

Workshop Solutions to Section 2.5

How to find the domain and range of the exponential function $f(x) = a^x$?

1- If $f(x) = c \cdot a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$

2- If $f(x) = -c \cdot a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

3- If $f(x) = c \cdot e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$

4- If $f(x) = -c \cdot e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

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| <p>1) Find the domain of the function $f(x) = 4^x$.</p> <p><u>Solution:</u></p> <p>From Step (1) above, we deduce that $D_f = \mathbb{R}$</p> | <p>2) Find the range of the function $f(x) = 4^x$.</p> <p><u>Solution:</u></p> <p>From Step (1) above, we deduce that $R_f = (0, \infty)$</p> |
| <p>3) Find the domain of the function $f(x) = 4^x - 3$.</p> <p><u>Solution:</u></p> <p>From Step (1) above, we deduce that $D_f = \mathbb{R}$</p> | <p>4) Find the range of the function $f(x) = 4^x - 3$.</p> <p><u>Solution:</u></p> <p>From Step (1) above, we deduce that $R_f = (-3, \infty)$</p> |
| <p>5) Find the domain of the function $f(x) = 5 - 3^x$.</p> <p><u>Solution:</u></p> <p>From Step (2) above, we deduce that $D_f = \mathbb{R}$</p> | <p>6) Find the range of the function $f(x) = 5 - 3^x$.</p> <p><u>Solution:</u></p> <p>From Step (2) above, we deduce that $R_f = (-\infty, 5)$</p> |
| <p>7) Find the domain of the function $f(x) = 3^{-x} + 1$.</p> <p><u>Solution:</u></p> <p>From Step (1) above, we deduce that $D_f = \mathbb{R}$</p> | <p>8) Find the range of the function $f(x) = 3^{-x} + 1$.</p> <p><u>Solution:</u></p> <p>From Step (1) above, we deduce that $R_f = (1, \infty)$</p> |
| <p>9) Find the domain of the function $f(x) = e^x$.</p> <p><u>Solution:</u></p> <p>From Step (3) above, we deduce that $D_f = \mathbb{R}$</p> | <p>10) Find the range of the function $f(x) = e^x$.</p> <p><u>Solution:</u></p> <p>From Step (3) above, we deduce that $R_f = (0, \infty)$</p> |
| <p>11) Find the domain of the function $f(x) = e^x - 3$.</p> <p><u>Solution:</u></p> <p>From Step (3) above, we deduce that $D_f = \mathbb{R}$</p> | <p>12) Find the range of the function $f(x) = e^x - 3$.</p> <p><u>Solution:</u></p> <p>From Step (3) above, we deduce that $R_f = (-3, \infty)$</p> |
| <p>13) Find the domain of the function $f(x) = e^x + 1$.</p> <p><u>Solution:</u></p> <p>From Step (3) above, we deduce that $D_f = \mathbb{R}$</p> | <p>14) Find the domain of the function $f(x) = \frac{1}{1-e^x}$.</p> <p><u>Solution:</u></p> <p>$f(x)$ is defined when $1 - e^x \neq 0$ $\Leftrightarrow e^x \neq 1 \Leftrightarrow \ln e^x \neq \ln 1$ $\Leftrightarrow x \neq 0$ $\therefore D_f = \mathbb{R} \setminus \{0\}$</p> |

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| <p>15) Find the domain of the function $f(x) = \frac{1}{1+e^x}$.</p> <p><u>Solution:</u></p> <p>$f(x)$ is defined when $1 + e^x \neq 0$. But there is no value of x makes $1 + e^x = 0$. Therefore, $D_f = \mathbb{R}$</p> | <p>16) Find the domain of the function $f(x) = \sqrt{1 + 3^x}$.</p> <p><u>Solution:</u></p> <p>$f(x)$ is defined when $1 + 3^x \geq 0$. But $1 + 3^x > 0$ always. Therefore, $D_f = \mathbb{R}$</p> |
| <p>17) If $4^{(x+1)} = 8$, then $x =$</p> <p><u>Solution:</u></p> $\begin{aligned} 4^{(x+1)} &= 8 \\ (2^2)^{(x+1)} &= 2^3 \\ 2^{2(x+1)} &= 2^3 \\ 2(x+1) &= 3 \\ 2x+2 &= 3 \\ 2x &= 3-2=1 \\ \therefore x &= \frac{1}{2} \end{aligned}$ | <p>18) If $4^{(x-1)} = 8$, then $x =$</p> <p><u>Solution:</u></p> $\begin{aligned} 4^{(x-1)} &= 8 \\ (2^2)^{(x-1)} &= 2^3 \\ 2^{2(x-1)} &= 2^3 \\ 2(x-1) &= 3 \\ 2x-2 &= 3 \\ 2x &= 3+2=5 \\ \therefore x &= \frac{5}{2} \end{aligned}$ |
| <p>19) If $9^{(x+1)} = 27$, then $x =$</p> <p><u>Solution:</u></p> $\begin{aligned} 9^{(x+1)} &= 27 \\ (3^2)^{(x+1)} &= 3^3 \\ 3^{2(x+1)} &= 3^3 \\ 2(x+1) &= 3 \\ 2x+2 &= 3 \\ 2x &= 3-2=1 \\ \therefore x &= \frac{1}{2} \end{aligned}$ | <p>20) If $9^{(x-1)} = 27$, then $x =$</p> <p><u>Solution:</u></p> $\begin{aligned} 9^{(x-1)} &= 27 \\ (3^2)^{(x-1)} &= 3^3 \\ 3^{2(x-1)} &= 3^3 \\ 2(x-1) &= 3 \\ 2x-2 &= 3 \\ 2x &= 3+2=5 \\ \therefore x &= \frac{5}{2} \end{aligned}$ |
| <p>21) If $5^{2(x-1)} = 125$, then $x =$</p> <p><u>Solution:</u></p> $\begin{aligned} 5^{2(x-1)} &= 125 \\ 5^{2(x-1)} &= 5^3 \\ 2(x-1) &= 3 \\ 2x-2 &= 3 \\ 2x &= 3+2=5 \\ \therefore x &= \frac{5}{2} \end{aligned}$ | <p>22) If $5^{2(x+1)} = 125$, then $x =$</p> <p><u>Solution:</u></p> $\begin{aligned} 5^{2(x+1)} &= 125 \\ 5^{2(x+1)} &= 5^3 \\ 2(x+1) &= 3 \\ 2x+2 &= 3 \\ 2x &= 3-2=1 \\ \therefore x &= \frac{1}{2} \end{aligned}$ |