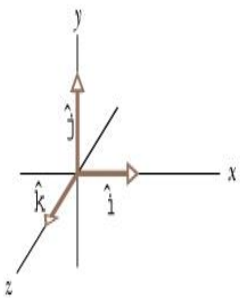
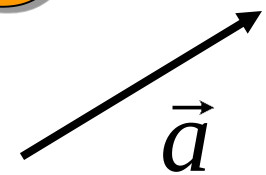
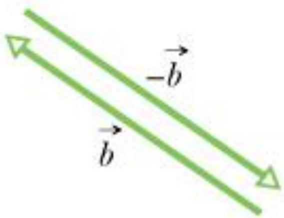
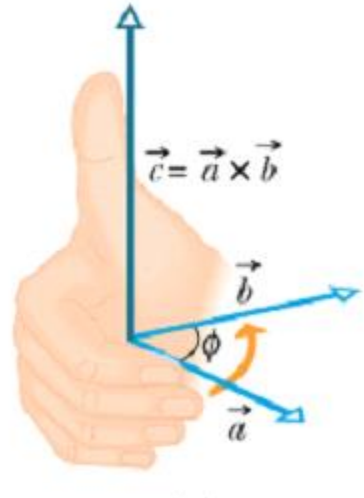
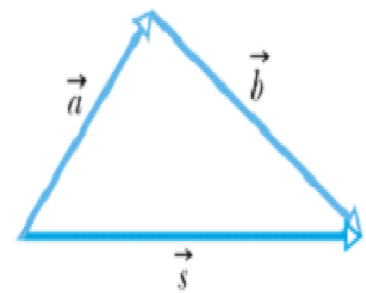
Chapter 3

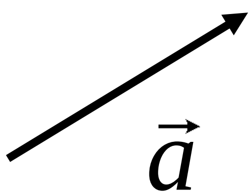












VECTORS

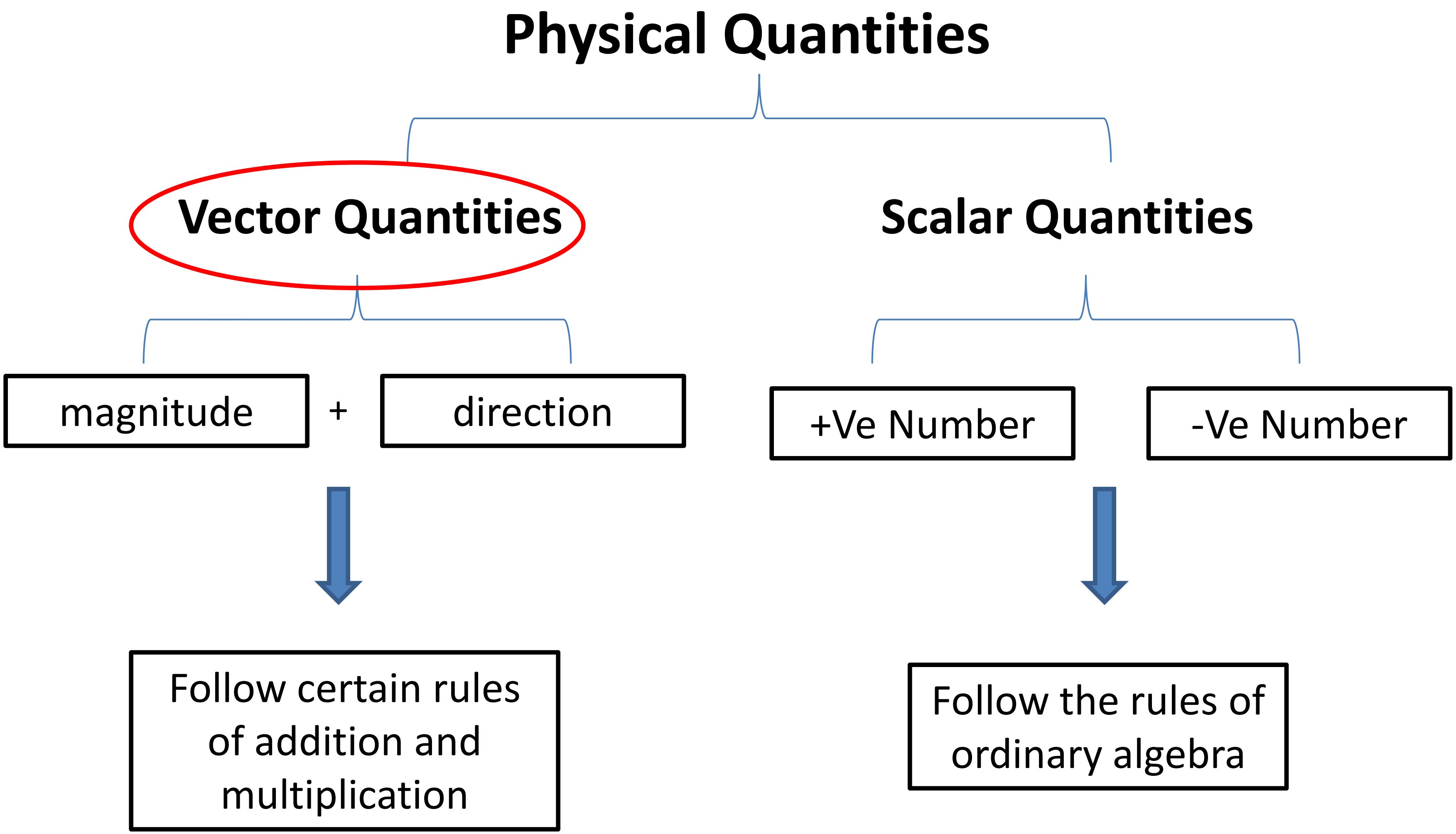
Learning Outcomes

After studying this chapter, you will be able:

- to define vector quantity and scalar quantity and differentiate between them.
- to add vectors geometrically and write the resultant equation.
- to identify vector addition properties: commutative law, associative law and vector subtraction.
- to find the inverse of any vector.
- to resolve any vector and find its x and y components.
- to calculate the magnitude and direction of vector.
- to identify the unit vector (magnitude and direction) on three axes.
- to write a vector in unit vector notations.
- to add vectors by components.
- to multiply vector by scalar (either +ve or - ve no.).
- to identify the two kinds of multiplication of a vector by another vector.
- to calculate the scalar product of two vectors in terms of the magnitude of the two vectors and angle between them.
- to calculate the scalar product of unit vectors.
- to calculate the vector product of two vectors in terms of the magnitude of the two vectors and the angle between them, in magnitude and direction.
- to use the right-hand rule to find the direction of the vector product.
- to calculate the vector product of unit vectors.
- to calculate the magnitude of the vector product of two vectors when they are written in unit-vector notation .

3-1 VECTORS AND THEIR COMPONENTS

Vectors and Scalars



Vectors Addition

Adding Vectors Geometrically

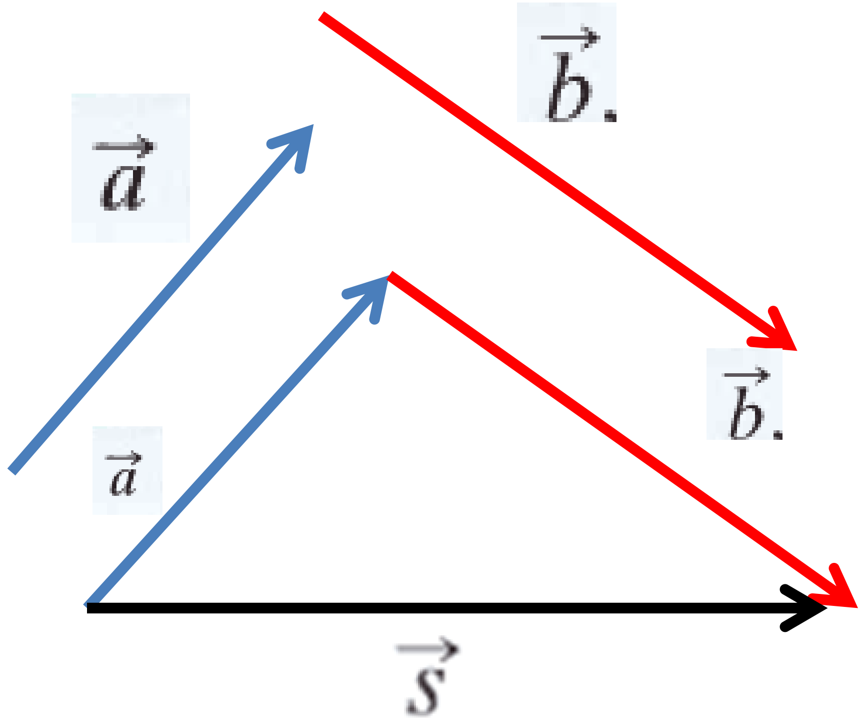
- Vector equation
- Commutative Law
- Associative Law
- Vector Subtraction

Adding Vectors by Components

- Components
- resolving the vector
- writing a vector in magnitude- angle notation

- Unit Vectors
- writing a vector in Unit vector notation

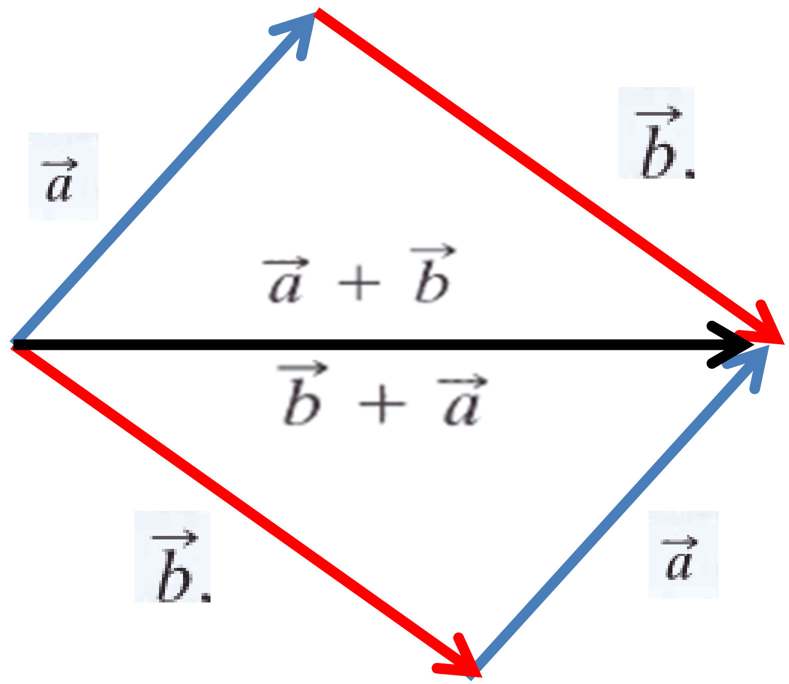
Adding Vectors Geometrically



- Vector equation

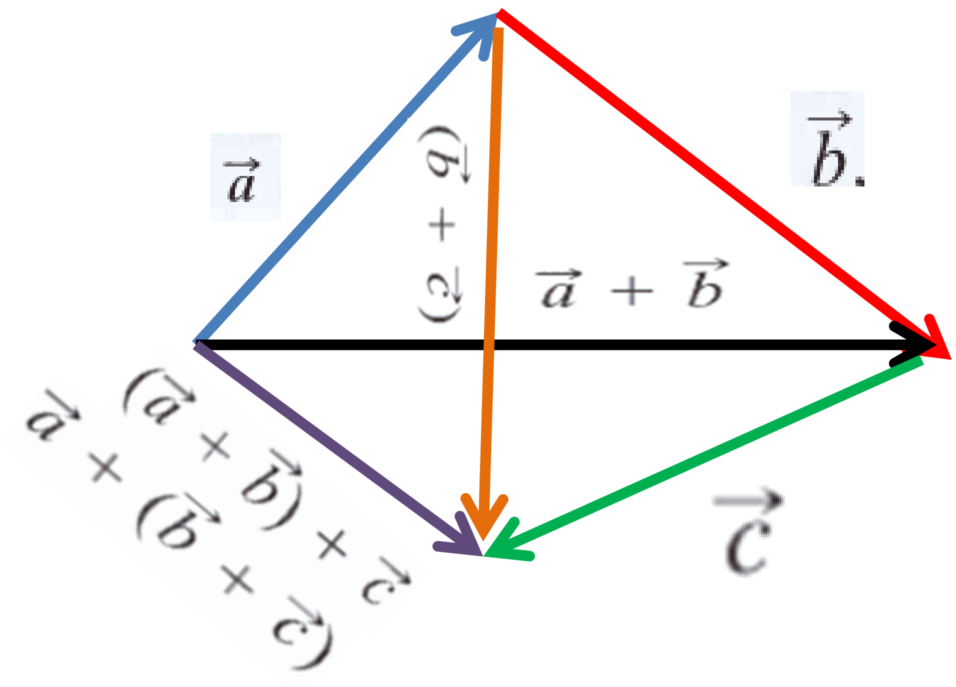
$$\vec{s} = \vec{a} + \vec{b},$$

• Commutative Law



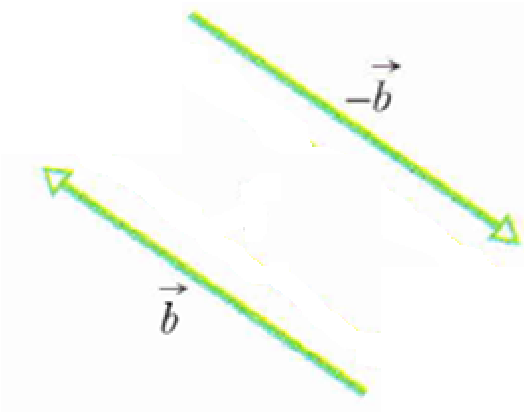
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

• Associative Law

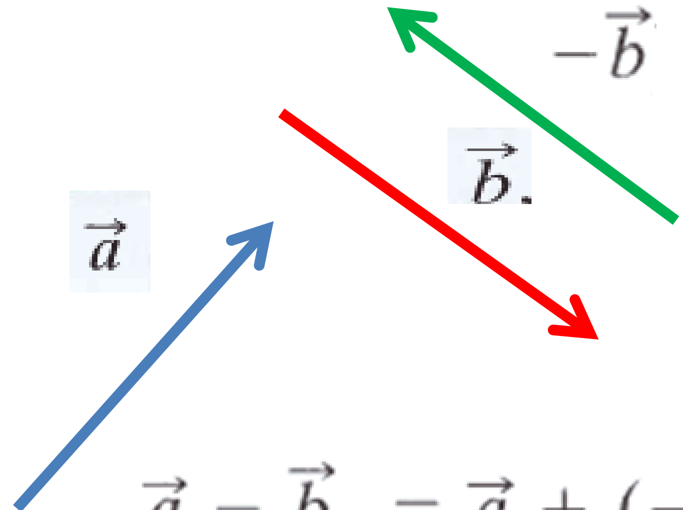


$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

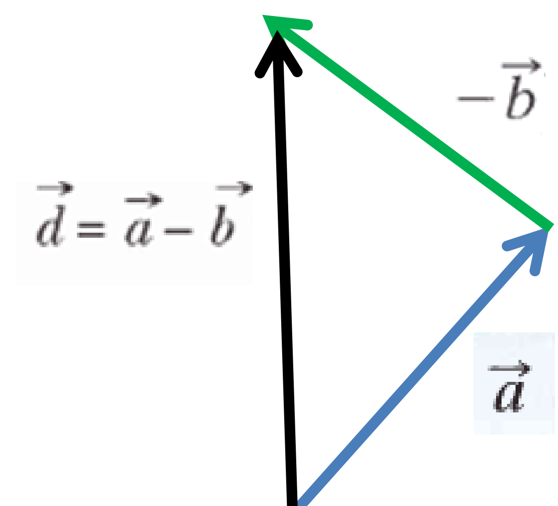
• Vector Subtraction



$$\vec{b} + (-\vec{b}) = 0$$



$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

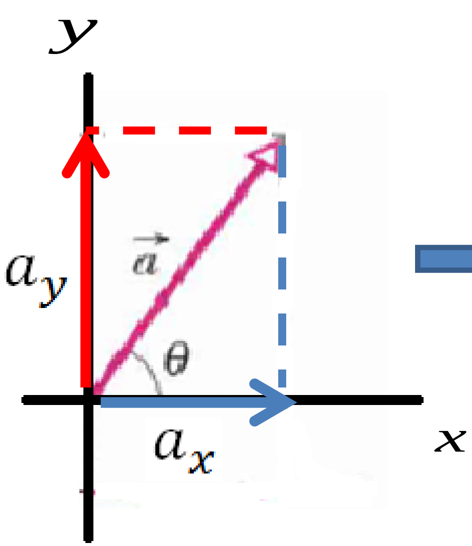


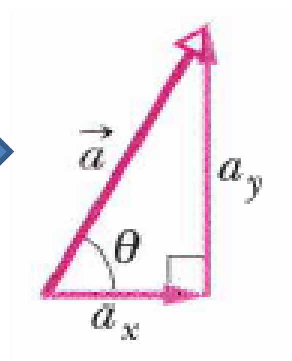
$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

Components of Vectors

- Resolving the vector is the process of finding the components

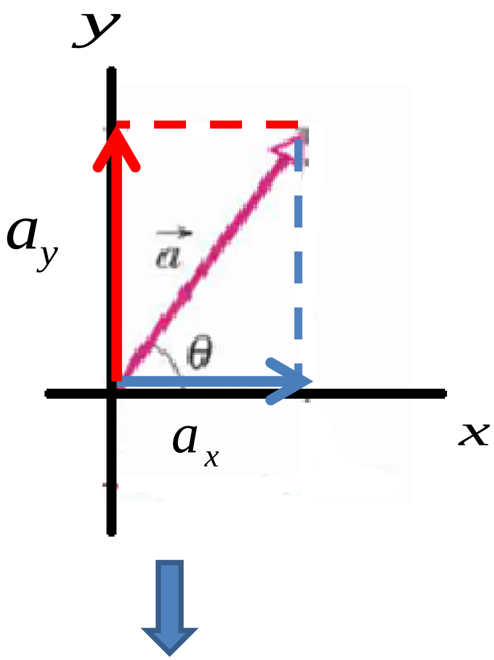
- Component is the projection of the vector on an axis



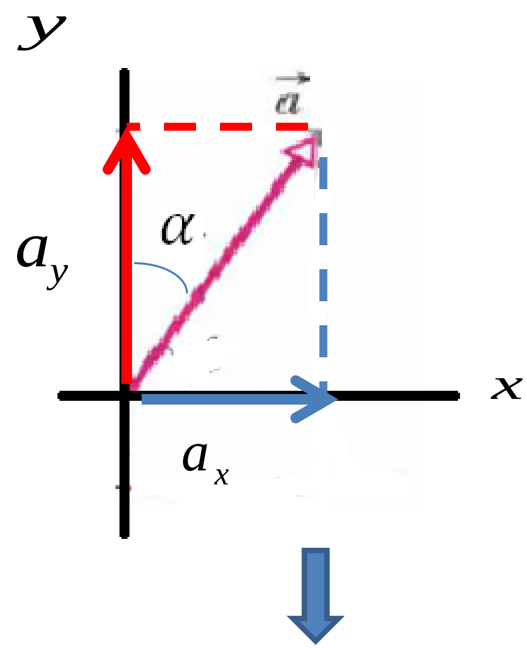


$$\cos\theta = \frac{a_x}{a} \quad , \quad \sin\theta = \frac{a_y}{a}$$

$$a_x = a\ \cos\theta \ \ and \ \ a_y = a\ \sin\theta$$



$$a_x = a\cos\theta \ \ and \ a_y = a\sin\theta$$



$$a_x = a \ \sin\alpha \ \ and \ a_y = a\cos\alpha$$

$$\vec{a}$$

a and θ

- Finding the components.

$$a_x = a \cos\theta$$

$$a_y = a \sin\theta$$

a_x and a_y

- Writing a vector in magnitude-angle notation

$$a = |a| = \sqrt{a_x{}^2 + a_y{}^2}$$

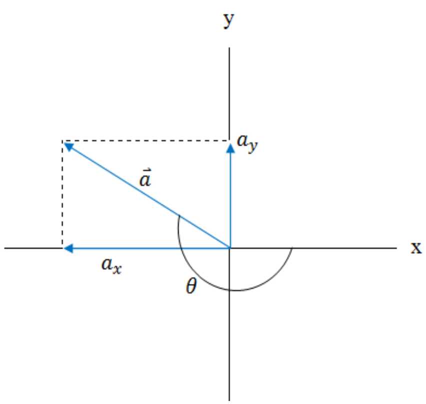
$$\tan\theta = \frac{a_y}{a_x}$$

Rem : When use these formulas to find the components, the angle must be measured from positive X-axis, if clockwise put θ -ve if counterclockwise put θ +ve.

How to find the components of a vector in different positions?

When the angle is from the +ve x-axis

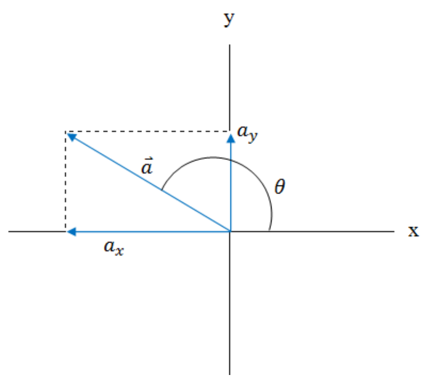
Clockwise



$$a_x = a\cos -\theta$$

$$a_y = a\sin -\theta$$

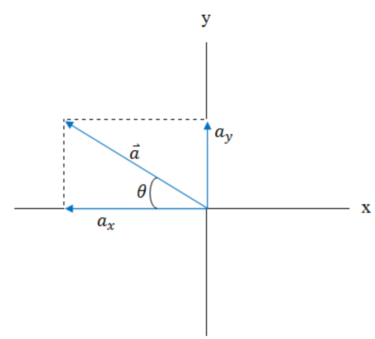
Counter-clockwise



$$a_x = a\cos\theta$$

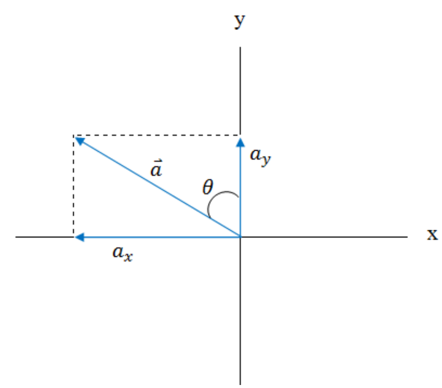
$$a_y = a\sin\theta$$

When the angle is from any different axis



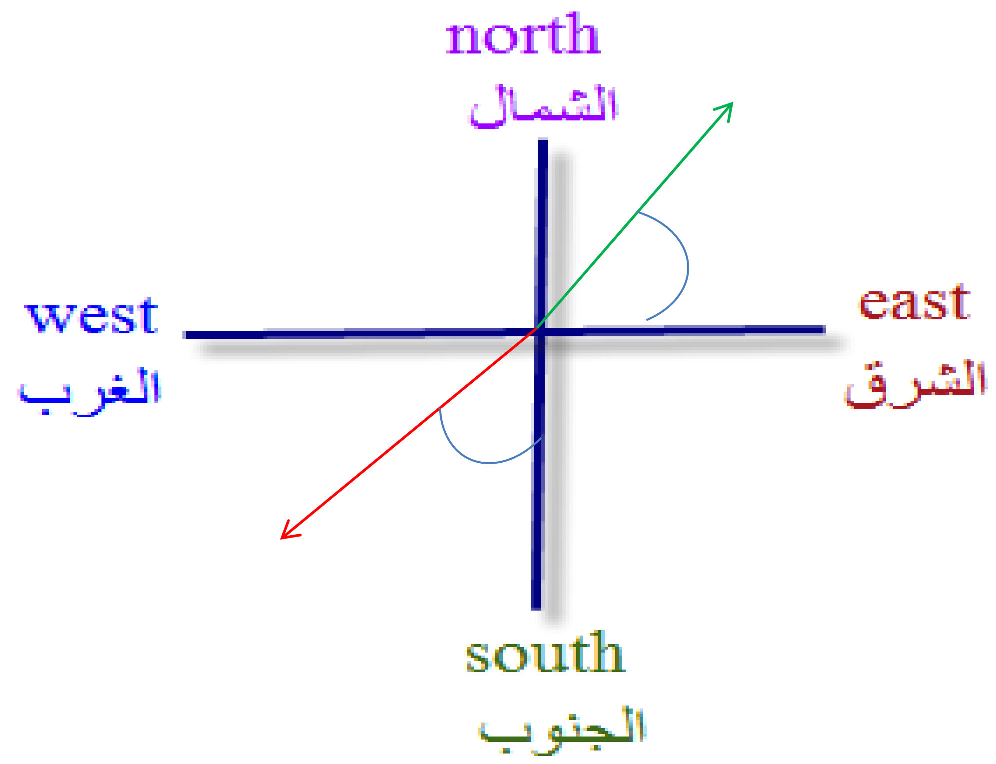
$$a_x = -\,a\cos\theta$$

$$a_y = +\,a\sin\theta$$



$$a_x = -\,a\sin\theta$$

$$a_y = +\,a\cos\theta$$

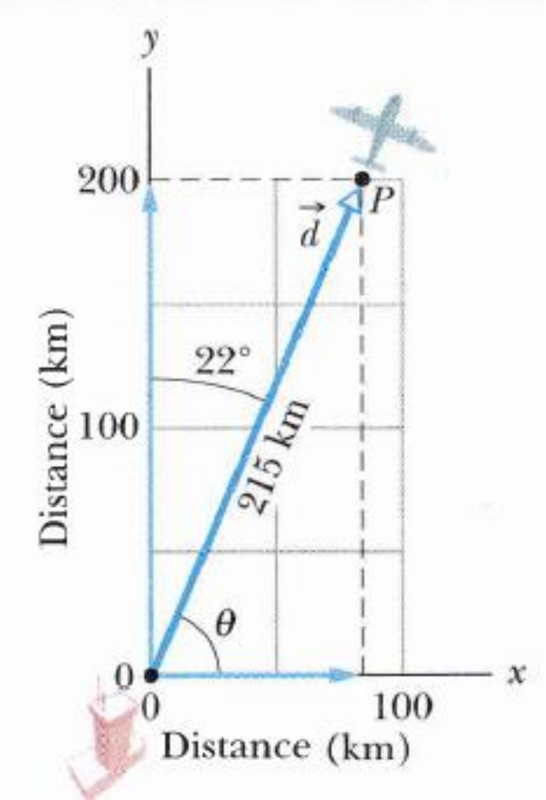


North of east = toward the north from due east

West of south= = toward the west from due south

Sample Problem 3.02

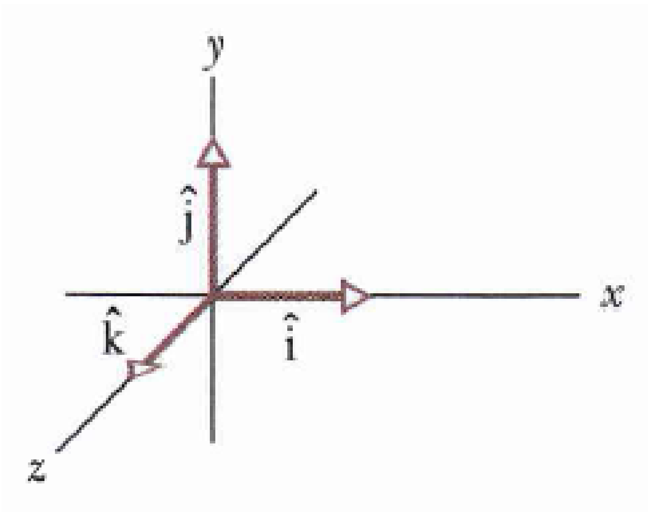
A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



3-2 UNIT VECTORS, ADDING VECTORS BY COMPONENTS

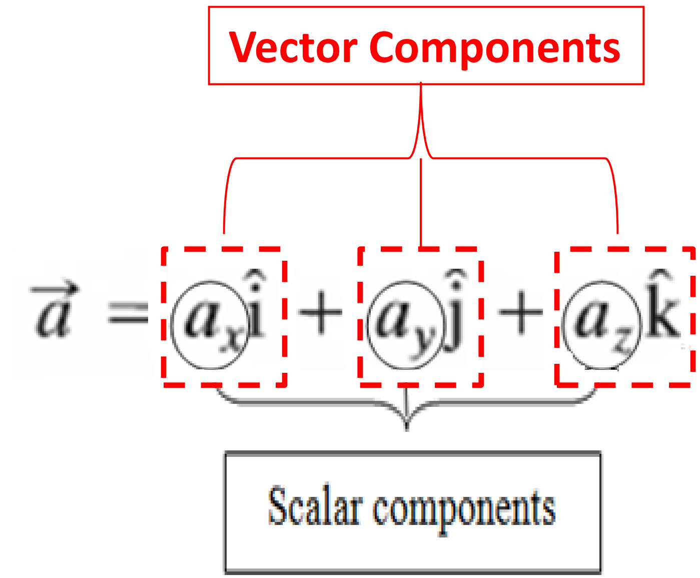
Unit Vectors

- Unit vector is a vector of magnitude 1 and points in a particular direction



- **Writing a vector in Unit vector notation**

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$



Adding vectors by Components

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{r} = \vec{a} \mp \vec{b}$$

$$\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}$$

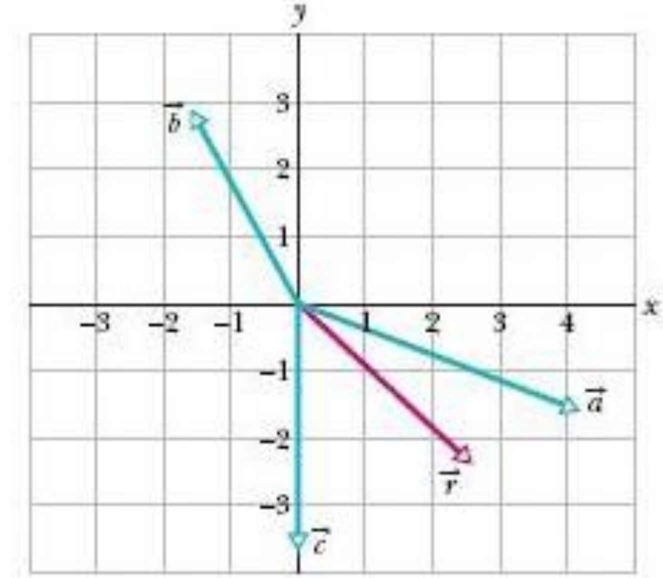
$$r_x = a_x \mp b_x \qquad r_y = a_y \mp b_y \qquad r_z = a_z \mp b_z$$

Sample Problem 3.04

Figure 3-17a shows the following three vectors:

$$\vec{a} = (4.2\ \text{m})\hat{\text{i}} - (1.5\ \text{m})\hat{\text{j}},$$
$$\vec{b} = (-1.6\ \text{m})\hat{\text{i}} + (2.9\ \text{m})\hat{\text{j}},$$
and
$$\vec{c} = (-3.7\ \text{m})\hat{\text{j}}.$$

What is their vector sum $\vec{r}$ which is also shown?



3-3 MULTIPLYING VECTORS

Multiplying vectors

Multiplying a vector by a scalar

- **+ve scalar** → will produce a new vector in the same direction as the started vector →

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

$$2\vec{a} = 4\hat{\imath} + 6\hat{\jmath}$$

- **-ve scalar** → will produce a new vector in the opposite direction of the started vector →

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

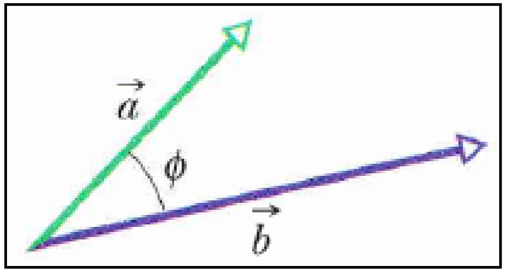
$$-2\vec{a} = -4\hat{\imath} - 6\hat{\jmath}$$

Multiplying a vector by a vector

- **Scalar product (or Dot product)** → will produce a scalar
- **Vector product (or cross product)** → will produce a new vector

The Scalar (Dot product)

If the two vectors are given in magnitude and the angle between them



$$\vec{a} \cdot \vec{b} = ab\cos\phi$$

If the two vectors are given in unit vector notation

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

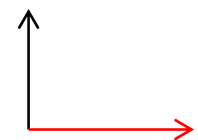
$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a} \cdot \vec{b} = ab \cos\phi$$

1- The scalar product is commutative ➡ $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

2- If the two vectors are parallel ➡ $\theta = 0 \Rightarrow \vec{a} \, . \vec{b} = a\, b$



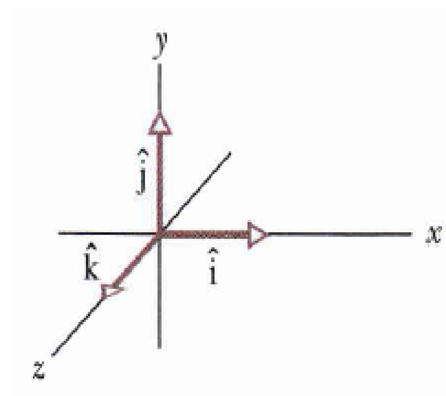
3- If the two vectors are perpendicular ➡ $\theta = 90 \Rightarrow \vec{a} \, . \vec{b} = 0$

4- If the two vectors are Antiparallel ➡ $\theta = 180 \Rightarrow \vec{a} \, . \vec{b} = -\, a\, b$

5- Multiplying Unit vectors

$$\hat{i} . \hat{i} = (1)(1)\cos 0 = 1 \;\Rightarrow\; \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} . \hat{j} = (1)(1)\cos 90 = 0 \;\Rightarrow\; \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$



Properties Of the scalar product

The scalar product is commutative

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

the angle between two vectors can be found

$$\vec{a} \cdot \vec{b} = ab \cos\theta$$

any two similar unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

any two different unit vectors

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

If $\theta = 0 \Rightarrow \vec{a} . \vec{b} = a\, b$ ⇨ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} . \vec{b} = -\, a\, b$ ⇨ vectors are anti parallel

$\theta = 90 \Rightarrow \vec{a} . \vec{b} = 0$ ⇨ vectors are perpendicular

Sample Problem 3.05

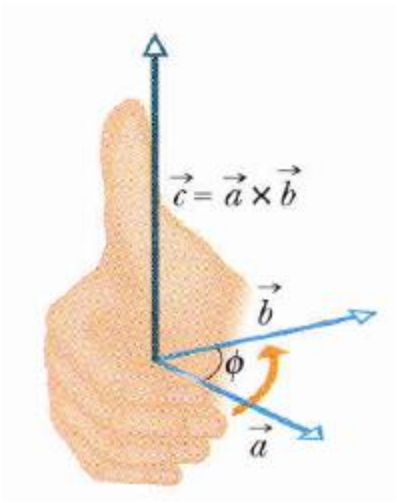
What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

Vector (Cross product)

If the two vectors are given in magnitude and angle between them

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

The direction of the result vector



If the two vectors are given in unit vector notation

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

1- The vector product is Anti commutative ➡ $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

2- If the two vectors are parallel ➡ $\theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$

3- If the two vectors are perpendicular ➡ $\theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = a\,b$

4- If the two vectors are Anti-parallel ➡ $\theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$

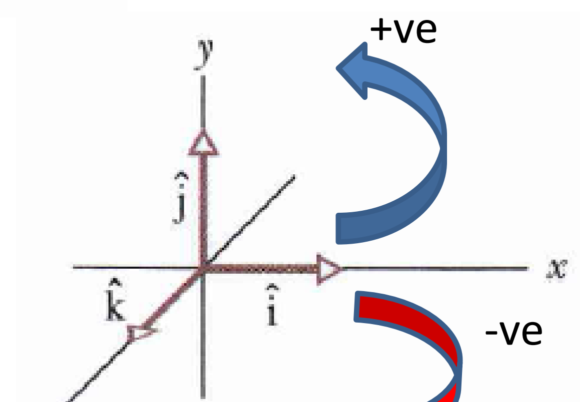
5- Multiplying Unit vectors

$$\left|\hat{i} \times \hat{i}\right| = (1)(1)\sin 0 = 0 \;\Rightarrow\; \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\left|\hat{i} \times \hat{j}\right| = (1)(1)\sin 90 = 1 \;\Rightarrow\; \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}, \qquad \hat{j} \times \hat{k} = \hat{i}, \qquad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \qquad \hat{k} \times \hat{j} = -\hat{i} \qquad \hat{i} \times \hat{k} = -\hat{j}$$



Properties of the Vector product

Anti- commutative

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

any two different unit vectors

$$\hat{i} \times \hat{j} = \hat{k}, \qquad \hat{j} \times \hat{k} = \hat{i},$$

$$\hat{k} \times \hat{i} = \hat{j}$$

any two similar unit vectors

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

The small angle between the two vectors must be used because the odd property of the sin function

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

If

$\theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$ ⇒ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$ ⇒ vectors are anti parallel

$\theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = a\,b$ ⇒ vectors are perpendicular

Sample Problem 3.07

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?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