The Doppler Effect

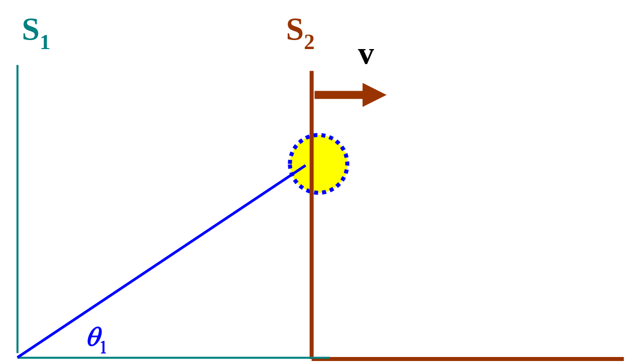
What is Doppler Effect?

- It is a phenomenon named after Austrian physicist **Christian Doppler** who proposed it in 1842 .
- It is the change in the ***measured frequency*** of a source, due to the motion of the source (and/or) the observer.
- ***The Doppler effect*** for sound waves travel in a medium, depends on **two** velocities: **the source velocity** and **the observer velocity** with respect to that medium.
- However, light and other electromagnetic waves require no medium. Therefore ***the Doppler effect*** for electro-magnetic waves depends on only **one** velocity: **the relative velocity** between the source and the observer.

Suppose that there is a source of frequency υ_o (and corresponding period T_o) at rest on the Y axis in S_2 .

And the source is moving with constant velocity v along the x-axis .

Making an angle θ_1 between the source position vector & + x-axis.



If we are at rest at the origin of S_1, we would measure the frequency of the source to be

$$\nu = \frac{\nu_0}{\gamma\left[1+\beta\cos\theta_1\right]} = \nu_0 \frac{\sqrt{1-v^2/c^2}}{1+\left(v/c\right)\cos\theta_1} \qquad \mathbf{1}$$

In this relation, υ_o is measured in a coordinate system at rest with respect to the source (zero relative speed, thus the subscript zero). The observers measure the frequency υ, the relative speed v, and the angle θ_1 between the source position vector r_1 & + x-axis .

At S_2 ;

$x_2 = 0$ & $x_2` = 0$

$T_0 = t_2` - t_2$

At S_1 ;

$x_1 = \gamma v t_2$ & $x_1` = \gamma v t_2`$

$T = (t_1` - t_1) + (r_1` - r_1)/c$

But ;

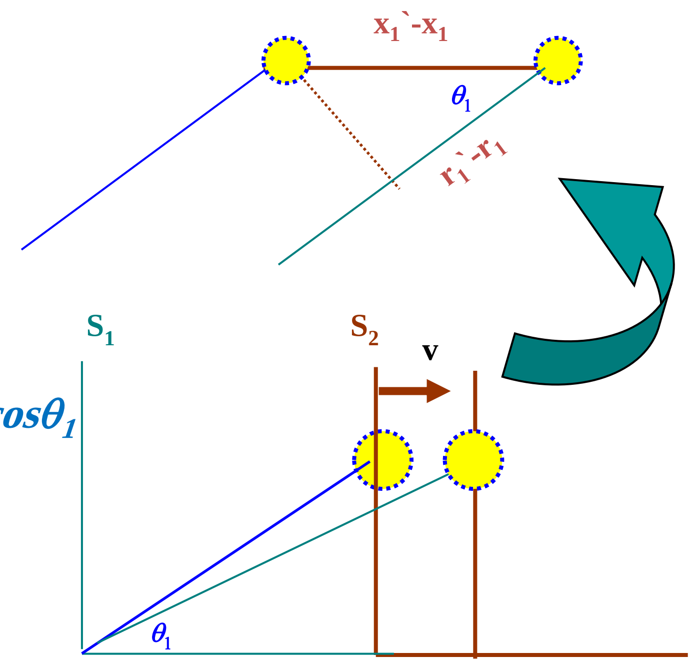
$r_1` - r_1 = (x_1` - x_1)\cos\theta_1 = \gamma v T_0 \cos\theta_1$

Hence;

$T = \gamma(t_2` - t_2) + (\gamma v T_0 \cos\theta_1)/c$

$= \gamma T_0 + (\gamma v T_0 \cos\theta_1)/c$

$= \gamma T_0[1+\beta\cos\theta_1]$



Some Special Cases

Equation 1 represents a general case, from which one may derive expressions for some other ***special cases***.

Assume that you are at the origin of S_1, what the measured frequency of the source would be:

1- ***If the source of light were moving directly away from you:***

In this case $\theta_1 = 0$, hence;

$$\boxed{\nu = \nu_0 \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}}$$

2- ***If the source of light were moving directly toward you:***

In this case $\theta_1 = 180^o$, hence;

$$\boxed{\nu = \nu_0 \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}}$$

The numerator now is larger than the denominator, giving the expected *increase in frequency* when the source is moving toward you.

Some Special Cases

3- ***If the source were moving perpendicular to a line from you:***

In this case (which known as ***the transverse Doppler effect***) $\theta_1 = 90^o$, At this angle there is no relative motion toward or away from you, so the **classical Doppler effect** for mechanical waves would give $\upsilon = \upsilon_o$.

However, the same **is not true** for electromagnetic waves. With $\theta_1 = 90^o$, Eq. (1) becomes;

$$\boxed{\nu = \nu_0\sqrt{1-v^2/c^2} = \nu_0/\gamma}$$

The frequency **decreases** for this relativistic transverse Doppler effect.

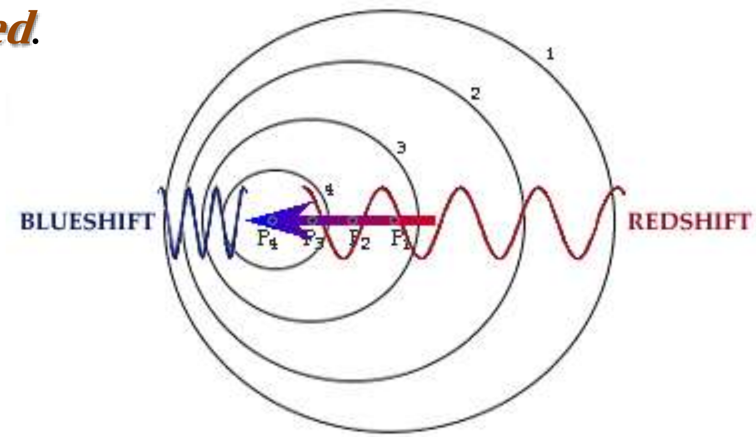
So…

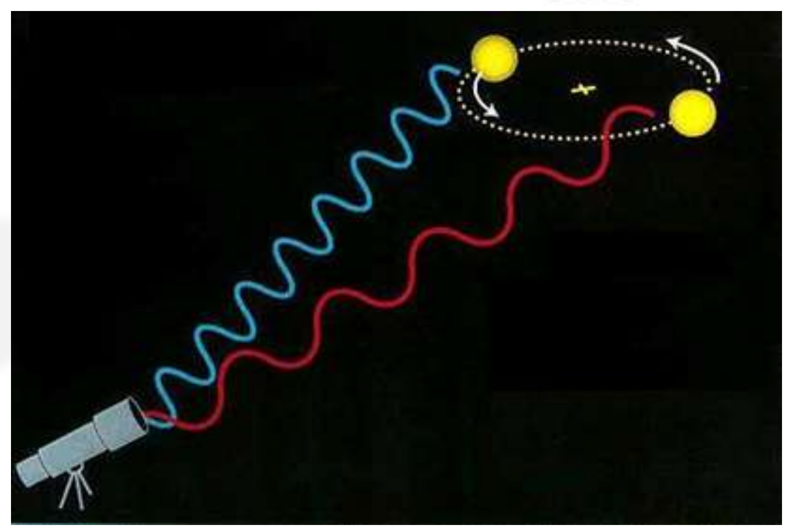
Doppler shifts in the frequencies of electromagnetic waves occur *not only for* **relative motion** toward or away from an observer, *but also for* **transverse motion**.

Astronomical Doppler effect

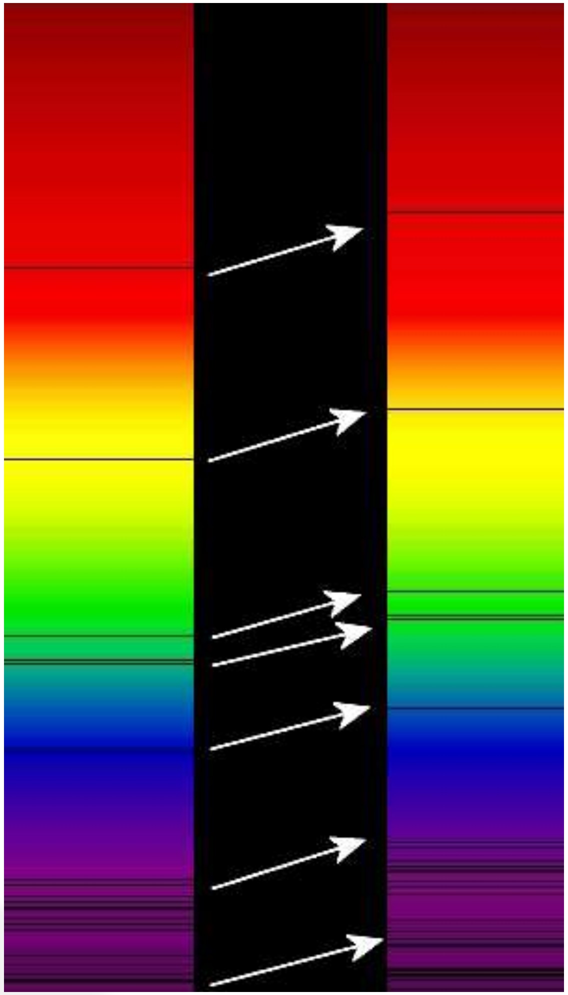
According to the Doppler Effect, the radiation emitted by an object moving toward an observer is squeezed; its frequency appears to increase and is therefore said to be ***blueshifted***. In contrast, the radiation emitted by an object moving away is stretched or ***redshifted***.

Blueshifts and redshifts exhibited by stars, galaxies and gas clouds indicate their motions with respect to the observer.



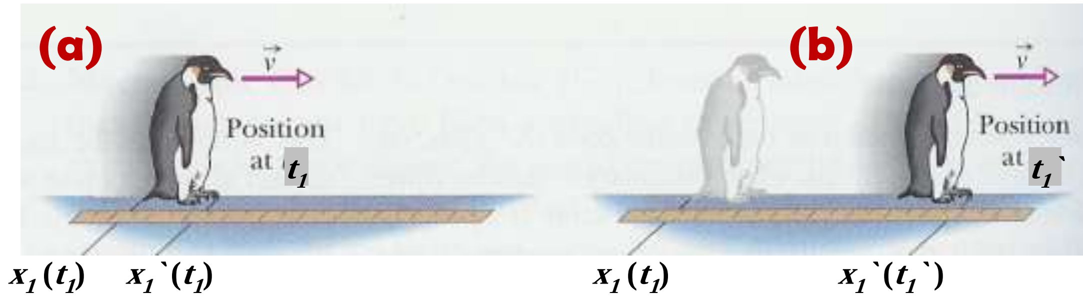


Astronomical Doppler effect



Redshift of spectral lines in the optical spectrum of a supercluster of distant galaxies (right), as compared to that of the Sun (left).

Length Contraction



If you want to measure the length of a penguin while it is moving, you must mark the positions of its front and back ***simultaneously*** (in your reference frame), as in (a), rather than ***at different times***, as in (b).

Applying the Lorentz transformations to our two distances, we obtain;

$$x_2 = \gamma\,(x_1 - vt_1) \qquad \text{and} \qquad x_2` = \gamma\,(x_1` - vt_1)$$

Subtracting, we obtain;

$$(x_2` - x_2) = \gamma\,(x_1` - x_1)$$

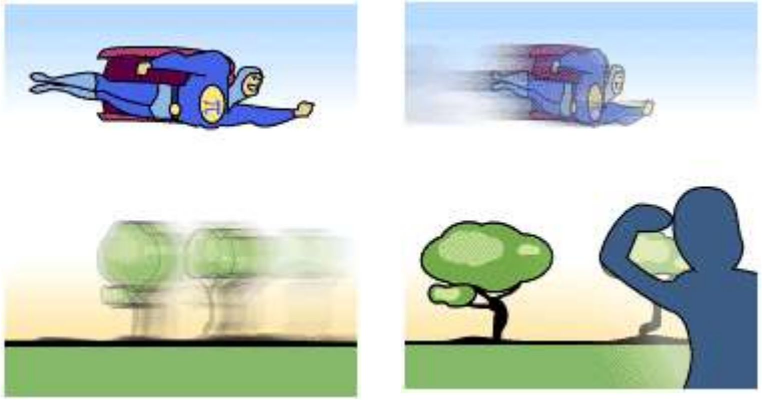
Note that $(x_2` - x_2)$ is the length as measured in S_2. Since the object is at rest with respect to S_2, let's call this length L_o. This gives us

$$\boxed{L = L_0\sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma}}$$

Length Contraction

➢Because the Lorentz factor **γ** is always **greater than unity**, then L **is always less than** L_o.
I.e. ***The relative motion causes a length contraction.***

➢Because **γ** increases with speed v, the length contraction also increases with v.



Because $y_2 = y_1$ and $z_2 = z_1$;
Length contraction occurs only along the direction of the relative motion.

Time Dilation

Suppose we travel inside a spaceship and watch a light clock. We will see the path of the light in simple **up-and-down motion**.

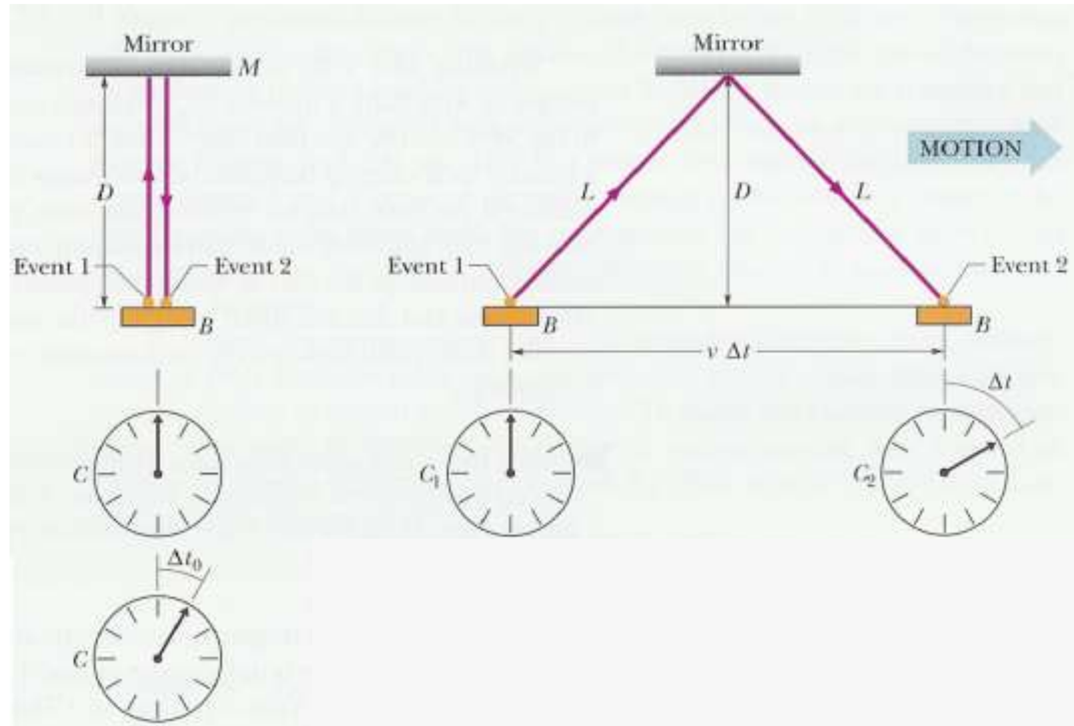
If, instead, we stand at some relative rest position and observe the spaceship passing us by $0.5c$. Because the light flash keeps up with the horizontally moving light clock, we will see the flash following **a diagonal path**.

I.e. according to us the flash travels a ***longer distance*** than it does in the reference frame of an observer riding with the ship.

Since the *speed of light is the same* in all reference frames (Einstein's second postulate), the flash must travel for a ***longer time*** between the mirrors in our frame than in the reference frame of an observer on board.

This stretching out of time is called ***time dilation***.

Time Dilation



$$\Delta t_0 = \frac{2D}{c}$$

$$\Delta t = \frac{2L}{c}$$

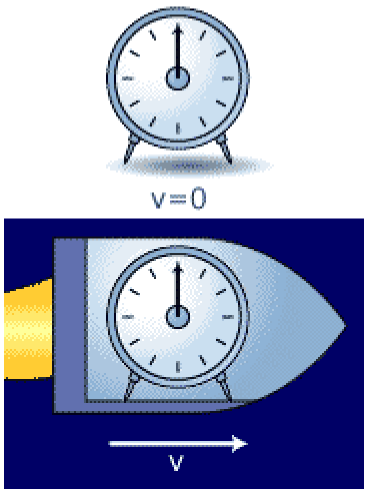
$$L = \sqrt{\left(\tfrac{1}{2} v\Delta t\right)^2 + D^2}$$

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} = \gamma \Delta t_0}$$

Time Dilation

Some numerical values:

- Assume that $v = 0.5c$,then $\gamma = 1.15$, so $T = 1.15\ T_0$ .This means that if we viewed a clock on a spaceship traveling at half the speed of light, we would see the second hand take 1.15 minutes to make a revolution, whereas if the spaceship were at rest, we would see it take 1 minute.



- If the spaceship passes us at 87% the speed of light, g = 2; and $T = 2\ T_0$. We would measure time events on the spaceship taking **twice the usual intervals. i.e.** the hands of a clock on the ship would turn only half as fast as those on our own clock.

- If it were possible to make a **clock fly** by us at **the speed of light**, **the clock would not appear to be running at all**. We would measure the interval between ticks to be infinite.

- ***Time dilation*** has been confirmed in the laboratory countless times with particle accelerators. The lifetimes of fast-moving radioactive particles increase as the speed goes up, and the amount of increase is just what Einstein's equation predicts.

Lorentz Velocity Transformations

Suppose we wish to use the **Lorentz transformation** equations to compare the ***velocities*** that two observers in different inertial reference frames S_1 and S_2 would measure for the same moving particle.

Let S_2 moves with velocity v relative to S_1, and there is a particle in S_1 moving with constant velocity v_{1x} parallel to the x-axis. If the particle position is (x_1, y_1, z_1) at the instant t_1 , then;

Lorentz coordinate transformations		***Differentials of those equations***
$x_2 = \gamma(x_1 - vt_1)$	$\Rightarrow$	$dx_2 = \gamma(dx_1 - vdt_1)$
$y_2 = y_1 , \quad z_2 = z_1$	$\Rightarrow$	$dy_2 = dy_1 , \quad dz_2 = dz_1$
$t_2 = \gamma\left(t_1 - \frac{v}{c^2}x_1\right)$	$\Rightarrow$	$dt_2 = \gamma\left(dt_1 - \frac{v}{c^2}dx_1\right)$

Dividing dx_2 , dy_2 and dz_2 by dt_2 to obtain the velocity components gives;

$$dx_2 = \gamma(dx_1 - v\,dt_1)$$

$$dy_2 = dy_1\,, \quad dz_2 = dz_1$$

$$dt_2 = \gamma\left(dt_1 - \frac{v}{c^2}dx_1\right)$$

$$v_{2x} = \frac{\qquad}{\qquad} = \frac{(dx_1 - v\,dt_1)/dt_1}{(dt_1 - v/c^2 dx_1)/dt_1}$$

Hence;

$$\boxed{v_{2x} = \frac{v_{1x} - v}{1 - v_{1x}.v/c^2}}$$

Similarly we can find that;

$$v_{2y} = \frac{v_{1y}}{\gamma[1 - v_{1x}.v/c^2]}$$

$$v_{2z} = \frac{v_{1z}}{\gamma[1 - v_{1x}.v/c^2]}$$

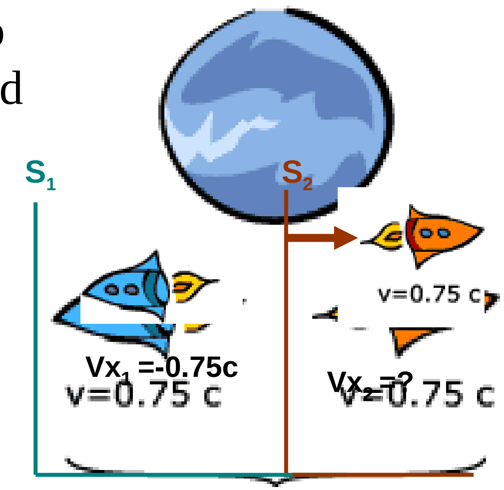
Using ***Lorntz velocity transformations*** **No** speed could be **greater than** ***c***.

Example (1)

Imagine that you are standing between two space-ships moving away from you. One space-ship moves to the left with a speed of 0.75 c (relative to you) and the other one moves to the right also with a speed of 0.75 c (relative to you).
At what speed will each space-ship see the other moving away?

In classical Newtonian mechanics, two different velocities ***v1*** and ***v2*** are added together by the formula

$$\boxed{v_{2x} = v_{1x} - v}$$



So, can the speed we see be ;

-0.75 *c* - 0.75 *c* = -1.5 *c* ***??***

No,
the speed cannot, of course, be faster than the speed of light c.

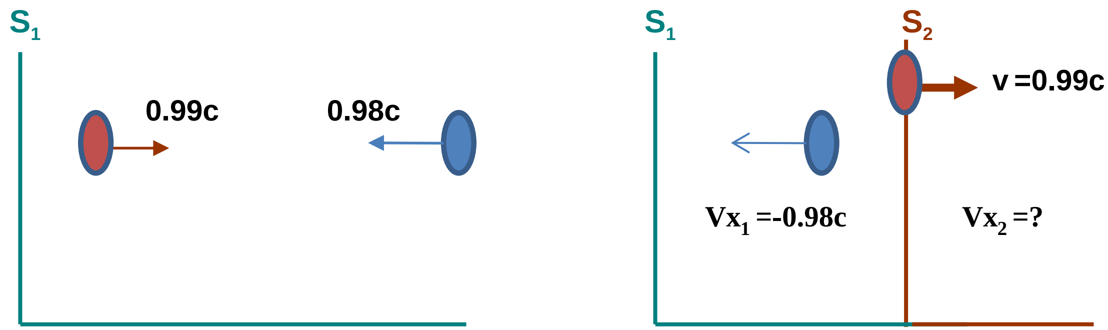
However, in special relativity, the velocities are added together as

$$\boxed{v_{2x} = \frac{v_{1x} - v}{1 - v_{1x}.v/c^2}} \qquad = \frac{-0.75c - 0.75c}{1 - (-0.75 \times 0.75)c^2/c^2}$$

Hence, their ***relative speed*** will be **0.96 *c***.

Example (2)

Two subatomic particles moving at 0.99c and 0.98c in a laboratory collide head-on. What was their relative velocity?



*Using **Lorntz velocity transformations***

$$\boxed{v_{2x} = \frac{v_{1x} - v}{1 - v_{1x}.v / c^2}} \qquad = \frac{-0.98c - 0.99c}{1 - (-0.98 \times 0.99)c^2 / c^2}$$

Hence, their ***relative speed*** will be **0.9999 *c*** .

Even if the head-on collision had been between light beams,
Their relative speed will be;

$$= \left| \frac{-c - c}{1 - (-1)c^2 / c^2} \right| = \left| \frac{-2c}{2} \right| = c$$

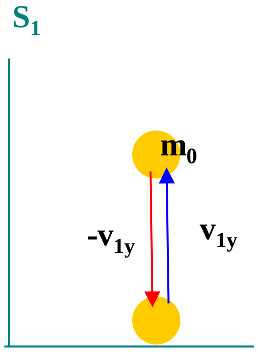
In classical physics, c + c = 2c,
but in special relativity, "c + c = c".

Relativistic Mass & Momentum

In classical physics when two bodies collide together, the total ***mass, energy** and **momentum*** before and after the collision are equal.

Let us apply conservation laws to viewers from two different inertial reference frames S_1 and S_2.

If someone in S_1 throws a ball with mass m_0 to make an elastic collision with the ground, then the *conservation law of momentum* in his frame requires that



$$\Delta P_{1x} = 0$$

$$\Delta P_{1y} = m_0 v_{1yf} - (-m_0 v_{1yi}) = 2 m_0 v_{1y}$$

For an observer in S_2 the ***conservation law of momentum*** *will* requires that:

$$\Delta P_{2x} = m v_{2xf} - (m v_{2xi}) = 0$$

$$\Delta P_{2y} = m v_{2yf} - (-m v_{2yi}) = 2 m v_{2y}$$

Using Lorentz coordinate transformations

$$v_{2y} = \frac{v_{1y}}{\gamma[1 - v_{1x}.v / c^2]}$$

Recall:

v_{1x}=0

$$\Delta P_{2y} = 2mv_{2y} = 2mv_{1y}\sqrt{1 - v^2 / c^2}$$

But **the principle of relativity** demands that *the laws of physics are the same in all inertial reference frames*. Hence;

$$\Delta P_{1y} = \Delta P_{2y}$$

$$2m_0 v_{1y} = 2mv_{1y}\sqrt{1 - v^2 / c^2}$$

Or;

$$\boxed{m = m_0 / \sqrt{1 - v^2 / c^2} = \gamma\, m_0}$$

This is the ***relativistic mass transformation***.

Since $\boldsymbol{p} = m\boldsymbol{v}$,
the ***relativistic linear momentum*** can be written as;

$$\boxed{p = m_0 v / \sqrt{1 - v^2 / c^2} = \gamma\, m_0 v}$$

Notes:

1- If $v \ll c$ then m is effectively equal to the rest mass m_0 (***the classical limit***). When we refer to the **mass of an electron** as 9.1×10^{-31} kg we mean its ***rest mass***.

2- As the velocity of a body increases the relativistic mass becomes significantly greater than the rest mass. The relativistic mass of a body traveling at about 0.99c is roughly **seven** times its rest mass.

3- As **v** $\rightarrow$ **c** , *the mass* $\rightarrow$ *infinity*.

This huge increase in **inertia** makes it **impossible to accelerate bodies of non-zero rest mass** up to the velocity of light.

Relativistic Force

We can use Newton's second law to define ***force*** by the relation F = dp/dt. So we have;

$$\boldsymbol{F} = \frac{d}{dt} m v = m\boldsymbol{a} + \boldsymbol{v}\frac{dm}{dt}$$

If the force is **perpendicular** to the velocity, the force can't do any work on the particle, so the speed won't change. This happens in ***uniform circular motion***. The direction of *v* changes, but the magnitude of *v* doesn't.

Therefore *m* doesn't change and **dm/dt=0**. Substituting for $m = \gamma m_0$, we have

$$\boxed{F_\perp = \gamma m_0 a}$$

However, if the force is **parallel** to the velocity, the ***particle's speed*** and ***mass*** will change. Then;

$$\frac{dm}{dt} = \frac{d}{dt}\frac{m_0}{\left(1 - v^2/c^2\right)^{1/2}} = \frac{m_0}{\left(1 - v^2/c^2\right)^{3/2}}\frac{v}{c^2}\frac{dv}{dt}$$

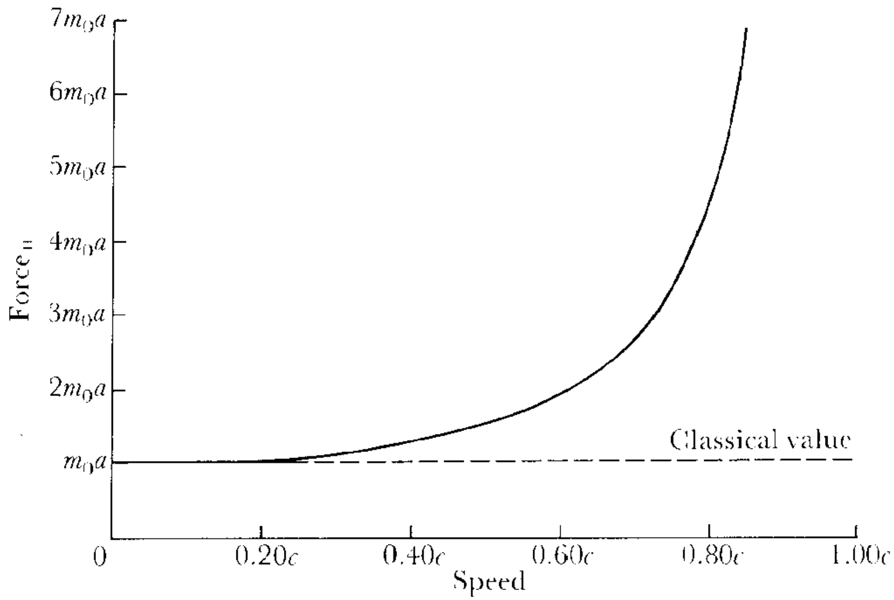
and;

$$\boxed{F_\parallel = \gamma^3 m_0 a}$$

Notes:

1- $F_{\parallel}$ is much larger than $F_{\perp}$

2- $F_{\parallel}$ increases rapidly as v gets close to c.



To approach c we need an infinite force to accelerate an infinite mass.

Relativistic Energy

What is the expression of *KE* in relativistic physics?
Let's start an object from rest with a net external force *F* in the (+ve) x direction. Then the work done by *F* will be stored in the form of *kinetic energy*.

That is,

$$KE = \int F_{\parallel} dx = \int_0^v \underbrace{m_0\left(1 - v^2/c^2\right)^{-3/2}}_{m_0\gamma^3} \underbrace{v\frac{dv}{dx}}_{a} dx$$

which integrates to

$$KE = m_0 c^2 \left(1 - v^2/c^2\right)^{-1/2} - m_0 c^2 = \gamma m_0 c^2 - m_0 c^2$$

or most simply;

$$\boxed{KE = mc^2 - m_0 c^2 = \Delta mc^2} \qquad \textbf{(1)}$$

where $\Delta m = m - m_0$ is *the relativistic mass increase*.

Suppose a body with a rest energy of E_0 undergoes a work that increases its *KE*, then its total energy will be $E = E_0 + KE$, or;

$$\boxed{KE = E - E_0} \qquad \textbf{(2)}$$

Equivalence of Mass and Energy

Comparing (1) & (2), we can say that:

$$\boxed{E = mc^2} \quad \text{and} \quad \boxed{E_0 = m_0 c^2} \qquad \textbf{(3)}$$

This famous equation states that;

Energy and mass are just two equivalent ways of describing the same thing.

- This law unified two things which appeared to be completely unrelated in the old model (classical physics).

- The decrease of mass in the sun by the process of thermonuclear fusion bathes the solar system with radiant energy. **There is sufficient hydrogen fuel for fusion to last another 5 billion years !!**

- When we strike a match, phosphorus atoms in the match head rearrange themselves and combine with oxygen in the air to form new molecules. The resulting molecules have very slightly less mass than the separate phosphorus and oxygen molecules.

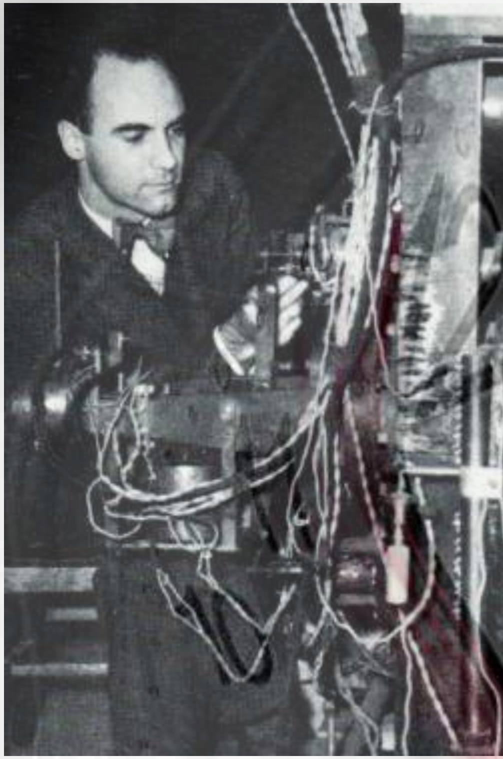
❐ The equation $E = mc^2$ **is not restricted to chemical and nuclear reactions. It applies to ALL** forms of energy and mass. **For example**;

- **The filament of a light bulb energized with electricity has more mass than when it is turned off.**
- **A hot cup of tea has more mass than the same cup of tea when cold.**
- **A wound-up spring clock has more mass than the same clock when unwound.**

But these examples involve incredibly small changes in mass-too small to be measured.

❐ Nuclear energy involves larger changes in energy, because the rest mass of nuclei converted into kinetic energy.

1 gram of mass ~ energy released in an atomic bomb

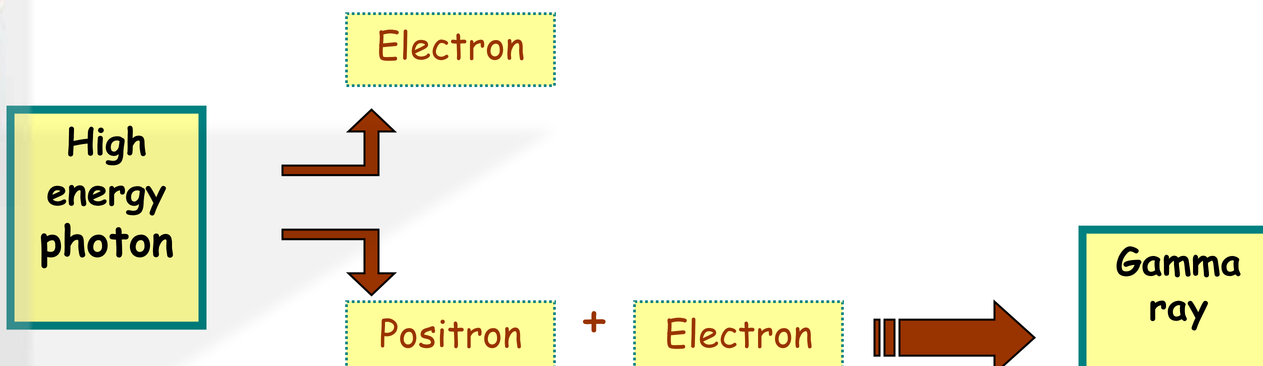


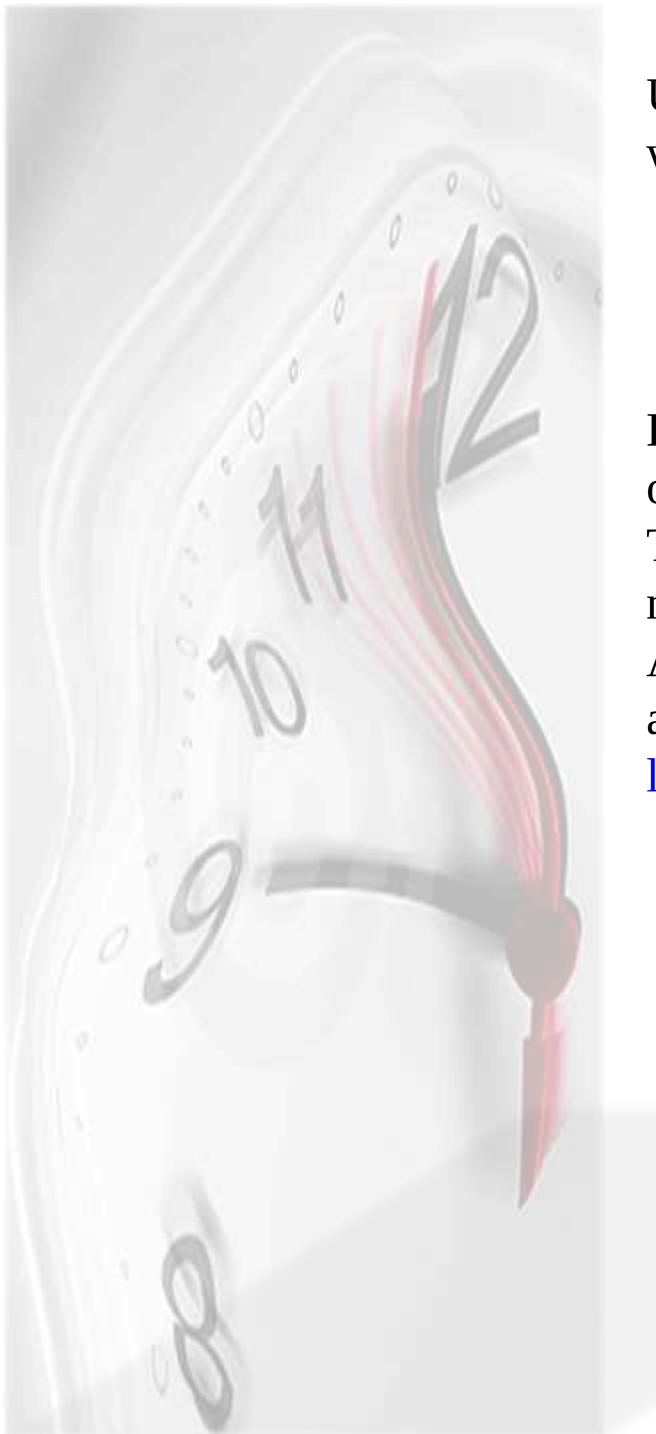
❐ The first evidence for the conversion of radiant energy to mass was provided in **1932** by the American physicist **Carl Anderson, who discovered the *positron*** by the track it left in a cloud chamber.

❐ ***The positron*** *is the antiparticle of the electron, equal in mass and spin to the electron but opposite in charge*.

❐ When a photon comes close to an atomic nucleus, it can create an electron and a positron together as a pair, thus creating mass. The created particles fly apart.

❐ The positron is not part of normal matter because it lives such a short time in the presence of matter. As soon as it encounters an electron, the pair is annihilated, sending out two gamma rays in the process. Then mass is converted back to radiant energy.





Using the relation of *the relativistic mass* along with $E = mc^2$, we obtain

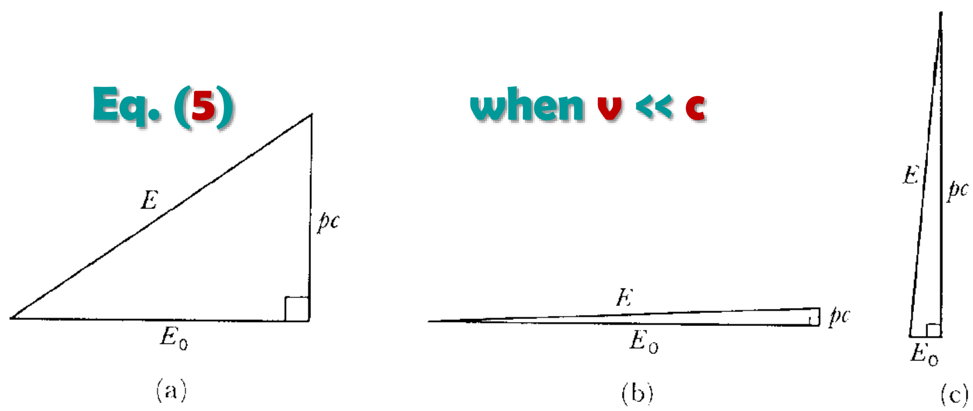
$$\boxed{E = \gamma m_0 c^2 = \gamma E_0} \quad \textbf{(4)}$$

Equation (4) as other *relativistic* expressions shows that for objects with a nonzero rest mass, ***c*** **is the upper limiting speed**. This doesn't forbid the existence of particles that have zero rest mass and which can only move at $v = c$.

Again starting with $m = m_0/(1 - v^2/c^2)^{1/2}$, if we square both sides and rearrange terms recognizing mv as the magnitude of the linear momentum p, we get;

$$\boxed{E^2 = p^2c^2 + E_0^2} \quad \textbf{(5)}$$

Eq. (5) **when v << c** **when v → c**



The Unit Conversions

Since it's easier to determine the work done and thus KE in **electron volts** for charged atomic and subatomic particles, **eV-** based units are most often used as following;

1- When a particle is described as a **1-MeV** particle ⇨ This means that the ***kinetic energy*** of this particle is **1-MeV**.

$$E = mc^2$$

2- According to **(3)** we can specify the **mass units** as ⇨ **eV/c^2, KeV/c^2, MeV/c^2, ect**

$$E^2 = p^2c^2 + E_0^2$$

3- According to **(5)** we can specify the **momentum units** as ⇨ **eV/c , KeV/c, MeV/c, ect**