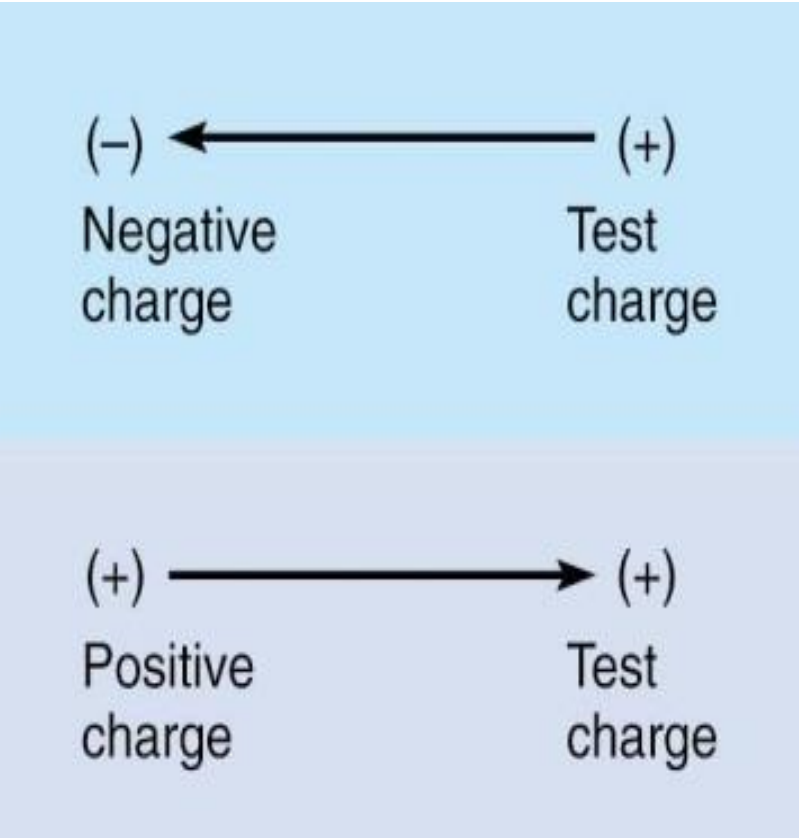
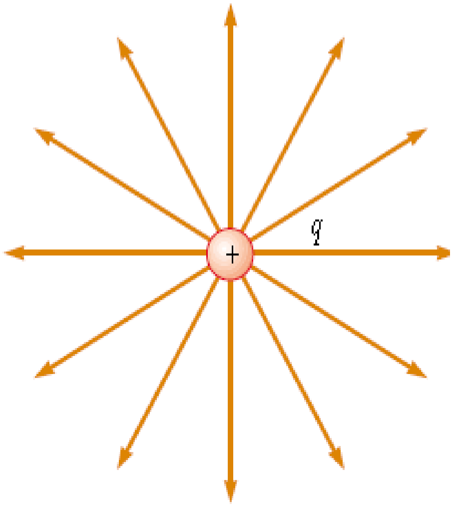
ELECTRIC FIELD

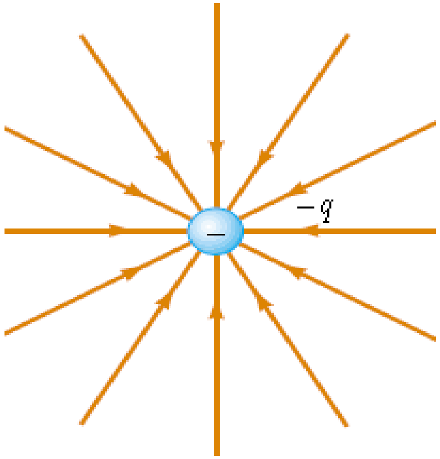
- **Electric field** is the space around the electric charge.
- Electric field is represented by lines extending away **from positive charge and towards negative charge**. These lines are also called **the lines of force**.
- **A positive test charge** is conventionally used to identify the properties of an electric field. The vector arrow points in the direction of the force that the test charge would experience.

ELECTRIC FIELD





(a)



(b)

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

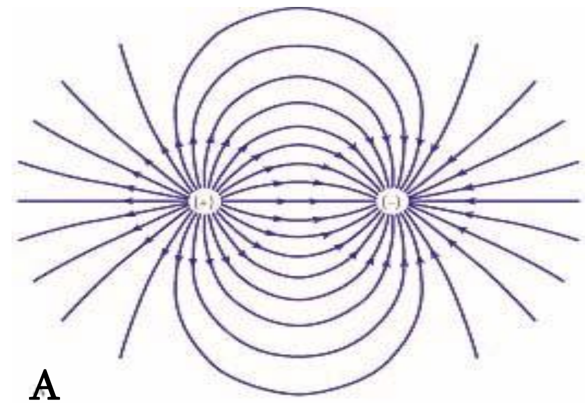
ELECTRIC FIELD LINES OF POINT CHARGES

Lines of force 'Electric Field' for

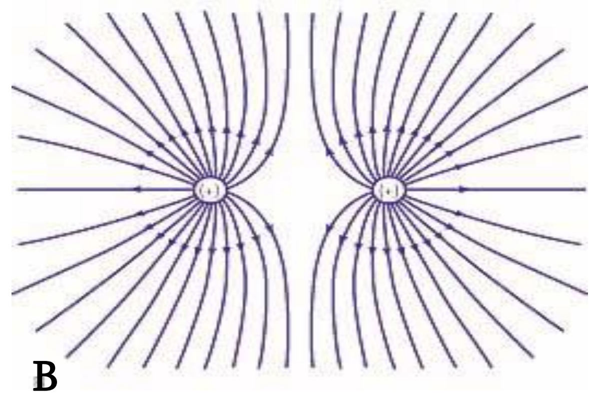
(A) **different signed charges**

(B) positive signed charges

The charges have the same magnitude as the test charge.



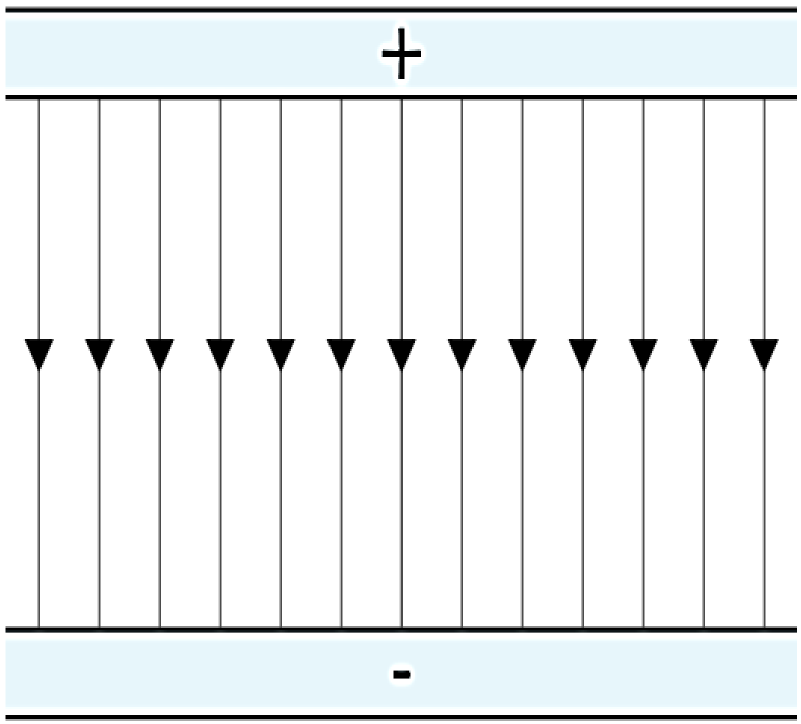
A



B

Electric Field Lines of Continuous Distribution of Charges

Lines of force 'Electric Field' for a uniformly distributed charges are straight lines with same magnitude of electric field. This field is then called a uniform electric field.



ELECTRIC FIELD DUE TO A POINT CHARGE

- The magnitude of the electric field due to any point charge q at any distance r is given by

$$E = \frac{k\,|q|}{r^2}$$

- The electric force, therefore, on another point charge Q due to this electric field is

$$\vec{F} = Q\vec{E}$$

- The SI unit of the electric field is N/C.

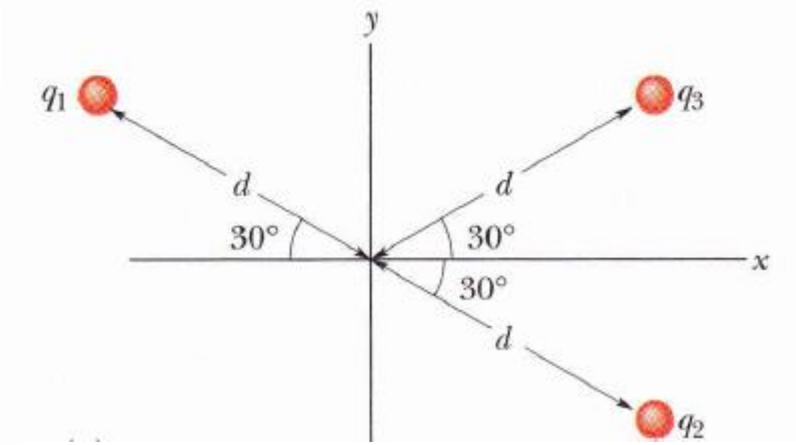
<u>NOTE</u>: The above equation suggests that the electric force has the same direction of the electric field unless the charge Q is negative.

$$\mathbf{F}_g = m\mathbf{g}$$

$$\boxed{\mathbf{F}_e = q\mathbf{E}}$$

Sample Problem

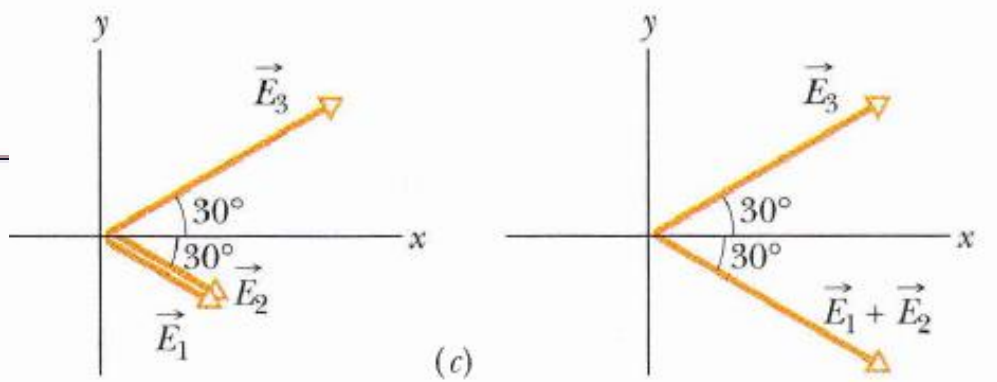
Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?



Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}$$



$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

$E(net) = E_y + E_x$, $E_y = 0$, $E_x = E_{3x}\cos 30 + (E_{1x} + E_{2x})\cos 30$

$$E_{1x} + E_{2x} = E_{3x}$$

$$E = 2E_{3x} = 2E_3 \cos 30^\circ$$
$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

Electric Field Due to an Electric Dipole

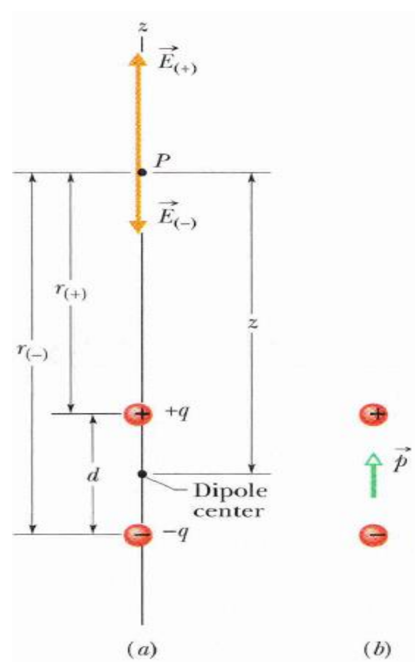
- For any two equal and opposite-signed charges q and $-q$ separated by a distance d, the magnitude of their electric dipole is

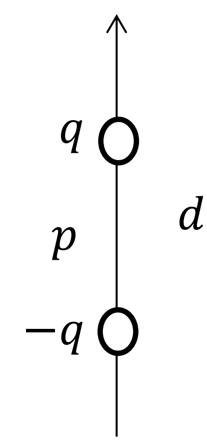
$$p = q\,d$$

- The unit of the electric dipole is C.m and its direction is from negative to positive charge.
- The magnitude of the electric field at a distance z from the midpoint of an electric dipole p is given by

$$E = \frac{1}{2\pi\varepsilon_0}\cdot\frac{p}{z^3}$$

$$E = \frac{q}{2\pi\varepsilon_0 z^3}\,\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}. \qquad z \gg d \rightarrow$$





- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

$$\rho \equiv \frac{Q}{V}$$

where ρ has units of coulombs per cubic meter $(\mathrm{C/m^3})$.

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ (lowercase Greek sigma) is defined by

$$\sigma \equiv \frac{Q}{A}$$

where σ has units of coulombs per square meter $(\mathrm{C/m^2})$.

- If a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density** λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

where λ has units of coulombs per meter $(\mathrm{C/m})$.

- If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge dq in a small volume, surface, or length element are

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

ELECTRIC FIELD DUE TO A LINE OF CHARGE

The linear charge density λ is defined as λ=q/L where L is the length of the line (or circumference of the circ

For a ring having a total charge q and radius R, the electric field at a distance z from the cente

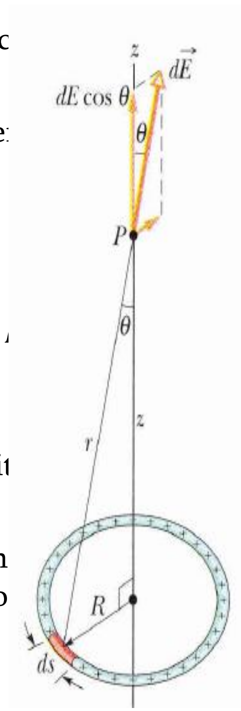
$$E = \frac{q\,z}{4\pi\varepsilon_0 (z^2 + R^2)^{\frac{3}{2}}}$$

The electric field at large distance from the center $z \gg$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}$$

From this we note that the ring appears like a point charge at large distance from it

The electric field at the center of the ring $z = 0$ is E=0, because the electric force on a test charge located at the cen due to any point on the ring will be cancelled by the opposite-sided po



Electric Field Due to a Surface of Charge (Disk)

- The surface charge density σ is defined as σ =q/A where A is the area of the surface.

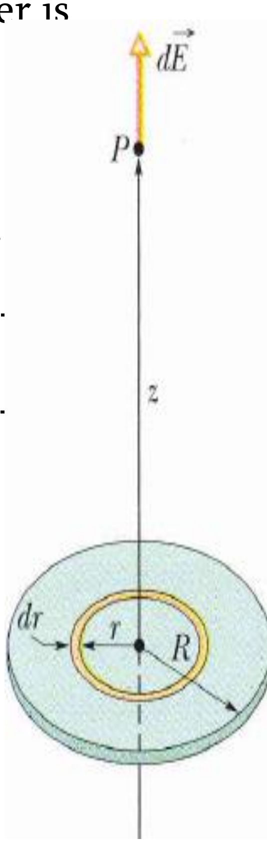
- For a disk having a surface charge σ and radius R, the electric field at a distance z from the center is

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

The electric field at large distance from the center of the disk $z \gg R$ is

The electric field at the center of the disk $z = 0$ is $E =$

The electric field for infinite sheet $R \rightarrow \infty$ is $E =$



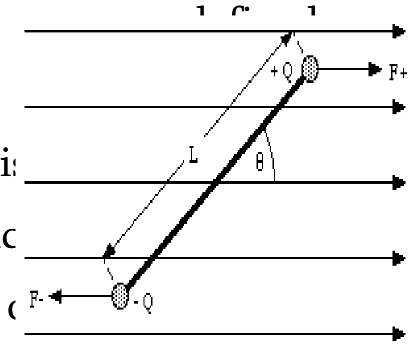
Electric Dipole in an Electric Field - Torque

- If a dipole $\vec{p}$ is place in a uniform electric field $\vec{E}$, the field will exert a torque $\vec{\tau}$ which is

$$\vec{\tau} = \vec{p} \times \vec{E} = pE\ \sin\theta\ \hat{n}$$

The unit of the torque is N.m and its direction i

diploe and electric



The torque will act on the diploe and rotate it to be in the direction

(reducing θ to zero).

- The minimum torque occurs when p and E are parallel $\theta = 0$ or anti-parallel $\theta = 180$ with $\tau = 0$
- The maximum torque occurs when p and E are perpendicular $\theta = 90$ with $\tau = pE$.

ELECTRIC DIPOLE IN AN ELECTRIC FIELD - POTENTIAL

- If a dipole $\vec{p}$ is place in a uniform electric field $\vec{E}$, the dipole has a potential energy U which is defined as

$$U = -\vec{p} \cdot \vec{E} = -pE \cos\theta$$

The unit of the potential is Joule (J)

- The maximum potential occurs when p and E are anti-parallel $\theta = 180$ with $U = pE$.
- The minimum potential occurs when p and E are parallel $\theta = 0$ with $U = -pE$.
- If the diploe rotates from initial orientation θ_i to final orientation θ_f, the work done is

$$W = -\Delta U = -\left(U_f - U_i\right)$$

- The work done by an external torque is the negative of the above work.

Worked Exercises

1. Calculate the electric field at a distance 5 cm from a point charge 2.5 nC.

Solution

The magnitude of the electric field at any distance from a point charge is given by

$$E = \frac{k\,|q|}{r^2}$$

Hence the electric field at 5 cm from the charge is

$$E = \frac{k\,q}{r^2} = \frac{9\times10^{9}\times2.5\times10^{-9}}{0.05^2} = 9000\ \mathrm{N/C}$$

WORKED EXERCISES

2. A point charge produces an electric field of 180 N/C at 2 cm. Calculate the magnitude of the electric field at 4 cm.

Solution

The magnitude of the electric field due to a point charge is given by

$$E = \frac{k\,|q|}{r^2}$$

Therefore to find the magnitude of the electric field at any point, we have to determine the charge, firstly.

$$q = \frac{E\,r^2}{k} = \frac{180\times 0.02^2}{9\times 10^9} = 8\times 10^{-12}\mathrm{C}$$

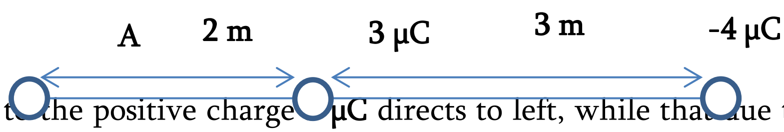
Hence the electric field at 4 cm from the charge is

$$E = \frac{k\,q}{r^2} = \frac{9\times 10^9\times 8\times 10^{-12}}{0.04^2} = 45\ \mathrm{N/C}$$

WORKED EXERCISES

3. As shown in the arrangement below, calculate the magnitude of the electric field at the point A. If a third charge 2 μC is placed at point A, calculate the force acting on it.

Solution



- The electric field at point A due to the positive charge 3 μC directs to left, while that due to the negative charge **-4 μC** directs to right. That means the magnitude of the electric field at A is their difference . Firstly, the magnitude of the electric field at A due to **2 μC** is

$$E_1 = \frac{k\,q}{r^2} = \frac{9\times10^{9}\times3\times10^{-6}}{2^2} = 6750\ \text{N/C}$$

WORKED EXERCISES

Secondly, the magnitude of the electric field at A due to -4 μC is

$$E_2 = \frac{k\,q}{r^2} = \frac{9\times10^{9}\times4\times10^{-6}}{5^2} = 1440\ \text{N/C}$$

Therefore the magnitude of the electric field at point A due to both charges is

$$E = E_1 + E_2 = 6750 - 1440 = 5310\ \text{N/C}$$

- We know that the electric force on any charge is defined as

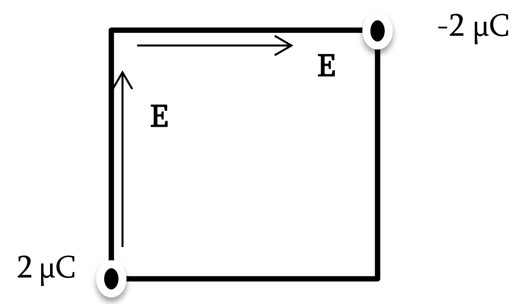
$$F = QE = 2\times10^{-6}\times5310 = 0.01062\ \text{N}$$

NOTE: Please revisit the previous chapter and compare this solution with that of worked exercise 6.

WORKED EXERCISES

4. Two charges (2 μC and -2 μC) are located at the two corners of a square of side 5 m as shown in the figure. Calculate the magnitude of the electric field at one of the other centres of the square.

Solution



Taking the upper corner, the magnitudes of the electric field due to each charge are similar but directions are different.

WORKED EXERCISES

Therefore we can calculate the electric field of one of them as

$$E_1 = \frac{k\,q}{r^2} = \frac{9\times10^{9}\times2\times10^{-6}}{5^2} = 720\ \mathrm{N/C}$$

Since the electric fields of these charges are perpendicular, we can find the magnitude as

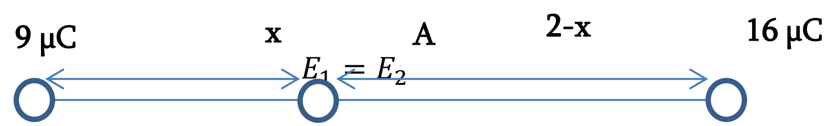
$$E = \sqrt{E_1^2 + E_2^2} = \sqrt{E_1^2 + E_1^2} = E_1\sqrt{2} = 720\sqrt{2} = 1018\ \mathrm{N/C}$$

WORKED EXERCISES

5. Two charges 9.0 and 16.0 μC are separated by a distance of 2 m. Where should a third charge be placed for a net force on it zero?

Solution

As the charges are of same sign, the resultant electric field due to both will vanish (be zero) in somewhere ONLY between them and nowhere else. Consider a point A at distance x from the smaller charge where the electric field vanishes, the



$$\frac{9 \times 10^9 \times 9 \times 10^{-6}}{x^2} = \frac{9 \times 10^9 \times 16 \times 10^{-6}}{(2-x)^2}$$

WORKED EXERCISES

$$\frac{9}{x^2} = \frac{16}{(2-x)^2}$$

Taking the square root of the above, we get

$$\frac{3}{x} = \frac{4}{2-x}$$

This leads to

$$x = \frac{6}{7} = 0.86\ m$$

NOTE: Please revisit the previous chapter and compare this solution with that of worked exercise 8.

WORKED EXERCISES

6. Two equal and opposite charges 6.0 μC and -6.0 μC are separated by a distance of 2 cm. What is the magnitude of the electric field at 30 cm from their midpoint?

Solution

The magnitude of the electric field due to a dipole is

$$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{p}{z^3} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{q\,d}{z^3} = \frac{1}{2\pi \times 8.85 \times 10^{-12}} \cdot \frac{6 \times 10^{-6} \times 0.02}{0.3^3} = 8.0 \times 10^4 \mathrm{N/C}$$

Worked Exercises

7. The magnitude of the electric field at 5 cm from an infinite sheet is 200 N/C, what is the surface charge density?

Solution

The magnitude of the electric field at any distance from an infinite sheet is

$$E = \frac{\sigma}{2\varepsilon_0} \quad \rightarrow \quad \sigma = 2\varepsilon_0 E = 2 \times 8.85 \times 10^{-12} \times 200 = 3\ \mathrm{nC/m^2}$$

WORKED EXERCISES

8. An electron with initial speed of 9×10^6m/s enters a parallel, uniform electric field of magnitude 4500 N/C. Calculate the distance the electron travelled before stop?

Solution

The electron has negative charge, therefore the force acting on it will be in the opposite direction to the electric field which makes the electron stops. The magnitude of the force is

$$F = e\,E \;\; = 1.6 \times 10^{-19} \times 4500 = 7.2 \times 10^{-16}\ \mathrm{N}$$

From mechanics we know that the resultant force equals the mass times the acceleration (Newton's second law)

$$a = \frac{F}{m} = \frac{7.2 \times 10^{-16}}{9.11 \times 10^{-31}} = 7.9 \times 10^{14}\ \mathrm{m/s^2}$$

This is the magnitude but actually it is in minus because it is deceleration.

WORKED EXERCISES

From mechanics also we get

$$v^2 = v_0^2 + 2ax \quad \rightarrow \quad x = (v^2 - v_0^2)/2a = (0^2 - (9 \times 10^6)^2/(-2 \times 7.9 \times 10^{14}) = 5 \text{ cm}$$

WORKED EXERCISES

9. A ring of radius 5 cm carries a linear charge density of 5 × 10[illegible] C/m. Calculate the electric field at (i) 6 cm and (ii) 6 m from its center?

Solution

(i) The electric field due to a ring is

$$E = \frac{q\,z}{4\pi\varepsilon_0 (z^2 + R^2)^{\frac{3}{2}}}$$

But the charge is given by

q= λ L where L is the circumference of the ring (L = $2\pi R$)

Therefore

$$E = \frac{(\lambda 2\pi R)z}{4\pi\varepsilon_0 (z^2 + R^2)^{\frac{3}{2}}} = \frac{\lambda R z}{2\varepsilon_0 (z^2 + R^2)^{\frac{3}{2}}} = 1.8 \times 10^6 \mathrm{N/C}$$

WORKED EXERCISES

field could be given as

$$E = \frac{q}{4\pi\varepsilon_0 z^2} = \frac{\lambda R}{2\varepsilon_0 z^2} = 392\ \mathrm{N/C}$$

Worked Exercises

10. Calculate the magnitude of the torque and work needed to completely align a dipole of 5×10^{-6}C.m in a uniform electric field of 5000 N/C where the dipole initially was at an angle of 30 degrees with the field.

Solution

The torque is defined as

$$\vec{\tau} = \vec{p} \times \vec{E} = pE \ \sin\theta \ \hat{n}$$

Therefore the magnitude of the torque needed to align the diploe is

$$\tau = 5 \times 10^{-6} \times 5000 \times \sin 30 = 0.0125 \text{ N.m}$$

The work is

$$W = -\Delta U = -\left(U_f - U_i\right) = -PE\ (cos0 - cos30) = -5 \times 10^{-6} \times 5000 \times (cos0 - cos30) = -0.0034 \text{ J}$$