

Q1)- Mark true or false and justify your answer; (8marks)

1- The functions $f_1(x) = \cos 2x$, $f_2(x) = 1$ and $f_3(x) = \cos^2 x$ are linearly dependent . []

2- The functions $f_1(x) = e^{3x}$ and $f_2(x) = e^{-2x}$ are fundamental set of solutions of;
 $y'' - 9y = 0$,

3- The differential equation $2y'' - 5y' - 3y = 0$ has solution;
 $y = c_1 e^{3x} + c_2 e^{-\frac{x}{2}}$
 []

4-The function $f(x) = 1 + 7e^{2x}$ is annihilated by the operator $D(D - 7)$ []

5- The differential equation $y'' - y' - 4y = \sin^2 x$ can be solved by the undetermined coefficients method . []

6 - If the Auxiliary equation of the DE $ay'' + by' + cy = g(x)$ has roots $\alpha \pm \beta i$ then the general solution is $y = c_1 e^{\alpha x} \cos \beta x + c_2 e^{\alpha x} \sin \beta x$ []

7-Two Functions y_1, y_2 are linearly dependent if and only if $w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$ []

8- The linear independent functions ;
 $\sin \sqrt{3} x$ or $\cos \sqrt{3} x$ are annihilated by $(D^2 + 3)$ []

9- The following problem $y'' + 9y' + 3y = 0$, $y(3) = 2, y(2) = 4$ is called the Boundary Value problem. []

10- If $3 \pm \sqrt{5} i$ are the roots of the auxiliary equation corresponding to $ay'' + by' + cy = 0$, then the general solution is;
 $y = c_1 e^{\sqrt{5}x} \cos 3x + c_2 e^{\sqrt{5}x} \sin 3x$ []

11-The DE $x^2 y''' + xy' + 4y = x$ Is Cauchy-Euler equation []

12- The linearly independent functions that are annihilated by the operator $D^3 - 10D^2 + 25$ are x, e^{5x}, xe^{5x} []

Q2) Find the general solution of the following;

2) $x^2y'' - 4xy' = x^4$ (5marks)

3) $y'' - 4y' + 5y = 35e^{-4x}$ $y(0) = 3, y'(0) = 1$ (By undetermined coefficient method)
(5marks)

Q3) a) Determine the form of a particular solution of ;

$$y''' - 2y'' + y' = 3x^2 - \sin x + 7xe^x \quad (2 \text{ marks})$$

b) Solve the system of differential equations; (4 mark)

$$\begin{aligned} \frac{dx}{dt} &= -5x - y & x(1) &= 0, y(1) = 1 \\ \frac{dy}{dt} &= 4x - y \end{aligned}$$

