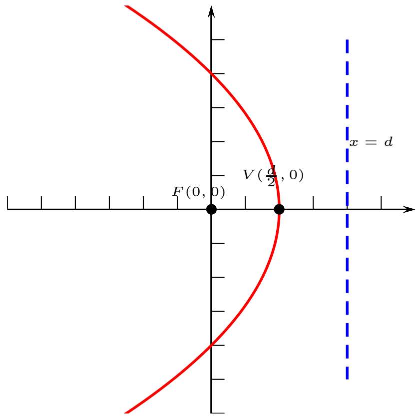
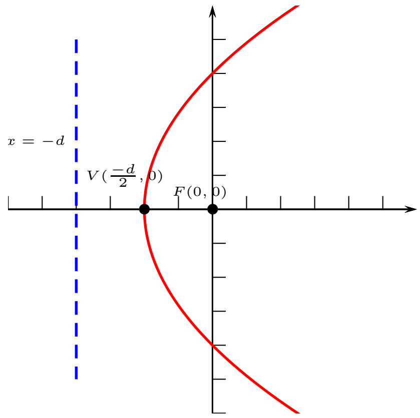
Conic Sections in Polar Coordinates

$e = 1$, parabola with horizontal axis:

Equation:	$r = \dfrac{d}{1 + \cos\theta}$
Eccentricity:	$e = 1$
Conic:	parabola
Open side:	left
vertex	$(\frac{d}{2}, 0)$, $@\theta = 0$, $(\frac{d}{2}, 0)$
focus	$(0, 0)$
directrix	$x = d$
rectangular Eq.	$y^2 = -2d(x - \frac{d}{2})$

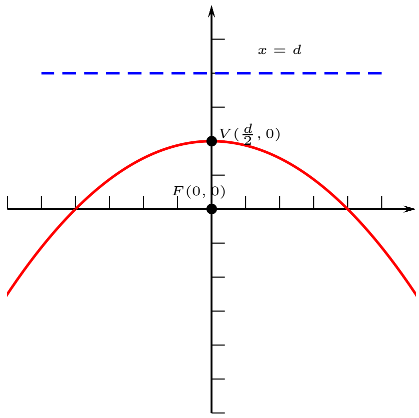


Equation:	$r = \dfrac{d}{1 - \cos\theta}$
Eccentricity:	$e = 1$
Conic:	parabola
Open side:	right
vertex	$(\frac{-d}{2}, 0)$, $@\theta = \pi$, $(\frac{d}{2}, \pi)$
focus	$(0, 0)$
directrix	$x = -d$
rectangular Eq.	$y^2 = 2d(x + \frac{d}{2})$

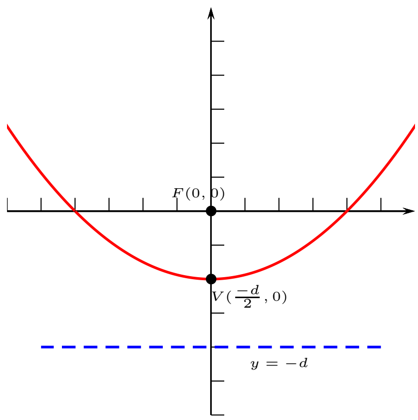


$e = 1$, parabola with vertical axis:

Equation:	$r = \dfrac{d}{1+\sin\theta}$
Eccentricity:	$e = 1$
Conic:	parabola
Open side:	down
vertex	$(0, \frac{d}{2})$, $@\theta = \frac{\pi}{2}$, $(\frac{d}{2}, \frac{\pi}{2})$
focus	$(0, 0)$
directrix	$y = d$
rectangular Eq.	$x^2 = -2d(y - \frac{d}{2})$

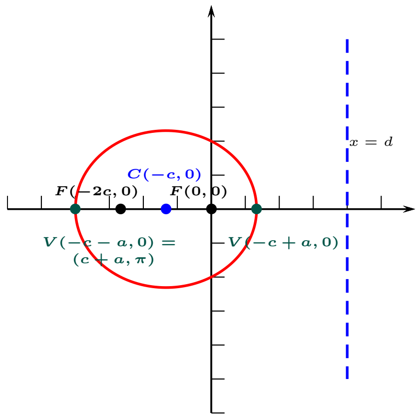


Equation:	$r = \dfrac{d}{1-\sin\theta}$
Eccentricity:	$e = 1$
Conic:	parabola
Open side:	up
vertex	$(0, \frac{-d}{2})$, $@\theta = \frac{3\pi}{2}$, $(\frac{d}{2}, \frac{3\pi}{2})$
focus	$(0, 0)$
directrix	$y = -d$
rectangular Eq.	$x^2 = 2d(y + \frac{d}{2})$

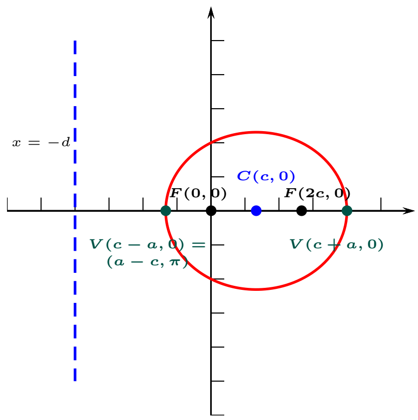


$0 < e < 1$, ellipse with horizontal axis:

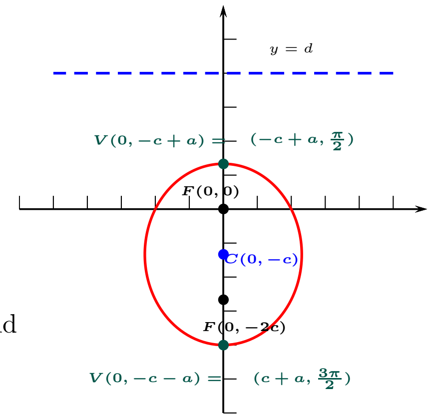
Equation:	$r = \dfrac{ed}{1 + e\cos\theta}$
Eccentricity:	$e < 1$
Conic:	ellipse
Major axis:	x-axis
Coefficients	$c = \dfrac{d}{e^{-2} - 1}$, $a = \dfrac{c}{e}$, $b = \sqrt{a^2 - c^2}$
Center	$(-c, 0)$
Vertices	$(-c - a, 0)@\theta = \pi$, $(c + a, \pi)$ and
	$(-c + a, 0)@\theta = 0$, $(a - c, 0)$
Foci	$(0, 0)$ and $(-2c, 0)$
directrix	$x = d$
Rectangular Eq.	$\dfrac{(x + c)^2}{a^2} + \dfrac{y^2}{b^2} = 1$



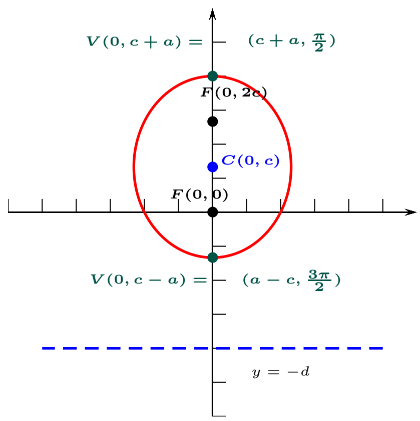
Equation:	$r = \dfrac{ed}{1 - e\cos\theta}$
Eccentricity:	$e < 1$
Conic:	ellipse
Major axis:	x-axis
Coefficients	$c = \dfrac{d}{e^{-2} - 1}$, $a = \dfrac{c}{e}$, $b = \sqrt{a^2 - c^2}$
Center	$(c, 0)$
Vertices	$(c - a, 0)@\theta = \pi$, $(a - c, \pi)$ and
	$(c + a, 0)@\theta = 0$, $(c + a, 0)$
Foci	$(0, 0)$ and $(2c, 0)$
directrix	$x = -d$
Rectangular Eq.	$\dfrac{(x - c)^2}{a^2} + \dfrac{y^2}{b^2} = 1$



$0 < e < 1$, ellipse with vertical axis:



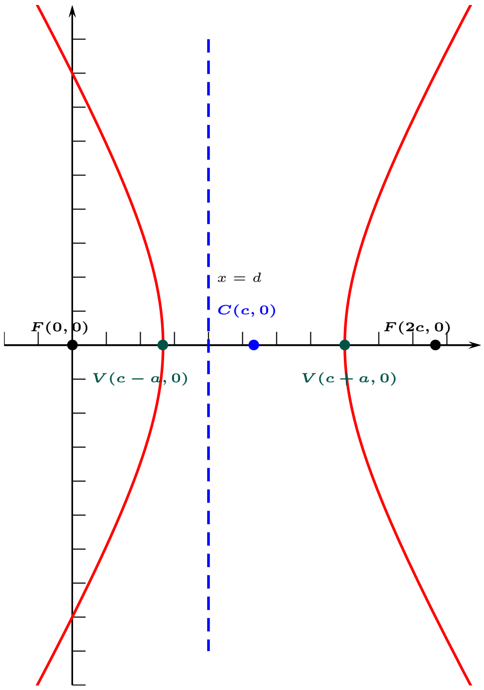
Equation:	$r = \dfrac{ed}{1 + e\sin\theta}$
Eccentricity:	$e < 1$
Conic:	ellipse
Major axis:	y-axis
Coefficients	$c = \dfrac{d}{e^{-2} - 1}$, $a = \dfrac{c}{e}$, $b = \sqrt{a^2 - c^2}$
Center	$(0, -c)$
Vertices	$(0, -c-a)@\theta = \dfrac{3\pi}{2}$, $(c+a, \dfrac{3\pi}{2})$ and
	$(0, -c+a)@\theta = \dfrac{\pi}{2}$, $(-c+a, \dfrac{\pi}{2})$
Foci	$(0,0)$ and $(0,-2c)$
directrix	$y = d$
Rectangular Eq.	$\dfrac{(y+c)^2}{a^2} + \dfrac{x^2}{b^2} = 1$



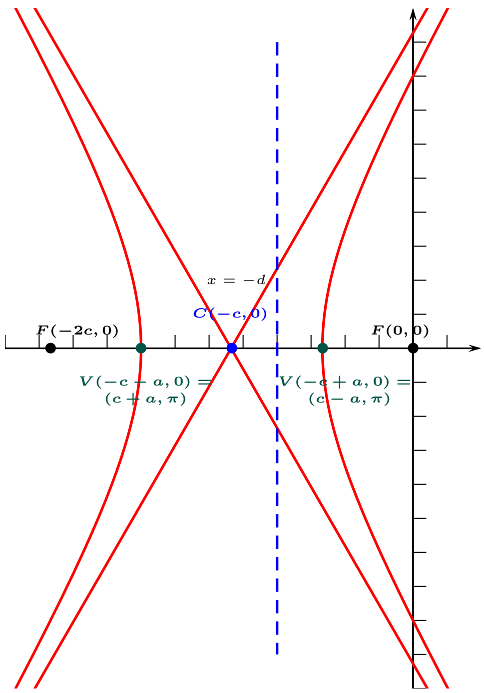
Equation:	$r = \dfrac{ed}{1 - e\sin\theta}$
Eccentricity:	$e < 1$
Conic:	ellipse
Major axis:	y-axis
Coefficients	$c = \dfrac{d}{e^{-2} - 1}$, $a = \dfrac{c}{e}$, $b = \sqrt{a^2 - c^2}$
Center	$(0, c)$
Vertices	$(0, c-a)@\theta = \dfrac{3\pi}{2}$, $(a-c, \dfrac{3\pi}{2})$ and
	$(0, c+a)@\theta = \dfrac{\pi}{2}$, $(c+a, \dfrac{\pi}{2})$
Foci	$(0,0)$ and $(0,2c)$
directrix	$y = -d$
Rectangular Eq.	$\dfrac{(y-c)^2}{a^2} + \dfrac{x^2}{b^2} = 1$

$e > 1$, hyperbola with horizontal axis:

Equation:	$r = \dfrac{ed}{1 + e\cos\theta}$
Eccentricity:	$e > 1$
Conic:	hyperbola
Transverse axis:	x-axis
Coefficients	$c = \dfrac{d}{1 - e^{-2}}$, $a = \dfrac{c}{e}$, $b = \sqrt{c^2 - a^2}$
Center	$(c, 0)$
Vertices	$(c + a, 0)@\theta = 0$, $(c + a, 0)$ and
	$(c - a, 0)@\theta = 0$, $(c - a, 0)$
Foci	$(0, 0)$ and $(2c, 0)$
directrix	$x = d$
asymptote	$y = \pm\dfrac{b}{a}(x - c)$
Rectangular Eq.	$\dfrac{(x - c)^2}{a^2} - \dfrac{y^2}{b^2} = 1$

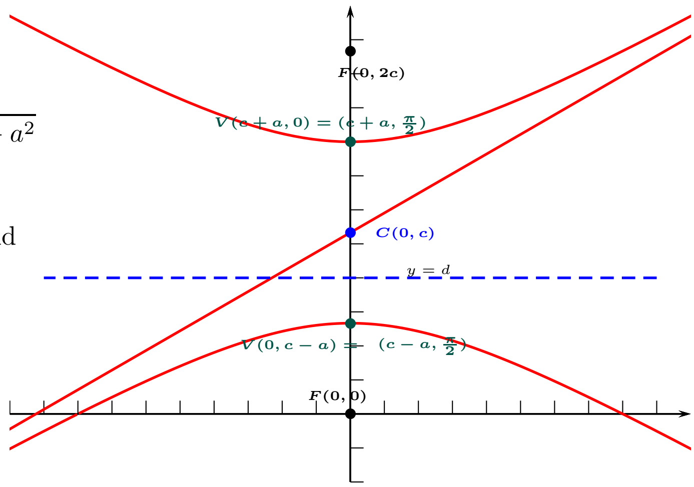


Equation:	$r = \dfrac{ed}{1 - e\cos\theta}$
Eccentricity:	$e > 1$
Conic:	hyperbola
Transverse axis:	x-axis
Coefficients	$c = \dfrac{d}{1 - e^{-2}}$, $a = \dfrac{c}{e}$, $b = \sqrt{c^2 - a^2}$
Center	$(-c, 0)$
Vertices	$(-c + a, 0)@\theta = \pi$, $(c - a, \pi)$ and
	$(-c - a, 0)@\theta = \pi$, $(c + a, \pi)$
Foci	$(0, 0)$ and $(-2c, 0)$
directrix	$x = -d$
asymptote	$y = \pm\dfrac{b}{a}(x + c)$
Rectangular Eq.	$\dfrac{(x + c)^2}{a^2} - \dfrac{y^2}{b^2} = 1$



$e > 1$, **hyperbola with vertical axis:**

Equation:	$r = \dfrac{ed}{1 + e\sin\theta}$
Eccentricity:	$e > 1$
Conic:	hyperbola
Transverse axis:	y-axis
Coefficients	$c = \dfrac{d}{1-e^{-2}}$, $a = \dfrac{c}{e}$, $b = \sqrt{c^2 - a^2}$
Center	$(0, c)$
Vertices	$(0, c+a)@\theta = \dfrac{\pi}{2}$, $(c+a, \dfrac{\pi}{2})$ and
	$(0, c-a)@\theta = \dfrac{\pi}{2}$, $(c-a, \dfrac{\pi}{2})$
Foci	$(0, 0)$ and $(0, 2c)$
directrix	$y = d$
asymptote	$y = \pm\dfrac{a}{b}x + c$
Rectangular Eq.	$\dfrac{(y-c)^2}{a^2} - \dfrac{x^2}{b^2} = 1$



Equation:	$r = \dfrac{ed}{1 - e\sin\theta}$
Eccentricity:	$e > 1$
Conic:	hyperbola
Transverse axis:	y-axis
Coefficients	$c = \dfrac{d}{1-e^{-2}}$, $a = \dfrac{c}{e}$, $b = \sqrt{c^2 - a^2}$
Center	$(0, -c)$
Vertices	$(0, -c-a) @ \theta = \dfrac{3\pi}{2}$, $(c+a, \dfrac{3\pi}{2}) =$ $(c+a, \dfrac{3\pi}{2})$ and $(0, -c+a) @ \theta = \dfrac{3\pi}{2}$, $(c-a, \dfrac{3\pi}{2})$
Foci	$(0,0)$ and $(0, -2c)$
directrix	$y = -d$
asymptote	$y = \pm \dfrac{a}{b} x - c$
Rectangular Eq.	$\dfrac{(y+c)^2}{a^2} - \dfrac{x^2}{b^2} = 1$