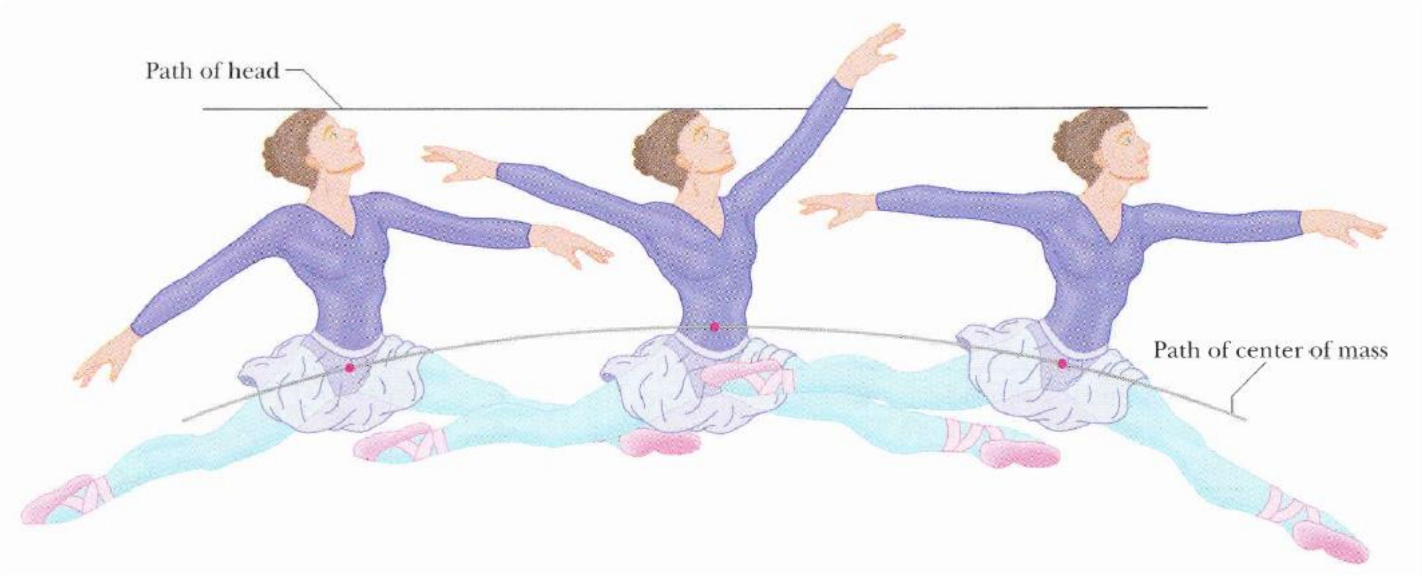
- Given the position of several particles along an axis or a plane, determine the location of their center of mass.
- Locate the center of mass of an extended, symmetric object by using the symmetry.
- For a two-dimensional or three-dimensional extended object with a uniform distribution of mass, determine the center of mass by (a) mentally dividing the object into simple geometric figures, each of which can be replaced by a particle at its center, and (b) finding the center of mass of those particles.
- Apply Newton s second law to a system of particles by relating the net force (of the forces acting on the particles) to the acceleration of the system s center of mass.
- Apply the constant acceleration to the motion of the particles in s system and to the motion of the system's center o mass.
- Given the mass and velocity of the particle in a system, calculate the velocity of the systems center of mass.
- Given mass and acceleration of the particle in a system, calculate the acceleration of the system's center of mass
- Given the position of a system's center of mass as a function of time, determine the velocity of the center of mass.
- Given the velocity of a system's center of mass as a function of time, determine the acceleration of the center of mass.

• Calculate the change of the velocity of a com by integrating the com’s acceleration function with respect to time.
• Calculate a com’s displacement by integrating the com’s velocity function with respect to time.
• When the particles in a two- particle system move without the system's com moving, relate the displacement of the particles and the velocities of the particles.
• Identify that momentum is a vector quantity and thus has both magnitude and direction and also components.
• Calculate the (linear) momentum of a particle as the product of the particle’s mass and velocity.
• Calculate the change in momentum (magnitude and direction) when a particle changes its speed and direction of travel.
• Apply the relationship between a particle’s momentum and the (net) force acting on the particle.
• Calculate the momentum of a system of particles as the product of the system’s total mass and its center-of- mass velocity.
• Apply the relationship between a system’s center-of-mass momentum and the net force acting on the system.
• For an isolated system of particles, apply the conservation of linear momenta to relate the initial momenta of the particles to their momenta at a later instant.
• Identify that the conservation of linear momentum can be done along an individual axis by using components along that axis, provided that there is no net external force component along that axis.

9-1 Center of Mass

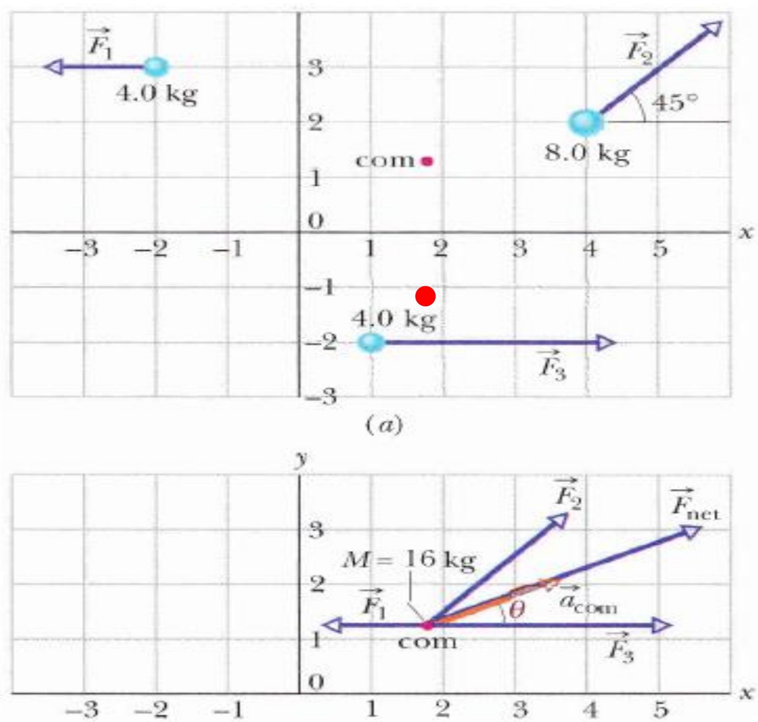
What is physics?

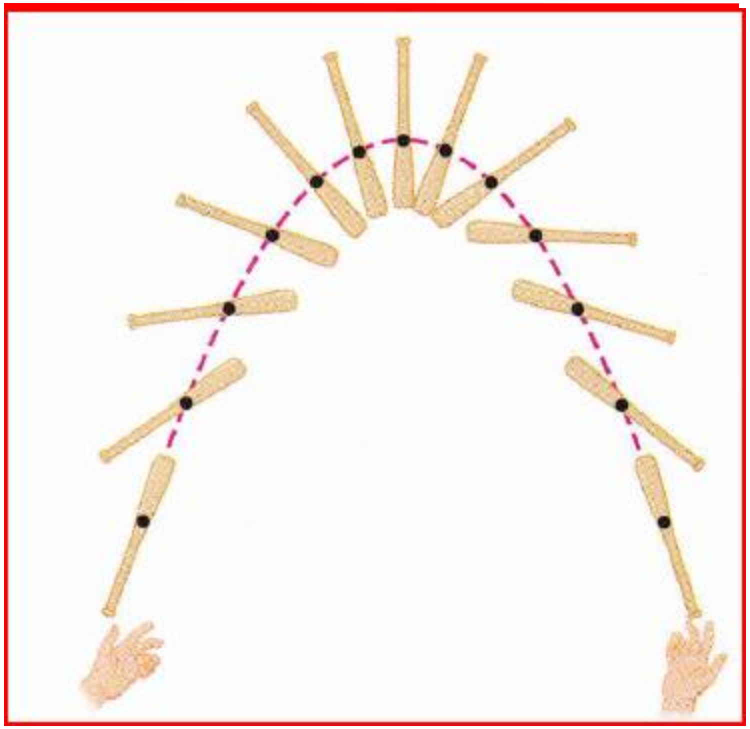
via an understanding of physics. In this chapter we discuss how the complicated motion of a system of objects, such as a car or a ballerina, can be simplified if we determine a special point of the system—the *center of mass* of that system.



Q. What is the center of mass (COM)?

The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.



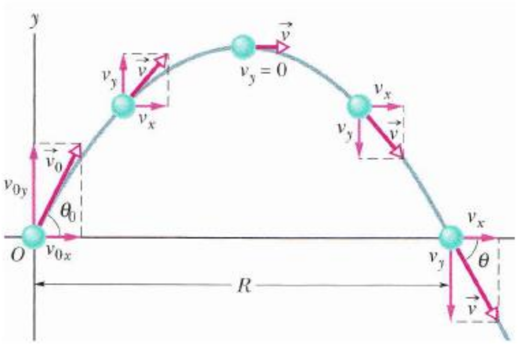


Q. Why do we study the center of mass (COM) of a system of particles?

A. We study the (COM) in order to predict the possible motion of the system.

When we study the motion we usually consider two kinds of systems

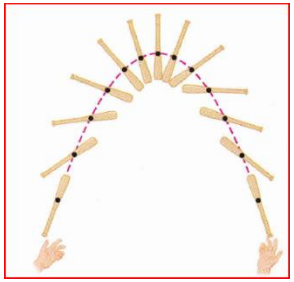
System contains one particle such as : ball – box - bead



It's motion is simple motion which we discussed before

System contains more than one particle such as : ballerina – car – baseball bat

1. It's motion is more complicated
2. Every parts of the bat moves differently
3. There are one point (COM) that moves in the simple parabolic path.



The Center of Mass

1. System of two particles on x-axis

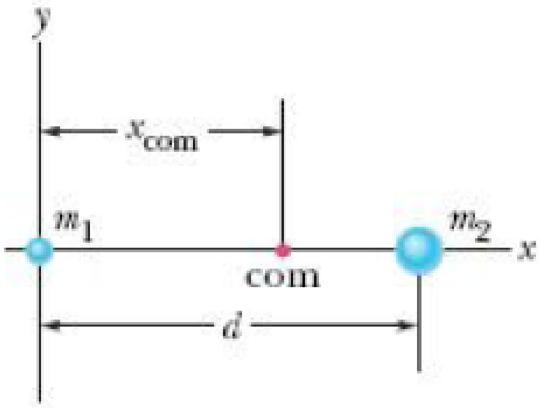
$$x_{com} = \frac{m_2}{m_1 + m_2} d$$

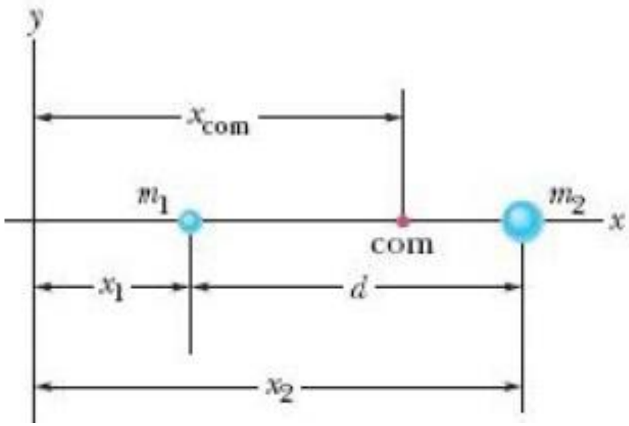
$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{M}$$

Where $M = m_1 + m_2.$

and x_1 , x_2 are the position of particles m_1 and m_2 respectively from the origin





2. System of n particles along x- axis:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M}$$

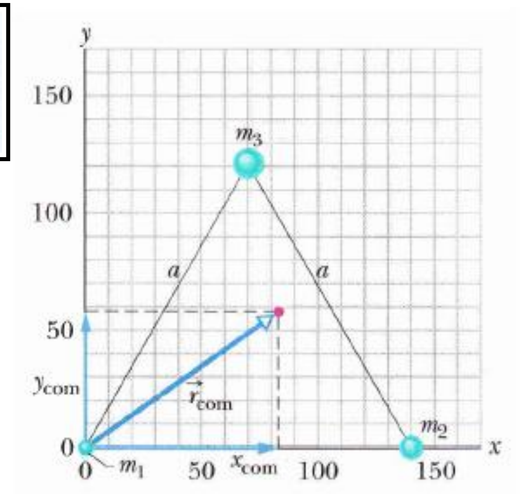
$$= \frac{1}{M}\sum_{i=1}^{n} m_i x_i .$$

Rem: put x_1 , x_2 ....etc, with their signs

3. System of n particles distributed in 3D:

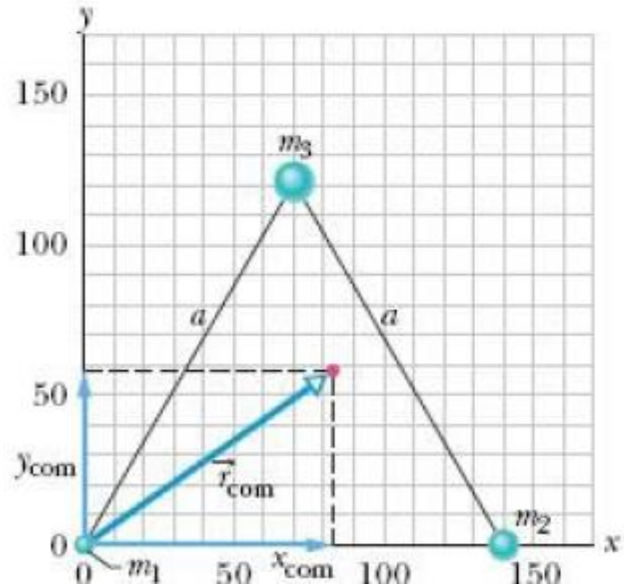
$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

$$\vec{r}_{com} = x_{com}\hat{\mathrm{i}} + y_{com}\hat{\mathrm{j}} + z_{com}\hat{\mathrm{k}}.$$



Sample Problem 9.01

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?



(9-3)

9-2 Newton's Second Law for a System of Particles

$$\vec{F}_{net} = M\vec{a}_{com}$$

1. $\vec{F}_{net}$ is the net force of *all external forces* that act on the system.

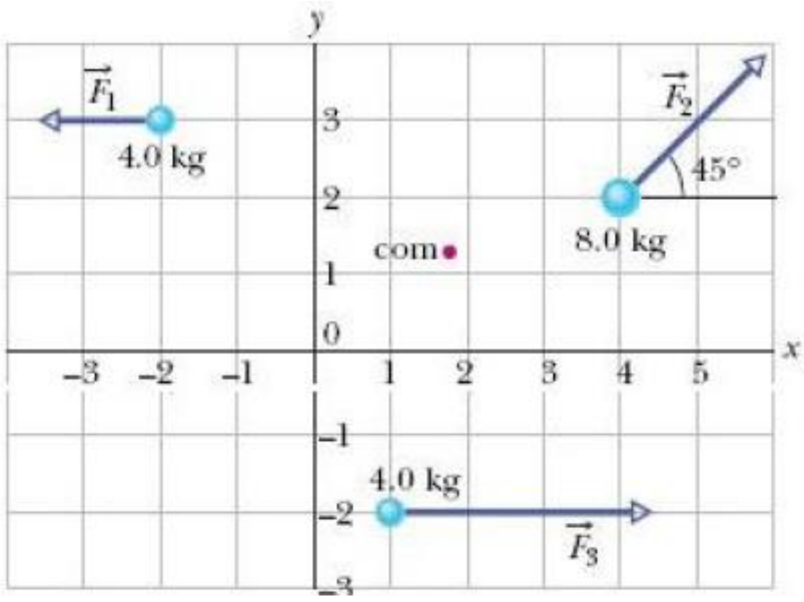
2. M is the *total mass* of the system. We assume that no mass enters or leaves the system as it moves, so that M remains constant. The system is said to be **closed.**

3. $\vec{a}_{com}$ is the acceleration of the *center of mass* of the system.

$$F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}$$

Sample Problem 9.03

The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?



(a) Fig. (9-7)

9-3 Linear Momentum

Linear Momentum of a single particle

Linear Momentum of a system of particles

Linear Momentum

$$\vec{p} = m\vec{v}$$

- $\vec{p}$ is a vector quantity
- SI unit is $(\text{kg} \cdot \text{m/s})$.

Newton's 2nd Law in terms of Momentum

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

In equation form

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

Thus, the relations $\vec{F}_{\text{net}} = d\vec{p}/dt$ and $\vec{F}_{\text{net}} = m\vec{a}$ are equivalent expressions of Newton's second law of motion for a particle.

Linear Momentum of a system of particles

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.$$

$$\vec{P} = M\vec{v}_{com}$$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

If we take the time derivative $$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{com}}{dt} = M\vec{a}_{com}.$$

9-5 Conservation of Linear Momentum

The system is said to be

Isolated: When the net external forces acting on a system of particles is zero

Closed: When no particles leave or enter the system

then $\vec{F}_{net} = 0$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = 0$$

then $\vec{P} = \text{constant}$ (closed, isolated system).

In words,

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system).}$$

This result is called the **law of conservation of linear momentum.** It can also be written as

$$\vec{P}_i = \vec{P}_f \qquad \text{(closed, isolated system).}$$

In words, this equation says that, for a closed, isolated system,

$$\begin{pmatrix}\text{total linear momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum} \\ \text{at some later time } t_f\end{pmatrix}.$$

Rem:

Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. However,

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

External Example

One-dimensional explosion: A ballot box with mass $m = 6.0$ kg slides with speed $v = 4.0$ m/s across a frictionless floor in the positive direction of an x axis. The box explodes into two pieces. One piece, with mass $m_1 = 2.0$ kg, moves in the positive direction of the x axis at $v_1 = 8.0$ m/s. What is the velocity of the second piece, with mass m_2?

THE END