

Kinetic Energy and Work

Learning Objectives

After studying this chapter, you will be able:

- 1- Apply the relationship between a particle's kinetic energy, mass, and speed.
- 2- Identify that kinetic energy is a scalar quantity.
- 3- Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.
- 4- Calculate work by taking a dot product of the force vector and displacement vector, in either magnitude-angle or unit-vector notation.
- 5- If multiple forces act on a particle, calculate the net work done by them.
- 6- Apply the work-kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.
- 7- Calculate the work done by the gravitational force when an object is lifted or lowered.
- 8- Apply the work-kinetic energy theorem to situations where an object is lifted or lowered.
- 9- Apply the relationship (Hooke's law) between the force on an object due to a spring, the stretch or compression of the spring, and the spring constant.
- 10- Identify that a spring force is a variable force.

- 11- Calculate the work done on an object by a spring force by integrating the force from the initial position to the final position of the object or by using the known generic result of that integration.
- 12- Calculate work by graphically integrating on a graph of force versus position of the object.
- 13- Apply the work-kinetic energy theorem to situation in which an object is moved by a spring force.
- 14- Given a variable force as a function of position, calculate the work done by it on an object by integrating the function from the initial to the final position of the object, in one or more dimensions.
- 15- Given a graph of force versus position, calculate the work done by graphically integrating from the initial position to the final position of the object
- 16- Convert a graph of acceleration versus position to a graph of force versus position.
- 17- Apply the work-kinetic energy theorem to situation where an object is moved by a variable force.
- 18- Apply the relationship between average power, the work done by a force, and the time interval in which that work is done.
- 19- Given the work as a function of time, find the instantaneous power.
- 20- Determine the instantaneous power by taking a dot product of the force vector and an object's velocity vector, in magnitude-angle and unit-vector notations.

7.1 kinetic Energy

What is physics

One of the fundamental goals of physics is to investigate something that everyone talks about: energy.

What is energy

energy is a scalar quantity associated with the state (or condition) of one or more objects.

Energy is a number that we

associate with a system of one or more objects. If a force changes one of the objects by, say, making it move, then the energy number changes. After countless

Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is *conserved*).

kinetic Energy

Kinetic energy K is energy associated with the *state of motion* of an object.

For an object of mass m whose speed v is well below the speed of light

$$K = \frac{1}{2}mv^2 \quad (\text{kinetic energy}).$$

For example, a 3.0 kg duck flying past us at 2.0 m/s

The SI unit of kinetic energy is the **joule** (J).

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2.$$

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When accelerated along a straight line at $2.8 \times 10^{15} \text{ m/s}^2$ in a machine, an electron (mass $m = 9.1 \times 10^{-31} \text{ kg}$) has an initial speed of $1.4 \times 10^7 \text{ m/s}$ and travels 5.8cm.

Find

- (a) the final speed of the electron and
- (b) the increase in its kinetic energy.

7.2 Work and kinetic energy

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Work is a Scalar Quantity

+ve Work



energy transferred to the
object

-ve Work



energy transferred from the
object

- Work has unit of energy (Joule J)

Work

Work Done



energy transferred to the
object by the force

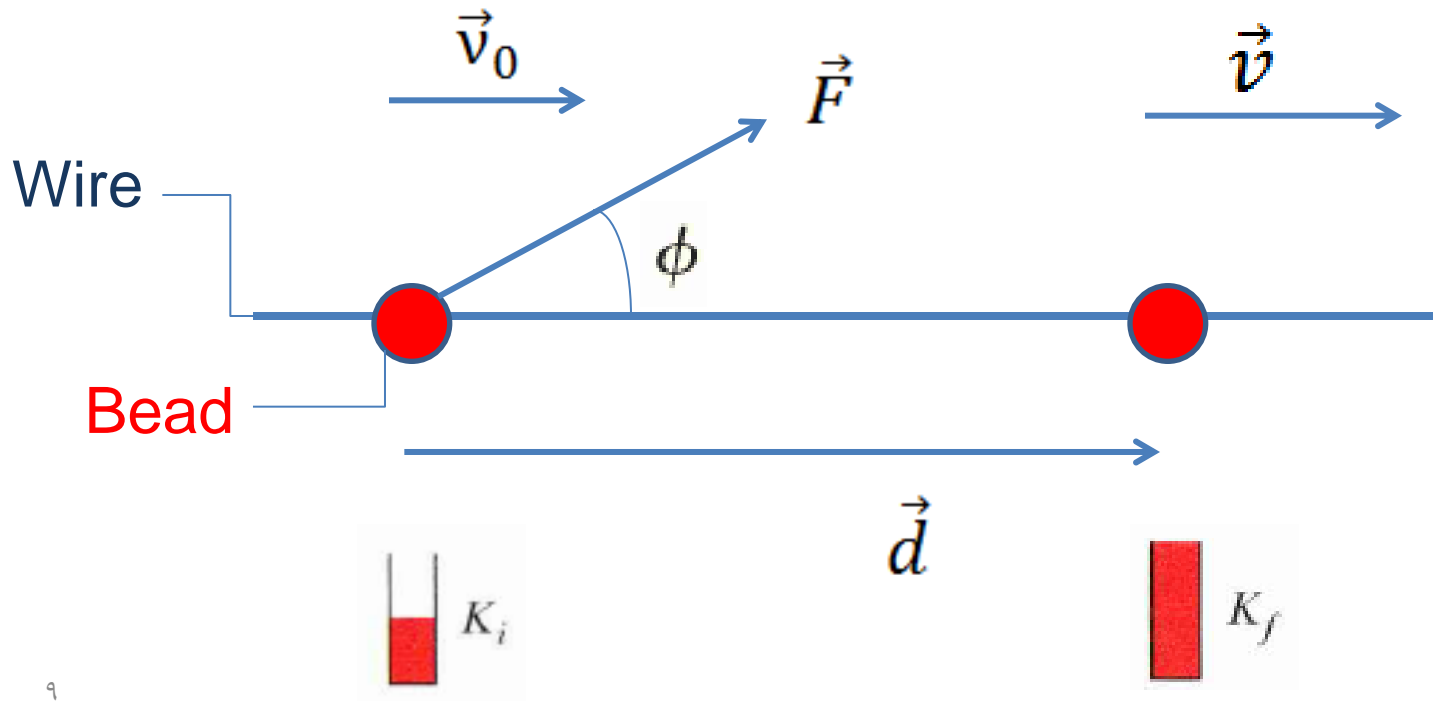
Doing Work



Is the act of transferring the
energy

Finding an Expression for Work

$$F = \text{const} \Rightarrow \vec{a} = \text{const}$$



Work done by a constant force is

$$W = F_x d.$$

$$W = (F \cos \phi) d$$

where ϕ is the angle between \vec{d} and \vec{F}

$$W = Fd \cos \phi$$



$$W = \vec{F} \cdot \vec{d}$$

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

when $\phi = 90^\circ \Rightarrow \cos \phi = 0 \Rightarrow W = 0$

when $\phi < 90^\circ \Rightarrow \cos \phi = +ve \Rightarrow W = +ve$

when $\phi > 90^\circ$ (up to 180°) $\Rightarrow \cos \phi = -ve \Rightarrow W = -ve$

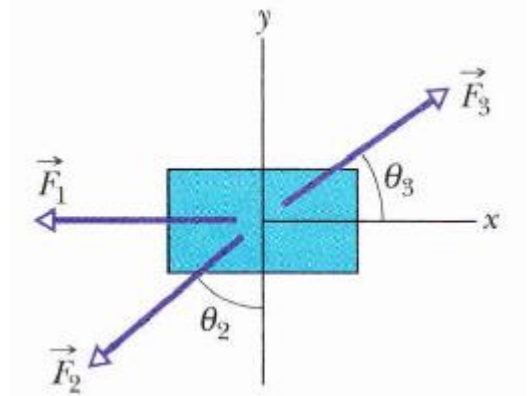
- Work has another unit

$$W = \vec{F} \cdot \vec{d}$$

$$1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m}$$

How to find the net Work done by several forces?

Net work done by several forces.



Find the work done by each force and then sum those works

$$W_1 = F_1 d$$

$$W_2 = F_2 d$$

$$W_3 = F_3 d$$

$$W_{net} = W_1 + W_2 + W_3 + \dots$$

Find the net force \vec{F}_{net} then



$$W_{net} = (F_{net})d \cos \phi,$$

where ϕ is the angle between \vec{F}_{net} and \vec{d}

Work–Kinetic Energy Theorem

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$



$$\Delta K = K_f - K_i = W,$$

which says that

$$\left(\begin{array}{l} \text{change in the kinetic} \\ \text{energy of a particle} \end{array} \right) = \left(\begin{array}{l} \text{net work done on} \\ \text{the particle} \end{array} \right).$$

We can also write

$$K_f = K_i + W,$$

$$K_f = K_i + W, \quad (7-11)$$

which says that

$$\left(\begin{array}{c} \text{kinetic energy after} \\ \text{the net work is done} \end{array} \right) = \left(\begin{array}{c} \text{kinetic energy} \\ \text{before the net work} \end{array} \right) + \left(\begin{array}{c} \text{the net} \\ \text{work done} \end{array} \right).$$

These statements are known traditionally as the **work–kinetic energy theorem**

For example, if the kinetic energy of a particle is initially 5 J and there is a net transfer of 2 J to the particle (positive net work), the final kinetic energy is 7 J. If, instead, there is a net transfer of 2 J from the particle (negative net work), the final kinetic energy is 3 J.

Sample Problem 7.02

Figure 7-4a shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement \vec{d} of magnitude 8.50 m, straight toward their truck. The push \vec{F}_1 of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull \vec{F}_2 of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces \vec{F}_1 and \vec{F}_2 during the displacement \vec{d} ?

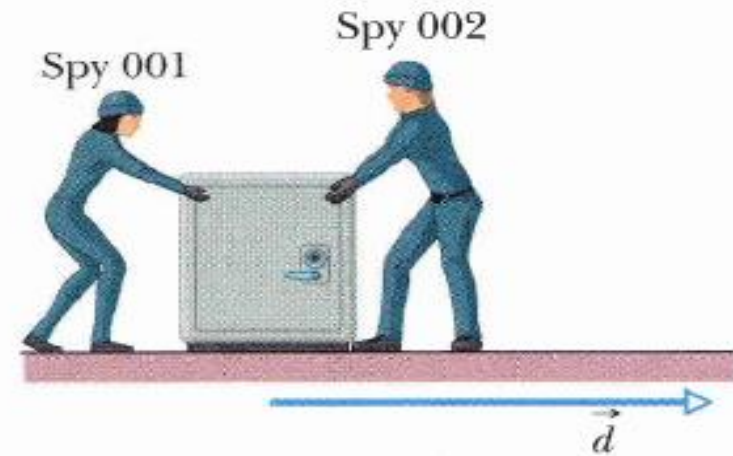


Fig. 7-4 (a)

(b) During the displacement, what is the work W_g done on the safe by the gravitational force \vec{F}_g and what is the work W_N done on the safe by the normal force \vec{F}_N from the floor?

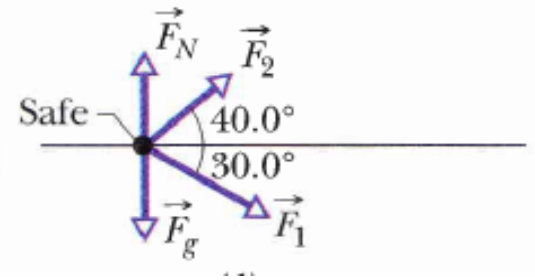
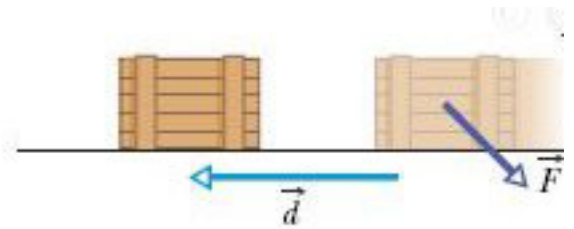


Fig 7-4 (b)

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

Sample Problem 7.03

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0 \text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0 \text{ N})\hat{i} + (-6.0 \text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig 7-5



(a) How much work does this force do on the crate during the displacement?

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement \vec{d} , what is its kinetic energy at the end of \vec{d} ?

7-3 Work done by the gravitational force

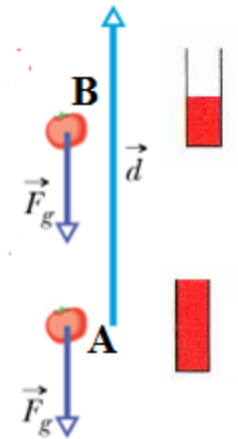
$$W = Fd \cos \phi$$

$$W_g = F_g d \cos \phi$$

$$W_g = mgd \cos \phi \quad (\text{work done by gravitational force})$$

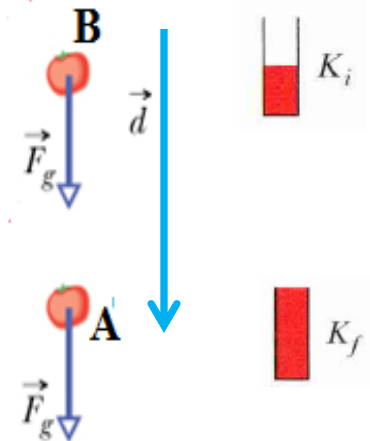
For a rising object, force \vec{F}_g is directed opposite the displacement \vec{d} .

$$W_g = mgd \cos 180^\circ = mgd(-1) = -mgd.$$



For a falling object, force \vec{F}_g is directed along the displacement \vec{d} .

$$W_g = mgd \cos 0^\circ = mgd(+1) = +mgd$$



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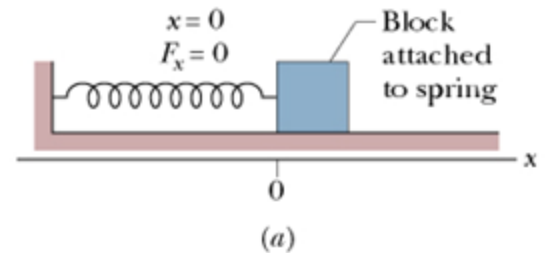
In 1975 the roof of Monteria's Velodrome, with a weight of 360 kN, was lifted by 10 cm so that it could be centered.

-How much work was done on the roof by the forces making the lift?

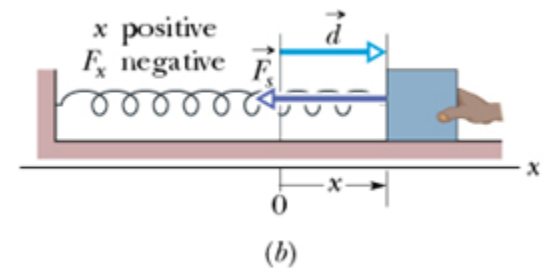
7-4 Work done by a spring force

The Spring Force

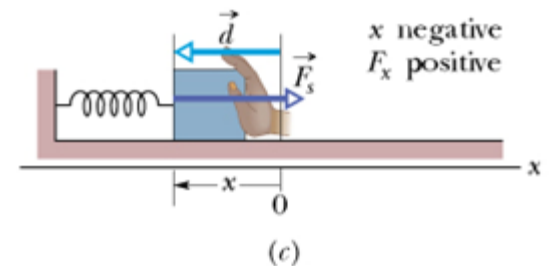
Fig. a shows a spring in its relaxed state.



In fig. b we pull one end of the spring and stretch it by an amount d . The spring resists by exerting a force F on our hand in the opposite direction.



In fig. c we push one end of the spring and compress it by an amount d . Again the spring resists by exerting a force F on our hand in the opposite direction.



The *spring force* is given by $\vec{F}_s = -k\vec{d}$ (Hooke's law)

$$\vec{F}_s = -k\vec{d} \quad (\text{Hooke's law})$$

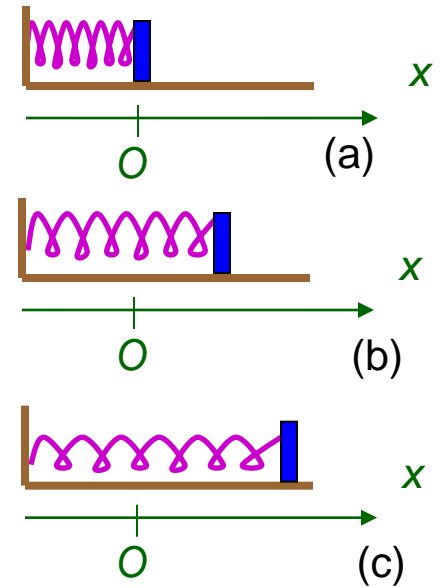
- The minus sign in Eq. 7-20 indicates that the direction of the spring force is always opposite the direction of the displacement of the spring.
- The constant k is called the **spring constant** (or **force constant**)
- The SI unit for k is the newton per meter.

$$d = x_2 - x_1 \quad , \quad \text{Let } x_1 = 0 \text{ and } x_2 = x$$

$$F_x = -kx \quad (\text{Hooke's law})$$

The Work Done by a Spring Force

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad (\text{work by a spring force}).$$



Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

$$\text{If } x_i > x_f \Rightarrow W_s = +ve$$

$$\text{If } x_f > x_i \Rightarrow W_s = -ve$$

If $x_i = 0$ and if we call the final position x ,

$$W_s = -\frac{1}{2}kx^2 \quad (\text{work b})$$

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A spring and block are in the arrangement of Fig. 7-10 when the block is pulled out to $x=+4.0$ cm, we must apply a force of magnitude 360 N to hold it there. We pull the block to $x=11$ cm and then release it. How much work does the spring do on the block as the block moves from $x_i = +5.0$ cm to (a) $x = +3.0$ cm, (b) $x = -3.0$ cm, (c) $x = -5.0$ cm, and (d) $x = -9.0$ cm ?

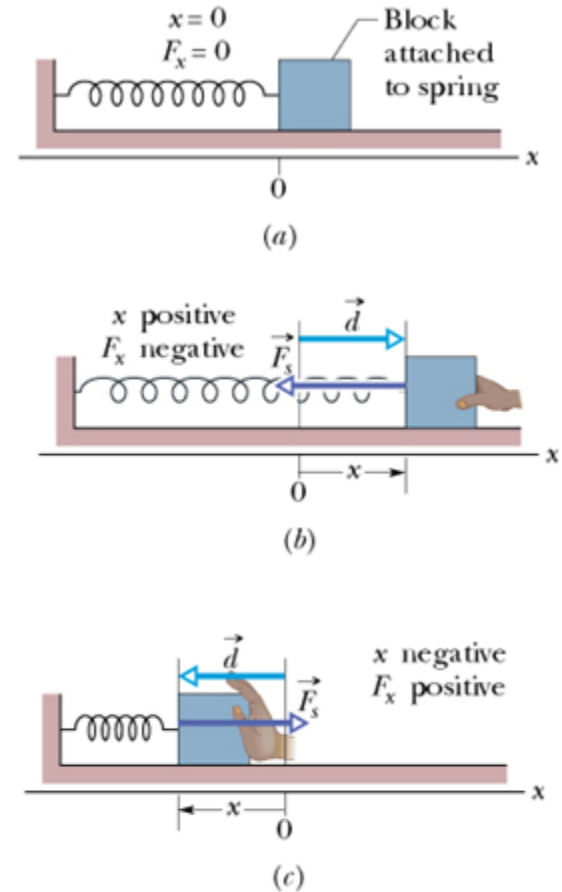


Fig 7-10

(b) Next, the package is moved leftward to $x_3 = -12$ mm. How much work does the spring force do on the package during this displacement? Explain the sign of this work.

7-6 Power

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt , the **average**

Power

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graph TD; Power --> AveragePower; Power --> InstantaneousPower;
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Average Power

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Instantaneous Power

$$P = \frac{dW}{dt}$$

The SI unit of power the joule per second. $\text{J/s} = \text{watt}$

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s}$$

$$P = \frac{dW}{dt} \quad \text{but}$$

$$W = Fd \cos \phi$$

or $W = F x \cos \phi$

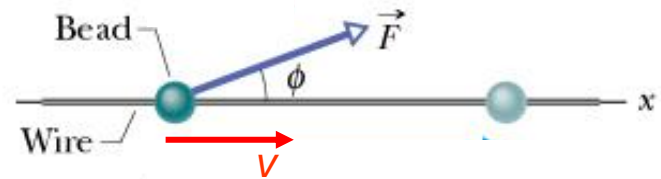
then

$$P = \frac{d(F \cos \phi dx)}{dt}$$

$$= F \cos \phi \left(\frac{dx}{dt} \right) \Rightarrow P = Fv \cos \phi$$

or

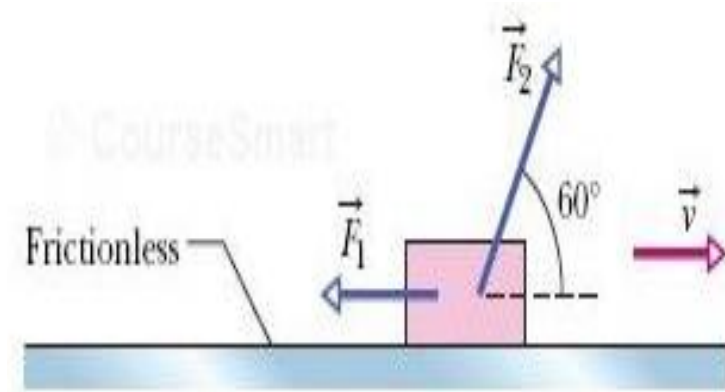
$$P = \vec{F} \cdot \vec{v}$$



Sample Problem 7.09

Here we calculate an instantaneous work—that is, the rate at which work is being done at any given instant rather than averaged over a time interval. Figure 7-15 shows constant forces F_1 and F_2 acting on a box as the box slides rightward across a frictionless floor. Force F_1 is horizontal, with magnitude 2.0 N; force F_2 is angled upward by 60° to the floor and has magnitude 4.0 N. the speed v of the box at a certain instant is 3.0 m/s.

what is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?



T H E

E N D